Adhesive contact and kinetics of adherence of a rigid conical punch on an elastic half-space (natural rubber)

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Abstract

The equilibrium contact of a rigid conical punch, applied against the flat and smooth surface of a soft elastomer sample (unfilled natural rubber), is studied with help of fracture mechanics concepts which can easily be introduced in this class of problems by using Sneddon's solution (1965) of Boussinesq's problem extended to all axisymmetric adhesive punches with a convex profile as a cone. The kinetics of adherence is measured when an imposed tensile force is applied in order to disturb the size of the contact area. Variations of the strain energy release rate $G$ and of the associated dissipation function $\Phi = (G - w)/w$, where $w$ is the Dupré energy of adhesion, are studied as a function of the crack propagation speed $V$ at the interface between a cone made of PMMA and the rubber sample (the limit of the contact is considered as a crack tip). As expected, a master curve $\Phi(V)$ is found, confirming the variation of $\Phi$ as the 0.55 power function of $V$, as recently established by Barquins et al. in adherence of flat ended spheres in pull-off/push-on tests, adherence and rolling of cylinders experiments and rebound of balls tests, with the same elastic rubber-like material.

1 Introduction

An earlier study [1], drawing upon Sneddon's 1965 generalisation of Hertz's problem to all axisymmetric punches [2], investigated the theoretical conditions of the equilibrium adhesive contact between a rigid cone and the smooth, flat surface of a elastic half-space as a soft natural rubber sample. To take into
account the molecular attraction forces, of van der Waals type in the case of elastomers, as has been previously demonstrated [3], the constant of integration \( \chi(1) \), corresponding to a rigid vertical displacement, is assumed non-zero. In this particular case, we give here the relation between load and penetration, and the adherence force for a conical punch. Experiments are occurred using a vulcanised rubber half-space (with modulus of elasticity \( E = 0.89 \) MPa and Poisson ratio's \( v \equiv 0.5 \)) and a rigid transparent cone optically smooth, made of plexiglass (PMMA), with a half-angle \( \alpha = \pi/2 - \beta = 85 \) degrees (Fig. 1). When the applied load \( P \) is changed, the equilibrium, which depends on van der Waals forces, is disturbed and such a situation will lead either to a separation of the two bodies or to a new equilibrium, depending on the intensity of the new applied load.

Since 1996, Barquins et al. [4-8] have conducted various experiments into adherence kinetics and the adhesive behaviour of cylindrical rollers. Moreover they have performed rebound tests of rigid balls on smooth natural rubber half-spaces. This research has shown that the dissipation function \( \Phi \) for viscoelastic loss located at the edge of the contact zone, which can be inferred from the strain energy release rate \( [9] \), \( \Phi = (G - w)/w \), where \( w \) denotes the Dupré adhesion energy, varies over a wide range of propagation speeds as a power function of speed \( \Phi \propto V^{0.55} \). This paper's main goal is to describe the equilibrium condition of the adhesive contact of a rigid conical punch with an elastomer surface, to study the separation kinematics with an imposed load and to show that the dissipation function \( \Phi \) remains a power function (0.55) of the variation with time of contact area's radius (crack propagation speed) for the specific type of rubber utilised (unfilled natural rubber).

2 Adhesive contact of a cone and kinetics of adherence

Sneddon [2] has derived a solution of the axisymmetric Boussinesq problem from which he deduced simple formulae for the depth of penetration \( \delta \) of the tip of a punch of convex arbitrary profile, for the total load \( P \) which must be applied to the punch to achieve this penetration, for distribution of pressure \( \sigma_x(r, 0) \) under the punch at the distance \( r \) of the symmetry axis inside the contact area, and for the displacement \( u_x(r, 0) \) of the surface at the distance \( r \) of the contact centre outside the contact area:

\[
\delta = \int_0^1 \frac{f'(x)}{\sqrt{1 - x^2}} \, dx + \frac{\pi}{2} \chi(1)
\]

\[
P = \frac{\pi \cdot a \cdot E}{1 - \nu^2} \int_0^1 \chi(t) \, dt = \frac{2E \cdot a}{1 - \nu^2} \left[ \delta - \int_0^1 \frac{x \cdot f(x)}{\sqrt{1 - x^2}} \, dx \right]
\]
Figure 1: Schematic views of the profile between a rigid conical punch and the flat and smooth surface of an elastic half-space, (a) as soon as the mass \( m_i \) (corresponding force \( P_i = m_i g \)) is applied, (b) at equilibrium under the constant mass \( m_i \) and (c) at the beginning of the detachment when a constant mass \( m_a \) (corresponding load \( P_a = m_a g \)), inferior to \( m_i \), is applied.
In these formulae, $a$ is the radius of the contact area, $z = f(x) = f(r/a)$ describes the profile of the punch (with $f(0) = 0$), and $\chi(t)$ is defined by

$$\chi(t) = \frac{2}{\pi} \left( \delta - t \int_0^t \frac{f'(x)}{\sqrt{t^2 - x^2}} \, dx \right)$$

Moreover, in the case of a conical punch (Fig. 1) the shape function is given by

$$f(x) = a \cdot x \cdot \tan \beta$$

For punches with continuous and convex profile, Sneddon [2] lets $\chi(1) = 0$ in order to have a finite normal stress at the edge of the contact area and he uses this criterion to determine the penetration $\delta$, eqn. (5). For a conical punch, the classical result [10]

$$\delta = \frac{\pi}{2} \cdot a \cdot \tan \beta$$

is easily found.

One of aims of this paper is to show that the hypothesis $\chi(1) = 0$ is not imperative, and that $\chi(1) \neq 0$ allows one, as previously demonstrated [3], to describe the adhesive contacts of a axisymmetrical punches, as a cone in the present case, taking into account the Dupré energy of adhesion or the thermodynamic work of adhesion, $w = \gamma_1 + \gamma_2 - \gamma_{12}$, of the facing solids (the $\gamma_i$ and $\gamma_{ij}$ are the surface and interfacial energies). Letting

$$K_I = -\frac{E}{2(1 - \nu^2)} \cdot \frac{\pi}{a} \cdot \chi(1)$$

the stress $\sigma_z$ and the discontinuity of displacement $[u_z]$ at a distance $\rho$ of the edge of the contact area can be written:

$$\sigma_z(a - \rho, 0) \propto \frac{K_I}{\sqrt{2\pi} \cdot \rho}$$

$$[u_z(a + \rho)] \propto \frac{4(1 - \nu^2)}{E} \cdot \frac{\rho}{\sqrt{2\pi} \cdot K_I}$$
These formulae are found in fracture mechanics in mode I (opening mode) and plane deformation, and \( K_I \) is the stress intensity factor. They appear because the edge of the contact area can be considered as a crack tip that recedes or advances according as the load \( P \) increases or decreases. In this case the strain energy release rate \( G \) is given by

\[
G = \frac{1}{2} \cdot \frac{1 - \nu^2}{E} \cdot K_I^2
\]  

(9)

where \( E \) is Young's modulus and \( \nu \) Poisson's ratio for the elastic half-space. The coefficient \((1/2)\) makes allowance for the fact that the spherical punch undergoes no deformation when applied against the elastic solid.

If we denote by \( P_1 \) the load which gives rise to the same radius of contact area in the absence of molecular attraction force \((\chi(1) = 0)\), and we denote by \( P \) the actual applied load \((P_1 > P)\) when these forces do come into play \((\chi(1) \neq 0)\), it can be shown [3] that:

\[
P_1 - P = -\frac{\pi \cdot E \cdot a}{1 - \nu^2} \cdot \chi(1) = \sqrt{4\pi \cdot a^3} \cdot K_I
\]

thus the strain energy release rate \( G \), given by the eqn (9), may be written as

\[
G = \frac{1 - \nu^2}{E} \cdot \left( P_1 - P \right)^2
\]  

(10)

As the shape function for a conical punch is given by the eqn (6), the function \( \chi(t) \) derived from the eqn (5) is equal to:

\[
\chi(t) = \frac{2\delta}{\pi} - a \cdot t \cdot \tan \beta
\]  

(11)

hence

\[
\delta = \frac{\pi \cdot a}{2} \cdot \tan \beta - \sqrt{2\pi \cdot a \cdot \frac{1 - \nu^2}{E}}
\]  

(12)

and from the eqn (2)

\[
P = \frac{2E \cdot a}{1 - \nu^2} \left( \delta - \frac{\pi \cdot a}{4} \cdot \tan \beta \right)
\]  

(13)

which is the state equation of the system linking the actual applied load \( P \) to the radius \( a \) of the contact area and the penetration \( \delta \) in the elastic half-space.

For an adhesive contact of a rigid conical punch, eqns (3) and (4) may be written

\[
\sigma_z(r, 0) = \frac{P_1 - P}{2\pi \cdot a^2} \cdot \frac{1}{r^2} - \frac{P_1}{\pi \cdot a^2} \cdot \cosh \left(-\frac{1}{r} \cdot \frac{a}{r} \right), \quad r < a
\]
It can be verified that the connection of the elastic half-space to the cone is tangential if \( \chi (1) = 0 \) (Fig. 1a), and vertical if \( \chi (1) \neq 0 \) (geometry of fracture mechanics, Fig. 1b).

Due to the intervention of molecular attraction forces, a finite area of contact exists at equilibrium under zero load, the value of the corresponding radius \( a ( P=0) \) is obtained from eqns (12) and (13)

\[
a ( P=0) = \frac{1 - \nu^2}{2\pi \cdot E} \cdot w \cdot \left( \frac{8}{\tan \beta} \right)^2
\]

Moreover, these molecular attraction forces allow one to applied a tensile force, at equilibrium, whose the critical value \( P_c \) is given by \( G = w \) and \( (\partial G / \partial A)_P = 0 \), \( A \) being the contact area \( A = \pi a^2 \)

\[
P_c = -\frac{54 (1 - \nu^2) \cdot w^2}{\pi \cdot E \cdot \tan^3 \beta}
\]

\( P_c \) is the quasistatic force of adherence of a cone at fixed load.

As will be seen, we have used the equilibrium measurements in order to verify the value of Young’s modulus of the test material, as declared by the elastomer’s manufacturer, whereupon a precise value for the Dupré energy of adhesion and was obtained. The method is simple, the relation linking the equilibrium contact radius \( a \) as a function of the load \( P \) (eqn (13) with \( \delta \) given by eqn (12)) can be written with eqn (10):

\[
P = \frac{\pi \cdot E \cdot a^2 \cdot \tan \beta}{2 (1 - \nu^2)} - \sqrt{\frac{8\pi \cdot a^3 \cdot E \cdot w}{1 - \nu^2}}
\]

Dividing all the terms of the eqn (17) by \( a^{3/2} \), we obtain

\[
P \cdot a^{-3/2} = \frac{\pi \cdot E \cdot \tan \beta}{2 (1 - \nu^2)} \cdot a^{1/2} - \sqrt{\frac{8\pi \cdot E \cdot w}{1 - \nu^2}}
\]
Assuming that \( v = 0.5 \) for the soft natural rubber tested, the previous relation may be written

\[
P \cdot a^{-3/2} = \frac{2\pi \cdot E \cdot \tan \beta}{3} \cdot a^{1/2} - \frac{32\pi \cdot E \cdot w}{3}
\]  
(18)

From which we conclude that \( P \cdot a^{-3/2} \) varies linearly as a function of \( a^{1/2} \) and thus the measurement of the slope allows the calculation of \( E \), finally, the determination of the ordinate at the origin furnishes, once \( E \) known, the Dupré energy of adhesion \( w \).

One can examine the adherence kinetics at any equilibrium state (\( G = w \)) by changing the applied load \( P \). As an example, when \( P \) is suddenly lowered for a given contact radius \( a \), i.e. \( P_1 \) constant, eqn (14), the strain energy release rate \( G \) (eqn (10)) rises so that \( G > w \) and the solids begin to separate. Today one is quite well aware that the difference \( (G - w) \) represents the applied force per unit length of the crack [8], this the driving force or “motor” of the crack, whose speed limit entirely depends on temperature. If we suppose that the viscoelastic losses are proportional to \( w \) and that they are localised at the crack tip [11, 12], we may write:

\[
G - w = w \cdot \Phi(a_T \cdot V)
\]  
(19)

an identity in which the right hand term corresponds to the viscous drag resulting from losses within the crack tip. The dimensionless function \( \Phi(a_T \cdot V) \) entirely depends on the crack propagation speed \( V \) and on the temperature \( T \) through the shift factor \( a_T \) of the WLF transformation [13]. At each instant, the crack propagates at such a speed \( V \) that the corresponding viscoelastic losses precisely offset the shift effect \( (G - w) \), and the speed \( V \) varies if \( G \) is modified when the contact radius \( a \) evolves over time. The function \( \Phi \) is characteristic of the type elastomers tested for propagation in Mode I, and may be directly related to the frequency dependence of the imaginary component of Young’s modulus [14].

The main interest of eqn (19) is that surface properties \( (w) \) and viscoelastic losses \( (\Phi) \) are clearly dissociated from the loading conditions and the system geometry which only appear in the rate \( G \). Predictions assume only that 1) the kinetic energy is negligible, 2) the rupture is adhesive, i.e. the propagation occurs at the interface so that experiments at equilibrium \( (V = 0) \) give the Dupré energy of adhesion \( w \) and, 3) viscous losses are limited to those areas where stress and strain rates are high, which implies that gross displacements are purely elastic and the strain energy release rate \( G \) can still be calculated by the theory of linear elasticity during crack propagation. Moreover, one should note that the existence of a unique value assumed for the Dupré energy of adhesion, which appears in eqn (19) as a negative term on the right hand side and a multiplicative term on the left hand side, is a natural result from highly cross-linked material, so that hysteresis effects between loading and unloading are not observed.

Starting from the equilibrium state under the mass \( m_i \), (the corresponding applied force is \( P_i = m_i g \), where \( g \) is the intensity of the gravity), the study of the
kinetics of adherence consists in measuring the evolution over time $t$ of the radius $a$ of the contact area when another mass $m_a < m_i$ is imposed and remains constant. For each value of $a$, the strain energy release rate $G$ can be calculated with the help of eqns (10) and (14) and linked to the crack propagation speed $V = -\frac{da}{dt}$. Moreover, the determination of $w$ by means of a simple method, which will described here, we may plot the variation of the dissipation function $\Phi$, with the help of eqn (18), as a function of the speed $V$ and verify the previous results for natural rubber [4-8, 15] in regards to the existence of a master curve representing $\Phi$ in terms of speed $V$ raised to the power 0.55.

3 Experimental results, discussion and conclusion

Equilibrium and separation kinetics experiments were carried out at constant temperature $\theta = 26$ °C and relative humidity $RH = 53\%$, between a conical punch, made of plexiglass (PMMA), with the vertical half-angle $\beta = 5^\circ$ (Fig. 1a), and the smooth, flat surface of a sheet of vulcanized natural rubber (Young’s modulus $E = 0.89$ MPa and Poisson ratio $\nu = 0.5$), with a thickness $e = 6$ mm, of sufficient width with respect to the contact areas so as to be considered as an elastic half-space.

To study the equilibrium, the conical rigid punch was pressed by a mass $m_i$ (initial force corresponding to $P_i = m_ig$) during a constant time $t_i = 10$ min, duration necessary so the molecular attraction forces would clearly manifest themselves [16], (the area of contact increases and the surface takes a fracture mechanics profile (Figs. 1a and 1b)), against the smooth surface of elastomer, by means of an incorporated microscope balance fitted with a video camera to record both the contact areas and their immediate neighbourhood through the transparent conical punch (Fig. 2) and their precise measurements afterwards.
[17]. To examine the separation kinetics, after a time interval $t_i$, the initial force $P_i$ is removed, whereas various active masses $m_a < m_i$ were applied at the rear part of the balance (Fig. 2) and the decrease in contact area radius (Fig. 1c) was recorded during an interval $t_\delta \delta 30$ s, an interval deliberately very shortened with respect to $t_i$ so as to avoid the dwell time effects [16, 18].

Fig. 3 shows the equilibrium contact radii $a$ between the cone made of transparent plexiglass, with $\beta = 5^\circ$, and the elastomer surface (NR) as a function of the initial mass $m_i$. Due to the intervention of molecular attraction forces, of van der Waals type, a finite area of contact exists at zero applied load. Moreover, as well known for spherical indenters, flat punches, flat ended spheres [7, 19, 20, 21] equilibrium contact areas exist under negative loading, but in the case of a conical punch, these tensile loads are very small. As example, the quasistatic force of adherence of the rigid cone at fixed load, given by the eqn (16), is equal to $P_c = -40 \mu$N, with $w = 43$ mJ.m$^{-2}$.

We have used the equilibrium measurements (Fig. 3) and eqn (18) to verify the value of Young's modulus for the test material and to obtain a precise value for the Dupré adhesion energy. Fig. 4 shows the variation of $P.a^{3/2}$ as a function of the square root of the contact radius $a$. The linear regression gives the slope $s = (2/3)\pi E \tan \beta = 164470$ Pa, of which it can be deduced the value of the Young modulus $E = 897200$ Pa, which corresponds very good to the value declared by the elastomer's manufacturer ($E = 0.89$ MPa). The value at the y-axis at the origin $-\sqrt{(32/3)\pi E w} = -1135$ N.m$^{-3/2}$ furnishes, once $E$ known, the value of the Dupré energy of adhesion $w = 43$ mJ.m$^{-2}$. This value is not in line with the values previously obtained [6, 7, 15] probably due to the increase in the relative humidity, which is much higher in the present experiments.

Experiments on the separation of the conical punch were conducted starting with the same initial mass $m_i = 5$ gm, maintained for $t_i = 10$ min, which corresponds to the force $P_i = 49$ mN, when applying different active masses $m_a = -2$, -1, -0.5, 0, 1, 2, 3, and 4 gm. Fig. 5 illustrates how the intensity of the active mass $m_a$ influences the evolution of the separation of the conical punch from the natural rubber surface. When $m_a \geq 0$ gm, the contact area tends toward a new equilibrium with a decrease in the crack propagation speed $V = -da/dt$. When $m_a < 0$ gm, the evolution of the system leads to the rupture of the contact area. As expected, it is observed that the greater the tensile force associated with the load $m_a$, the earlier contact rupture.

Curves $a(t)$, in Fig. 5, corresponding to active masses -0.5 and -1 gm, clearly present inflexions, that is to say the crack first decelerates, then accelerates until contact is completely broken. At an inflexion point, the crack propagation speed $V$ is minimum and it the same thing for the strain energy release rate $G$, so the radius of contact corresponding to an inflexion point is obtained if the derivative $(\partial G / \partial A)_P = 0$, at given load $P$, is equal to zero. From the eqn (10) with $P_i$ given by eqn (14), one can write

$$\left(\frac{\partial G}{\partial A}\right)_P = \frac{1 - \nu^2}{16\pi^2 \cdot E \cdot a^5} \left(P_i - P\right)\left(P_i + 3P\right)$$

(20)
Figure 3: Diameter $2a$ of equilibrium contact areas between the conical rigid punch and the flat and smooth surface of an elastic solid (unfilled natural rubber, Young Modulus $E=0.89$ MPa, Poisson ratio $\nu=0.5$) as a function of the normal applied mass $m_i$.

Figure 4: Parameter $Pa^{3/2}$ as a function of the square root of the radius $a$ of the equilibrium contact area of the conical punch (data from fig. 3). The slope of the curve allows one to assess the Young Modulus $E$ and the $y$-axis intersection value provides the Dupré energy of adhesion $w$. 
So, the radius of contact $a_{inf}$ corresponding at an inflexion point on a curve $a(t)$ at constant negative load $P$ is given by $P_1 = -3P$, and it is equal to

$$a_{inf} = \sqrt{\frac{6P \cdot (1 - \nu^2)}{\pi \cdot E \cdot \tan \beta}}$$

(21)

When the applied active separation force corresponds to the mass $m_a = -0.5$ gm, the eqn (21) gives $a_{inf(-0.5)} = 300 \, \mu m$, as clearly showed on Fig. 5. Likewise, with $m_a = -1$ gm, the calculation provides $a_{inf(-1)} = 424 \, \mu m$, a value which is in good agreement with the corresponding curve on Fig. 5. Lastly, for $m_a = -2$ gm, $a_{inf(-2)} = 600 \, \mu m$, this value cannot be seen on Fig. 5, because it is closed at hand of the value of the initial contact radius $a_i = 642 \, \mu m$.

![Figure 5: Kinetics of unloading $a(t)$ from the initial mass $m_i = 5$ gm, applied for $t_i = 10$ min, to various active masses $m_a = -2, -1, -0.5, 0, 1, 2, 3$ and 4 gm.](image)

In order to study the adherence kinetics, as previously described [4-8], the strain energy release rate was calculated with the help of eqn (10) for each experimental point on the graph in Fig. 5, the force $P_1$ was evaluated by eqn (14), and the crack propagation speed, $V = -da/dt$, was deduced from the measured value of the local slope of the tangent of the curve $a(t)$ at the point under consideration. Fig. 6 presents the values calculated using the results from Fig. 5. One notes that whatever the imposed separation force $P_a = m_ag$, the points are found on the same graph $G(V)$, which proves that the evaluation from Fig. 4 of the Dupré adhesion energy in indeed correct.

Using the values of the graph presented on Fig. 6 and the value of $w$ ($w = 43 \, mJ/m^2$), the eqn (19) allows us to trace the variation of the viscoelastic
Figure 6: Strain energy release rate $G$ vs crack propagation speed $V = -\frac{da}{dt}$ for experimental data deduced from Fig. 4.

Figure 7: Master curve illustrating the dissipation function $\Phi$ vs crack propagation speed $V$. The variation as the 0.55 power function of $V$ is found for the natural rubber (data from Fig. 6 with $w = 43 \text{ mJ.m}^{-2}$).

dissipation function $\Phi$ for natural rubber as a function of the crack propagation speed $V$ at the interface between the rigid conical punch and the elastomer.
sample. As one would expect in light of recent results obtained concerning the same elastomer [4-8, 15] with regards to adherence of rollers, rebound behaviour of rigid balls, kinetics of adherence of flat-ended spheres, Fig. 7 clearly illustrates that the function $\Phi$ varies with excellent accuracy as a power function to 0.55 of the crack propagation speed $V$.

The results of this study incontestably demonstrate that a master curve can be drawn and that the variation $F = V^{0.55}$ is characteristic of the propagation in Mode I at the interface of our rubber material in as much as the viscoelastic losses remain confined to a small volume surrounding the crack tip at the spot where the deformation speeds are high in such a way that the strain energy release rate can be calculated by use of the theory of linear elasticity.

References