Elastic-plastic deformation near the contact surface of the circular disk under high loading

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Abstract

The forward flow generated near the contact surface in the rolling disks in high loading is numerically approached to study from the view points of the kinematics and the mechanics by using FE analysis for the first time. Beginning with the elementary study on the behavior of the contact surface of the disk, the influence of the disk rotation on the plastic flow is examined. The plastic flow in the disk is studied by the plastic deformation law and the stress distributions. Finally, the followings are found: the deviatoric plastic flow cannot be generated in the case that two disks are located just on the centerline, but that it can be generated in the case that the disk centers are deviated normal to the center line $O'O$ by $\delta$.

1 Introduction

The contact surfaces of the gear, rolling roll, ball bearing etc. can often be seen to be subjected to plastic flow. In the laboratory, the plastic flow has been observed on the condition that two circular disks in contact are rotating under the high loading, which is called as the forward flow. Merwin et al. [1] tried to explain it numerically by using the finite difference method. However, it is not based on the kinematics and the mechanics of the disk, but on the behaviors of the particles in the disk. Bhargava et al. [2] observed the forward flow by applying FE analysis and considering the material property. Both researchers discussed the forward flow only by using the residual stresses near the contact surface. However, the residual stresses cannot explain directly the plastic flow by nature; they only represent the stresses after the deformation. Therefore, the kinematic and mechanical analysis during loading must be inevitably needed to
explain the cause of the forward flow [3].

In this report, elastic-plastic analysis on the mechanical behavior of the contact surface of the disks is introduced for the first time. The elastically and plastically deformable disks in contact and remove are numerically analysed by the FE analysis as the following; the elementary analysis on the contact surface is introduced to explain the mechanical behavior of the disks under compression. Second, the deviation between the center lines of two disks under loading causes the plastic flow in one way. This phenomenon will be one of the strongest candidates of the causes of the forward flow.

2 Analytical Method

The disk models for the numerical analysis are discretised as shown in Figure 1. The rigid beams are attached on two diameters ab and cd of the half disks, the diameters of which are $\phi 40\text{mm}$. The upper disk is fixed at the point O and can be rotated around it, and the diameter cd of the lower disk is vertically translated by a given displacement $0.03\text{mm}$ upward along the centerline $oo'$. The axial loading is generated in the radius direction, from which the Hertzian pressure $1.0\text{GPa}$ is given at the contact area $a=0.22\text{mm}$ ($a$: the contact length). The friction coefficient is $\mu = 0.2$. The lower disk is elastic (Young's modulus is $200\text{GPa}$). The upper one presents elastic, plastic and linear hardening ($E=200\text{GPa}$, $\nu = 0.3$, $\sigma_y = 400\text{MPa}$, $H=5\text{GPa}$). ABAQUS code is applied by using 8 noded rectangular element with the reduced integration.

3 Numerical Results

3.1. Contact of two disks under the axial loading

Figure 2 shows the deformation of the contact surfaces between the upper and the lower disks before and under loading, the marks on which show the nodal points and are symmetric with respect to the centerline $oo'$. During loading the nodal points are displaced to the direction of the center line because the hydro-static pressure $p$ is generated and the material is shrunk elastically. The upper disk is dented by the lower one due to the soft material: it is yielded at the subsurface and the deformation is larger than that of the elastic lower disk. Accordingly the nodal points of the upper disk near the center line are displaced to the origin more than those of the lower disk. The frictional shear stresses are generated from the difference of the displacements at the contact surface between two disks, which are oriented to the opposite direction of the difference of the displacements, i.e. the outer side. They show a point symmetric with respect to the origin, so that they are cancelled each other. The frictional forces are calculated by integrating the frictional stresses in the $x$ direction and the resultant force is zero even if $\mu = 0.2$ (Figure 3).

Figure 4 is the distributions of stresses at $y=1.5a$ in the upper disk under loading, where the material is yielded in the range of $\sigma_y = 400\text{MPa}$. In the yielded area the deviatoric stress $\sigma_x'$ shows a linear symmetric with respect to
the y axis and the shear stress $\tau_{xy}$ shows a point symmetric with respect to the origin, which makes both of the resultant forces in equilibrium. These stresses generate the symmetric plastic deformation with respect to the centerline. Accordingly, the deviation of the plastic deformations cannot be observed.

3.2 Rotation of the disk

The upper disk is rotated clockwise around the point O by applying the same displacements upward on the point a and downward on the point b in the y direction under the axial loading. Figure 5 shows the distributions of the frictional shear stresses on the contact surface with the increase of the rotation angle of the disk. As the rotation angle increases, the minus value of the frictional shear stress decreases. The shear stress is integrated along the x coordinate from the point of zero stress value, the value of which represents the driving force to the x direction in rotating. As the angle of the rotation is $0.0003^\circ$, the minus value of the frictional stress is zero, which results in loosing the fixed point between the upper and the lower disks: it is expected that the upper disk begins to slip. In this case the deformation of the disk is little which is shown in Figure 6. This means the fact that the plastic flow is hardly influenced by the directions of driving or following the disk. Thus, the forward flow cannot be shown by the analytical model in Figure 1.

3.3 Deviation of the centers of two disks

The following model is applied to analyze numerically. Figure 7 shows that the center of the lower disk is horizontally moved to the left side by $\delta$ and that it is translated by a given displacement upward vertically to be loaded. The deviation $\delta$ can be generated by both of the clearances of the roll bearings and the accompanied motions in two dimension of the loading disk and so on. The specimen disk in testing must not be fixed in order to be loaded by nature when it is tested by using the four wheeled rolling machine. The two wheeled rolling machine have been applied to these kinds of experiments, the axles of which were mostly worked as the cantilever beam supported on one side. In both cases the specimen can move caused by the clearances of the disks: this means that they are subjected to the repeated frictional forces and loadings.

Figure 8 shows the deformation of the disk in $\delta=0.1\text{mm}$ after unloading, which represents the curved lines in the radius direction: when two centers of the disk O and O' are on the vertical line, the straight lines can be observed. The black circles on the left side after unloading deviates from the white triangles more than those on the right side. This means the deviation of the plastic flow in the direction of the rotation of the disk. The flow is physically verified by the following figures.

Figure 9 shows the distribution of the frictional shear stress at the contact surface in $\delta=0.1\text{mm}$, where the integrated value of the shear stress over the minus area of the x axis is larger than that over the plus area of the x axis. Therefore, the resultant force generated by these integrated values causes the
deviation of the plastic flow to the left side. Figure 10 shows the distributions of the stresses at y=1.5a (a=(a₂-a₁)/2) in the disk from the surface. The center of the yielded area in the disk is x=-0.05mm, which is deviated by δ/2=0.05mm. The distributions of the stresses MISES, \( \sigma_x' \) and \( \tau \) are nearly symmetric with respect to the line x=-0.05mm. The symbols a₁ and a₂ represent the edges of the contact area, which also shows the yielded area. The deviatoric stress is integrated along the x axis from 0 to a₁, and from 0 to a₂. The integrated value of the former area is larger than that of the latter one. Therefore, a part of the deviatoric plastic flow is caused in proportional to the difference of the integrated value, i.e. the strain determined by the Prandtl-Leuss law. In the case of the shear stress the above details are same and also the shear stress generates the plastic flow by being integrated in the y direction.

Figure 11 shows the frictional shear stress in the case that the upper disk is rotated under the axial loading. The resultant force (the driving force) is smaller than that in Figure 5 even if the disk is rotated. In this case the deformation of the disk is very small and the deviatoric plastic flow cannot be observed. In addition, the friction coefficient does not influence on the flow. Figure 12 shows the deviatoric plastic flow of the node at the 10\(^{th}\) element from the origin. This represents the plastic flow even if the deviation of the center of the disk is small.

**4 Concluding Remarks**

It is found that the deviatoric plastic flow in the subsurface of the disk is generated by the deviation of the centerline of two disks normal to the centerline under high axial loading, which is not influenced by the direction of driving or being followed: the plastic flow is not generated when two disks are located just on their center line.

**References**


Figure 1: Discretized model.

Figure 2: Deformation of the disk surface at the contact area.
Figure 3: Distributions of the frictional stress at the contact area under loading.

Figure 4: Distributions of the stresses near the contact surface under loading ($y=1.5a$).
Figure 5: Distributions of the frictional shear stress at the contact area.

Figure 6: Deformation of the disk near the contact surface caused by rotation.
Figure 7: Two disks the centers of which are deviated by $\delta$ in the $x$ direction.

Figure 8: Deformation of the upper disk after unloading ($u_x$ is multiplied by 1000, $\delta = 0.1$ mm).
Figure 9: Distribution of the frictional shear stress at the contact surface \((\delta = 0.1\, \text{mm})\).

Figure 10: Distributions of the stresses near the contact surface under loading \((\delta = 0.1\, \text{mm}, y = 1.5a)\).
Figure 11: Distributions of the frictional shear stresses on the contact area ($\delta = 0.1\text{mm}$).

Figure 12: Deviation of plastic flow near the yielded area in the disk versus $\delta$. 