Transients analysis of synchronous generator using the finite element technique

I. Jurić-Grgić, R. Lucić & M. Kurtović
Department of Electrical Engineering, University of Split, Croatia.

Abstract

The paper describes the use of the finite element technique when modeling electromagnetic transients in a power system in the time domain. The finite element matrices of electrical machines have been obtained in natural (a,b,c) coordinates. The time integration has been performed using $\mathcal{N}$ method. This paper presents a new method for power system analysis, based on the finite element technique. This method has been successfully applied on sudden short circuit synchronous generator analysis and voltage regulation of the generator under a sudden load. This method is competitive, even simpler, than well known nodal voltage method. The proposed method is suitable to the multimachine analysis, our main goal of research.

1 Introduction

The finite element method, as it is well known, has been widely used especially dealing with electromagnetic field in continuum. Electrical network with lumped parameters can be viewed as a set of the finite elements, so the finite elements technique can be used. The paper [3] deals with the finite element analysis of transmission lines and electrical circuits. Transient behavior of a synchronous machine, up to now, usually has been analyzed in (d-q) coordinates system using Runge-Kutta numerical procedure method. However, each part of the power system, such as synchronous generator, transformer, transmission line, circuit breaker, etc. can be considered as finite element. A local system of algebraic equations that represent numerical model of the power system parts, has been obtained using time integration of corresponding system of Cauchy's equations.
Assemblage of the global system of equations is based on satisfaction of the continuity equation (Kirchhoff's law for current) for each node in electrical network.

By imposing appropriate initial conditions, time step $\Delta t$, integration parameter $\vartheta$ and solving global system, nodal voltage and branch currents are obtained.

## 2 The finite element technique

### 2.1 Synchronous generator finite element

The three phase synchronous generator with field and damper-windings is represented as one finite element with three nodes namely 1, 2, 3.

![Synchronous generator finite element](image)

Figure 1: Synchronous generator finite element

The starting point to obtain a local matrix and vector of synchronous generator finite element is a system of ordinary differential equations of the first order in the natural coordinates according to [4]:

$$
\frac{d\{i\}}{dt} = -[L]^{-1} \left( [R] + \omega \frac{d[L]}{dy} \right) \{i\} + [L]^{-1} \{u\} 
$$

(1)

$$
\frac{dy}{dt} = \omega 
$$

(2)

where are:

$$
\{i\}^T = \begin{bmatrix} i_a & i_b & i_c & i_f & i_D & i_Q \end{bmatrix}
$$

$$
\{u\}^T = \begin{bmatrix} u_a & u_b & u_c & u_f & 0 & 0 \end{bmatrix}
$$

$[L]$ - (6,6) inductance matrix

$[R]$ - (6,6) resistance matrix

$\omega$ - electrical angular frequency
\( \gamma \) - electrical angle of rotor position

\( t \) - time

\( i_r \) - field coil current

\( i_D \) - damper-winding current in axes d

\( i_Q \) - damper-winding current in axes q

\( u_f \) - field coil voltage

On system eqn (3) and eqn (4) obtained from eqn (1) and eqn (2), explicit/implicit mixed procedure, also known as \( \mathcal{G} \) - method, has been applied.

\[
\begin{align*}
\int_{t}^{t^+} \left( \frac{d}{dt} \left[ L \right]^{-1} \left( \frac{d}{d\gamma} \left[ L \right] \right) \right) \left[ L \right]^{-1} \{ u \} \ dt &= 0 \tag{3} \\
\int_{t}^{t^+} \left( \frac{d}{dt} \left[ A \right] \right) \ dt &= 0 \tag{4}
\end{align*}
\]

In this way, system of differential equations is transferred to system of algebraic equations:

\[
\{ i \}^+ = \left[ F \right]^{-1} \left( \left[ L^+ \right]^{-1} \mathcal{G} \Delta t \left[ u \right] + \left[ L \right]^{-1} \left( 1 - \mathcal{G} \right) \right) \tag{5}
\]

\[
\Delta t \left[ u \right] + \left( \left[ I \right] + \left[ A \right] \left( 1 - \mathcal{G} \right) \Delta t \right) \{ i \} = 0
\]

where are:

\[
\left[ I \right] \text{ - unit (6,6) matrix}
\]

\[
\left[ A \right] = -\left[ L \right]^{-1} \left( \left[ R \right] + \omega \frac{d}{d\gamma} \left[ L \right] \right)
\]

\[
\left[ F \right] = \left[ I \right] - \left[ A \right] \mathcal{G} \Delta t
\]

Variables marked with ‘+’ denote variables at the end of time interval, while variables without mark denote variables at the beginning of time interval. The optimum value of \( \mathcal{G} \) can be found by numerical experiments. The present numerical procedure is stable for \( 0 \leq \mathcal{G} \leq 1 \).
2.2 Switch as the finite element

Especial switch finite element to model three phase short circuit condition, has been shown in Figure 2.

\[
\begin{align*}
&\begin{array}{c}
1 \\
2 \\
3 \\
\end{array} \\
&\begin{array}{c}
U_1 \\
U_2 \\
U_3 \\
\end{array} \rightarrow \begin{array}{c}
l_1 \\
l_2 \\
l_3 \\
\end{array} \rightarrow \begin{array}{c}
r \\
r \\
r \\
\end{array}
\end{align*}
\]

Figure 2: Finite element switch

Switch has been modeled as a circuit with lumped resistance, so that very low value of resistance corresponds to the closed switch to model short circuit, while high resistance represent open switch to model no load.

The switch local system is:

\[
\begin{pmatrix}
\dot{i}_1^+ \\
\dot{i}_2^+ \\
\dot{i}_3^+
\end{pmatrix} = \frac{1}{3r} \begin{bmatrix} 1 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} \begin{pmatrix}
u_1^+ \\
u_2^+ \\
u_3^+
\end{pmatrix} + \frac{1}{3r} \begin{bmatrix} \frac{1}{1-\theta} & 0 & 0 \\ 0 & \frac{1}{1-\theta} & 0 \\ 0 & 0 & \frac{1}{1-\theta} \end{bmatrix} \begin{pmatrix}
u_1 \\
u_2 \\
u_3
\end{pmatrix} - \frac{1-\theta}{\theta} \begin{pmatrix}
i_1 \\
i_2 \\
i_3
\end{pmatrix}
\]

where are:

\( r \) – switch resistance

\[
[T] = \begin{bmatrix}
2 & -1 & -1 \\
-1 & 2 & -1 \\
-1 & -1 & 2
\end{bmatrix}
\]
2.3 Transmission line as the finite element

A transmission line as finite element has been shown in Figure 3.

![Diagram of transmission line]

Figure 3: Transmission line

The starting point to obtain a local matrix and vector of transmission line finite element is a system of ordinary differential equations of the first order:

$$\{\varphi_1\} - \{\varphi_2\} = [L] \left[\frac{di}{dt}\right] + [R] \{i\}$$

where are:

$$\{\varphi_1\}^T = [\varphi_{1a}(t) \, \varphi_{1b}(t) \, \varphi_{1c}(t) \, \ldots \, \varphi_{1n}(t)]$$

$$\{\varphi_2\}^T = [\varphi_{2a}(t) \, \varphi_{2b}(t) \, \varphi_{2c}(t) \, \ldots \, \varphi_{2n}(t)]$$

$$\{i\}^T = [i_a(t) \, i_b(t) \, i_c(t) \, \ldots \, i_n(t)]$$

$$[L]$$ - (n,n) inductance matrix

$$[R]$$ - (n,n) resistance matrix

On system eqn (9) obtained from eqn (8), explicit/implicit mixed procedure, also known as \(\vartheta\) - method, has been applied.
\[
\int_t^{t+}\left(\{\varphi_1\} - \{\varphi_2\} - [L]\frac{di}{dt} - [R]\{i\}\right)dt = 0
\] (9)

In this way, system of differential equations is transferred to system of algebraic equations:

\[
\{\varphi^+\} = \left[C\right]\{\varphi_1^+\} - \left[C\right]\{\varphi_2^+\} + \left[D\right]\{\varphi_1\} - \left[D\right]\{\varphi_2\} + \left[B\right]\{i\}
\] (10)

where are:

\[
\left[B\right] = \left[A\right]^{-1}\left(\left[L\right] - \left[R\right](1 - \vartheta)\Delta t\right)
\]

\[
\left[C\right] = \vartheta\Delta t\left[A\right]^{-1}
\]

\[
\left[D\right] = (1 - \vartheta)\Delta t\left[A\right]^{-1}
\]

\[
\left[A\right] = [L] + [R]\vartheta\Delta t
\]

2.4 Voltage regulator of synchronous generator as the finite element

Voltage regulator of synchronous generator as the finite element has been shown in Figure 4.

![Voltage regulator diagram](image)

Figure 4: Voltage regulator of synchronous generator

The starting point to obtain a local matrix and vector of voltage regulator finite element is a system of ordinary differential equations of the first order, according to paper [5].
\[
\frac{d\{s\}}{dt} = [K]\{s\} + [M]
\]  

(11)

where are:

\[
\{s\}^T = \{v_T(t), v_f(t), u_f(t), v_R(t)\}
\]

\[
[K] = \begin{bmatrix}
-\frac{1}{T_r} & 0 & 0 & 0 \\
0 & -\frac{1}{T_f} & -\frac{k_f}{T_fT_e} & \frac{k_kk_e}{T_fT_e} \\
0 & 0 & -\frac{1}{T_e} & \frac{k_e}{T_e} \\
-\frac{k_a}{T_a} & -\frac{k_a}{T_a} & 0 & -\frac{1}{T_a}
\end{bmatrix}
\]

\[
[M] = \begin{bmatrix}
\frac{k_r}{T_r}v_T(t) \\
0 \\
0 \\
\frac{k_a}{T_a}v_{REF} + \frac{1}{T_a}u_{f0}
\end{bmatrix}
\]

- \(k_s, k_r, k_f, k_e\) - regulator amplification factors
- \(T_a, T_r, T_f, T_e\) - time constants
- \(v_T(t), v_f(t), v_R(t)\) - regulator signals
- \(u_f(t)\) - field coil voltage
- \(u_{f0}\) - initial field coil voltage
- \(v_{REF}\) - referent phase voltage

On the system eqn (12) obtained from eqn (11), explicit/implicit mixed procedure, also known as \(\mathcal{D}\) - method, has been applied.
In this way, system of differential equations is transferred to system of algebraic equations:

\[
\{ s^+ \} = [R]^{-1} [P] [s] + [R]^{-1} [M]^T \vartheta \Delta t + [R]^{-1} [M] (1 - \vartheta) \Delta 
\]

where are:

\[
[P] = [I] + [K] (I - \vartheta) \Delta t
\]

\[
[R] = [I] - [K] \vartheta \Delta t
\]

\[
[I] \text{ - unit matrix}
\]

3 Test cases

Two simple test cases are used to illustrate the algorithms capability. The first case is a three phases sudden short circuit of synchronous generator, and second is voltage regulation of generator under sudden load.

3.1 Three phases sudden short circuit of the synchronous generator

On the basis of previous described procedure three phases sudden short circuit of the synchronous generator has been successfully applied as it is shown in Figure 5 and Figure 6.

![Figure 5: Current in a phase with no flux at t = 0](image-url)
The obtained results are in excellent agreement comparing to results obtained by Simulink simulations and results calculated in d-q system using Runge-Kutta method [2].

3.2 Voltage regulation of the generator under sudden load

This example shows the effect of the voltage regulation of the generator under sudden load.

From the Figure 7, it can be clearly seen the rise of generator voltage due to the voltage regulator, after sudden load. At first the generator is at no load state.
At a some moment the sudden load is connected to the generator and voltage drop is occurred. Thanks to the voltage regulator, the referent voltage of the generator is preserved.

4 Conclusion

The finite element method can be successfully implemented in stationary and transients analysis of a power system. Attracting side of this method is a possibility to make the simple algorithm for the multimachine system analysis. The presented numerical model is in natural (a,b,c) coordinate system, so symmetrical components or Parks coordinates are not necessary. As the power system has been built assembling the three phase finite elements, the solution of the multimachine system is natural and straightforward.

References


