Numerical simulation of the laboratory sound insulation determination experiments

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Abstract

This paper explores numerical simulations for the experimental evaluation of sound reduction index for building partitions in the low frequency range, i.e. the range in which the modal density is low in typical rooms. The simulations are based on the wave theoretical model of acoustic cavities, which is reviewed first. The governing (damped) wave equation is solved approximately by the Modal Approach and by the Finite Element Method. Absorption in rooms is modeled by introducing the flow resistivity, related to the reverberation time, which in turn is measured during the experiment. Two models are employed for the partition. In the first the partition is treated as locally reacting with mass impedance. In the second it is treated as a thick, finite plate. The simulation concerns experiments carried out in two different laboratories - one with the sending room having equal dimensions to that of the receiving room, and the other with the sending room and receiving room having considerably different dimensions. The reverberation also differs significantly between experiments. In one case both rooms are heavily damped, whilst in the other they have long reverberations. In both cases the test object is nominally the same - a single leaf gypsum board partition. The results of both simulations by numerical techniques are compared with the experimental results. In both cases the frame supporting the tested partition has considerable thickness and a niche is created on one or both sides of the test object. The influence of the position of the test partition in the supporting frame on the evaluation of the reduction index is studied in the simulations. The Finite Element analysis is carried out on a parallel computer - IBM SP2. Remarks concerning the application of computers of this type for the simulations involving steady state analysis in the frequency domain are given.

1. Introduction

During the last years low frequency noise has become increasingly important in building acoustics. The standards concerning sound insulation determination for building partitions are being extended from the traditional limit of 100 Hz down to 50 Hz. At such low frequencies the sound field in a typical dwelling room is not diffuse. Therefore, the statistical approach can not be applied and other methods, referring to wave theoretical approach have to be used for simulation of experiments. There has been some confusion caused by the fact, that the results of the sound insulation evaluation obtained for the same partition at different laboratories, differ significantly. To study the phenomena at low frequencies tools are necessary to describe the sound field and its interaction with the structure under test properly. Although the problem is not new, in the recent paper Atalla&Bernhard [1] complained about "relatively few realistic verification exercises" of numerical solutions for low-frequency structural-acoustic problems. The current paper can be treated as such verification. Two rooms coupled by a flexible partition constitute a classical task in building acoustics. Among early papers concerning the low frequency problem one can distinquish the work Mulholland&Lyons [2]. Attempts of modelling the low frequency sound insulation has been reported, e.g. by Gagliardini *et al.* [3]. Problems concerning the meaning of sound reduction index of partitions at low frequencies has previously been addressed by Kropp *et al.* [4]. In the current paper, simulations with several numerical methods will be compared to the measurement results for two different laboratory configurations: one with equal rooms and one with sending room and receiving room significantly different. Attention will be paid to the details of the placing of the partition under test (possible niche and supporting frame), as well as, to its construction. The accuracy of the simulations will be evaluated, and the computational effort will be discussed.

2. Measurements

The simulations concerned two sound transmission laboratories. The first one, belonging to the Department of Applied Acoustics, Chalmers University of Technology, Gothenburg, Sweden, consisted of two equal chambers. Its geometry is shown in Fig.1. The test partition was placed in a concrete frame 0.22 m thick. It was positioned nearly flush with the face of the frame on the sending room side, leaving a niche of 0.174 m on the side of the receiving room. The test opening had dimensions 4.80 m by 2.50 m, giving the area of 12 m^2. This laboratory will later be referred to as Case 1. The other laboratory, belonging to the DELTA Acoustics and Vibration, Aarhus, Denmark, consisted of two rooms that were significantly different from each other. Its geometry is shown in Fig.2. The test partition was placed in a concrete frame constructed with 2 leaves, each one 0.20 m thick, and with a 0.05 m gap between them. It was positioned so, that the niche at the sending room's side was 0.174 m, and the one on the receiving room's side was 0.25 m. The test opening was 4.20 m by 2.60 m, giving the area of 11 m^2. This laboratory will further be referred to as Case 2. In both cases, the measurements were carried out with a single leaf partition consisting of two layers of gypsum boards (total thickness 0.026 m) supported by steel studs. In both cases the partition under test did not cover the whole opening between rooms. In both cases the partition

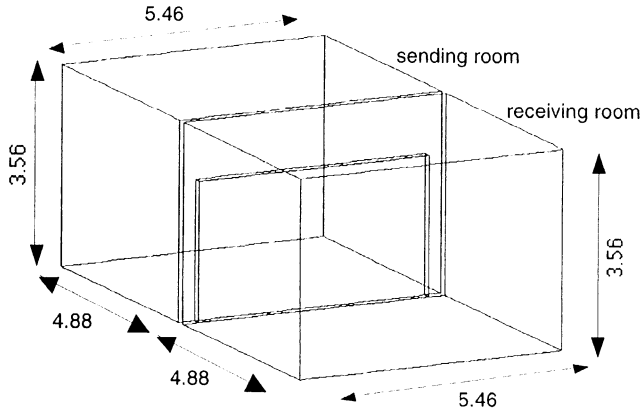


Figure 1. The layout of the "Case 1" laboratory

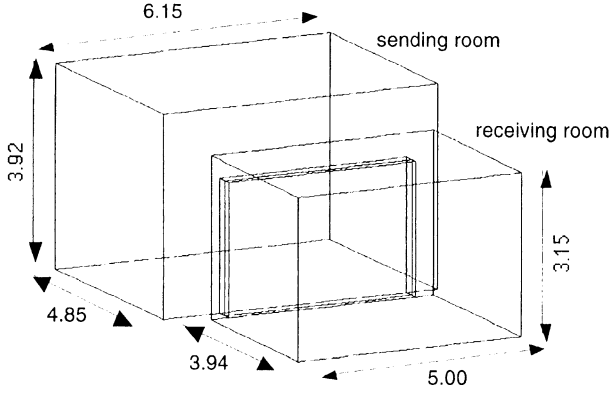


Figure 2. The layout of the "Case 2" laboratory

was placed in a niche in the supporting frame (i.e. at least at one side not flush with the frame's surface).

In Case 1 the loudspeaker was placed in the corner of the sending room and the sound pressure level (SPL) was measured in 7 positions specified according to the requirements of the standard (ISO 140 [5]). The points were located in the central part of the room, with at least 1.20 m distance from any of the walls and at least the same distance between any two points. Even the reverberation time was evaluated in those positions. Measurements were carried out employing both the MLS technique and the (more classical) digital filtering of the time signals.

In Case 2 the loudspeaker was also placed in the corner of the sending room and the SPL was measured by means of two rotating microphones in each room. Only 3rd octave band results were recorded, even for the reverberation time.

In both cases, the reduction index for the tested partition was calculated according to the formula:

$$R = L_{p,s} - L_{p,r} - 10\log\frac{A}{S} \tag{1}$$

which expresses the reduction index as the difference between the averaged SPLs in the sending room and in the receiving room, corrected for the absorption in the receiving room, and for the area of the test object.

3. Theoretical background for simulations

The simulations were based on the steady state frequency domain formulation of the linear acoustic approximation, described by the following system of equations:

$$\frac{j\omega}{c^2}\underline{\hat{p}} + \frac{\partial}{\partial x_i}\left(\rho_o \underline{\hat{v}}_i\right) = 0 \tag{2a}$$

$$\frac{\partial \underline{\hat{p}}}{\partial x_i} + \left(j\omega\rho_o + \Xi\right)\underline{\hat{v}}_i = 0 \tag{2b}$$

where: p - amplitude of acoustic pressure;
v - amplitude of particle velocity;
c - speed of sound;
ρ - ambient air density;
Ξ - flow resistivity.

From (2b) we can derive the amplitude of velocity as a function of the amplitude of pressure. Inserting it into (2a) we obtain:

$$\frac{\partial}{\partial x_i}\left(\frac{j\omega\rho_o}{\left(j\omega\rho_o + \Xi\right)}\frac{\partial \underline{\hat{p}}}{\partial x_i}\right) + \frac{\omega^2}{c^2}\underline{\hat{p}} = 0 \tag{3}$$

Hence, in the frequency domain we can obtain pressure formulation without assuming, that flow resistivity is constant in space or in time/frequency. In fact, it is dependent on frequency in the present application.

The ambient air density and the speed of sound were assumed according to the temperature during measurements. The flow resistivity for the cavities was calculated on the basis of measured reverberation time according to the following formula:

$$\Xi = 2.2\frac{2\pi\rho_o}{T_{60}} \tag{4}$$

As the reverberation time is usually dependent on frequency, so is also the flow resistivity.

The model for the acoustic cavities was complemented with the model for the partition. In the simplest case, the partition was modeled as a limp plate, i.e. locally reacting with mass impedance. This approximation was justified at low frequencies for the partition in question since its critical frequency was around 2500 Hz, and the first eigenfrequencies for a plate model of the partition were around 2 Hz. Hence, the partition was in the mass controlled

range. The only parameter is then mass per unit area, which was assumed as given by the manufacturer (a check by weighing a typical gypsum board gave the same result).

Another model for the partition was that of a thick elastic plate, as described by the Reissner-Mindlin theory. This model was only used in FEM analysis. The material parameters in that model were the elasticity constants and the damping coefficient. The partition consisted of two layers of gypsum boards screwed together to the steel studs. The layers were assumed to be in gliding contact. The elasticity constants were modified in order to obtain the appropriate stiffness and mass per unit area for the model using only one layer of boards. The damping in the plate did not influence the results, as the partition responded in the non-resonant way. The Rayleigh damping model was assumed, providing a least squares approximation of the constant damping ratio over the frequency range of interest.

4. Numerical models

4.1 The Modal Approach

The so called 'Modal Approach' follows the model used by Donner [6]. The sound field in both rooms is described by modal functions in the width (y-direction) and height (z-direction) of the rooms where the boundary conditions are given as rigid. This can be viewed as introducing waves with the wave fronts of the shape of modes in y- and z-directions, and unknown (complex) amplitudes, traveling along the x-direction. The solution to the set of equations (2) is assumed in the form:

$$p(x,y,z,\omega)=\sum_n\sum_m\left\{\left[p_{n,m+}(\omega)e^{-jk_{xn,m}x}+p_{n,m-}(\omega)e^{+jk_{xn,m}x}\right]\cos\left(\frac{n\pi}{l_y}y\right)\cos\left(\frac{m\pi}{l_z}z\right)\right\} \tag{5}$$

with the wave number in the x direction

$$k_{xn,m}^2=k_o^2-\left(\frac{n\pi}{l_y}\right)^2-\left(\frac{m\pi}{l_z}\right)^2 \tag{6}$$

$$k_o^2=\frac{\omega^2}{c^2}\left(1-\frac{j\Xi}{\rho\omega}\right) \tag{7}$$

The velocity component in the x-direction is obtained from the momentum conservation equation (2b) as:

$$v(x,y,z,\omega)=\frac{1}{\omega\rho-j\Xi}\sum_n\sum_m\left\{\left[k_{xn,m}\,p_{n,m+}(\omega)e^{-jk_{xn,m}x}-k_{xn,m}\,p_{n,m-}(\omega)e^{+jk_{xn,m}x}\right]\cos\left(\frac{n\pi}{l_y}y\right)\cos\left(\frac{m\pi}{l_z}z\right)\right\} \tag{8}$$

The damping is introduced by specifying a flow resistivity (eq.(7)). The unknown (complex) amplitudes are determined by the (velocity) boundary conditions. The boundary conditions in the length (x-direction) of the rooms are given by the conditions at the back walls (e.g. given velocity distribution) and by the conditions at the partition. There the sound fields in both rooms are coupled by the impedance of the partition.

The boundary conditions will be fulfilled in discrete points, the so called collocation points. The number of collocation points depends on the number of modes which one intends to consider. Although from the mathematical point of view the collocation method may not be satisfying (convergence is not guarantied by increasing the number of modes (Zurmuehl [7])), it is quite well tempered (see, e.g. Munjal [8]). For a partition with pure mass impedance the boundary conditions are:

$$p_I(x = l_x, y, z, \omega) - p_{II}(x = 0, y, z, \omega) = Z(\omega) v_p(y, z, \omega) \tag{9}$$

where v_p is the velocity component of the partition in the normal direction (i.e. x direction) and $Z(\omega) = j\omega m''$ the mass impedance. m'' is the mass per square meter. Additionally, the x-component of the partition's velocity equals the x-component of the particle velocity in the sound fields on both sides of the partition. This is the fastest method of those considered in the current paper. The results are satisfactory, at least for the examples presented here.

4.2 The Finite Element model

The application of the Finite Element method to the solution of the coupled structural-acoustic problem is discussed in the literature since the sixties (see, e.g. Gladwell [9]). By today, it is included in several commercially available FE packages. Single variable formulation with acoustic pressure as a primary variable is prevailing in those packages. Other formulations are possible, and may indeed be advantageous for coupled problems (e.g. Sandberg [10]), but they are not commercially available. It seems also, that the development today is focused on the implementation of more refined models describing the material behaviour, such as the poroelastic material as considered by Biot [11], rather then on the refinement of formulation for the standard acoustic problem in a fluid. It is, for instance, impossible to perform a transient (time domain) analysis for the fluid with flow resistivity varying in space with any of the packages known to the author. A certain solution in the frequency domain has been proposed by Craggs [12]. A somewhat more general analysis is given by Pietrzyk [13].

Consider the so called "weak" formulation (Zienkiewicz&Taylor [14]) of the boundary value problem of the partial differential equation as eq. (3), given by:

$$\int_{\Omega} w \left\{ \frac{\partial}{\partial x_i} \left(\frac{j\omega\rho_o}{(j\omega\rho_o + \Xi)} \frac{\partial \underline{\hat{p}}}{\partial x_i} \right) + \frac{\omega^2}{c^2} \underline{\hat{p}} \right\} d\Omega = 0 \tag{10}$$

where Ω is the entire domain of computation, and w is an arbitrary weighting function. By standard rearrangements we arrive at the following formulation, where the boundary conditions enter explicitly:

$$\int_{\Omega} w\left\{\frac{\omega^2}{c^2}\hat{\underline{p}}\right\}d\Omega + \int_{\Omega}\left\{\frac{j\omega\rho_o}{(j\omega\rho_o+\Xi)}\left(\frac{\partial w}{\partial x_i}\right)\left(\frac{\partial \hat{\underline{p}}}{\partial x_i}\right)\right\}d\Omega +$$
$$-\int_{\partial\Omega_2}\left\{w\omega^2\rho_o\underline{\hat{u}}_{si}n_i\right\}d(\partial\Omega_2) + \int_{\partial\Omega_3}\left\{wj\omega\rho_o Y_i\hat{\underline{p}}\right\}d(\partial\Omega_3) + \qquad (11)$$
$$+\int_{\partial\Omega_4}\left\{wj\omega\rho_o v_f\right\}d(\partial\Omega_4) = 0$$

The part over Ω_2 describes a fluid-structure interface with the structural displacement forcing the same displacement in fluid. The part over Ω_3 describes the partly absorbing surface with known mobility. The part over Ω_4 describes a velocity source.

For many types of partitions, especially for lightweight ones, the wavelength of the bending waves at low frequencies is substantially smaller then the wavelength of the sound waves in air. While modeling both the fluid and the structure with FE (FEM/FEM) one is faced with the problem of 3D mesh generation. The mesh in the fluid domain has to match the mesh at the partition, but the element size can than increase with the distance from the partition in order to reduce the number of unknowns. Unfortunately, generation of unstructured 3D meshes is not a solved problem and is not offered in the FE pre-processors. One possible solution might be to model the partition as locally reacting with mass impedance (FEM/LRMI), as described by eq. (9). This may seem to be a crude approximation, but it proves justified at low frequencies, and may be attractive while dealing with irregular rooms. This model is available e.g. in SYSNOISE. The FEM will be demonstrated with ABAQUS [15], and SYSNOISE [16].

5. Simulation results

5.1 Sound insulation evaluation

The results of the simulations are presented in terms of the sound reduction index (eq. (1)). In Fig.3 the measurement results are compared to the results of simulations for the Chalmers laboratory with both FEM/FEM and MA/LRMI techniques. The results are given as 3rd octave band spectra and cover the frequency range of up to 500 Hz. The agreement is fairly good - the deviation of simulation results from the measurement results is 4 dB at the most.. The FEM/FEM approach gave slightly better results, as expected. In this model the gypsum boards were treated as a thick flexible plate which enabled to account for the finite length of the plates. Nevertheless, the MA/LRMI approach gave similar results. The convergence study carried out for the MA/LRMI technique by solving the problem with gradually increased number of component waves showed, that the results were stable but for the highest frequency range. The result presented in Fig.3 corresponds to 25 and 16 component cosine terms in the width and height directions, respectively. The model consisted of 3 coupled chambers with the niche created in the supporting frame behind the partition treated as additional chamber. It took 11 min CPU for a SUN Ultra1 140 MHz workstation to execute a Matlab code. For the FEM/FEM model the mesh consisting of parabolic serendipity elements with nodal distance of 0.25 m was

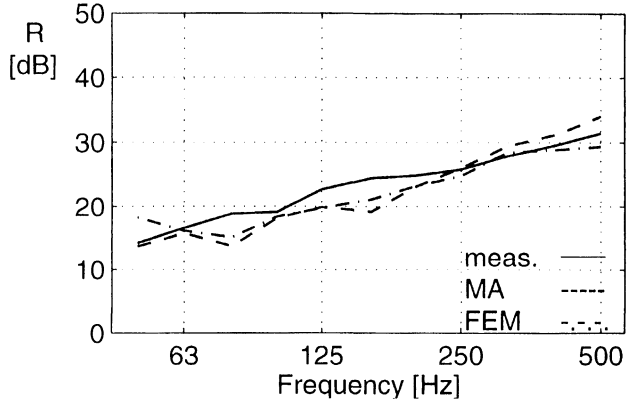


Figure 3. Simulation results for "Case 1"

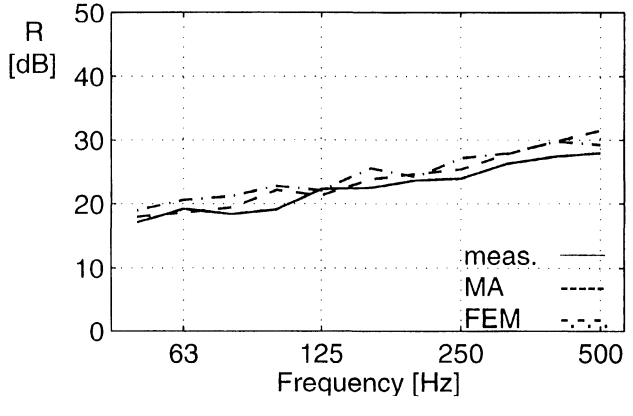


Figure 4. Simulation results for "Case 2"

used. It lead to a computational problem with 87000 unknowns which required 15 min CPU for one frequency step at a single node of an IBM SP2 computer. For both cases 520 frequency steps were calculated.

The results for the DELTA laboratory are shown in Fig.4. Again, the measurement results are compared to the results of simulation with the MA/LRMI, and the FEM/FEM model. The agreement is better than in Case 1, probably due to the higher damping (significantly shorter reverberation time). Especially the MA/LRMI technique seems successful. The model consisted of 4 chambers, including the niche on both sides of the partition. The maximum numbers of component cosine terms were 27 and 17. The calculation time was 26min CPU per frequency step on the same SUN Ultra1 140 MHz workstation, due to an extra chamber as compared with the Case 1. The FEM/FEM model was very similar in terms of computational effort to the Case 1, and the calculations were carried out on the same IBM SP2 computer.

5.2 Sensitivity to the partition location in the niche

It is often the case in the sound transmission laboratories, that the test object is being placed in a kind of rigid frame. Thus, it does not cover the whole area of the wall separating the two rooms. Moreover, such frames have sometimes considerable thickness, and particularly for single leaf partitions a niche is created on one or both sides of the test partition. The depth of such a niche is usually around 0.20 m. This may seem insignificant as compared to 4.00 to 5.00 m length of the room, but it changes considerably the conditions for the excitation of the partition, as well as, for the sound radiation to the receiving room. A study of the influence of the niche at the results of sound insulation was reported by Donner [6]. We considered Case 2, with the total thickness of the frame of 0.45 m, and the partition being originally positioned as described in chap. 2. We tried to move it in the niche in our simulations, so that in one case it was mounted flush with the frame at the sending room side, and in another case flush with the frame at the receiving room side. The results of such simulations are given in Fig.5. As it can be seen, the results are similar at high frequency end but diverge towards lower frequencies. At 50 Hz, the spread of the results is 4 dB, but already at 125 Hz the it is 7 dB.

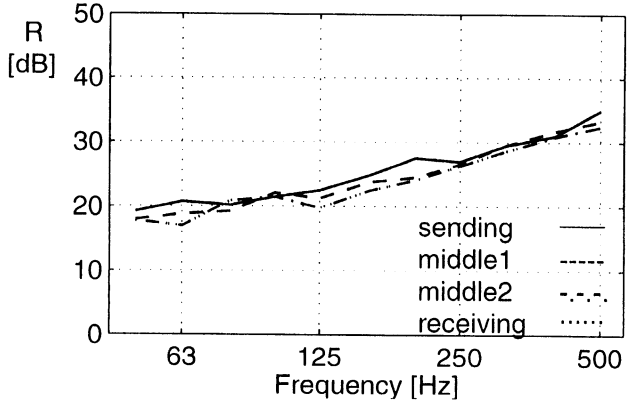


Figure 5. Results for different partition positions in the frame

6. Impact of parallel processing

Having access to a parallel machine or at least a number of separate workstations is more and more a reality today. Typical steady state analysis can benefit in several ways from being run on multiple processors. The easiest way is of course to run separate analysis cases on separate machines simultaneously. This does not require any modifications in the software. It is appropriate, in the first hand, for small jobs that easily fit into a working space of a moderate workstation. The slightly more complicated way, yet still possible to implement on separate workstations, is to divide one analysis case into several smaller frequency ranges and run each range in parallel. This is suitable for bigger jobs, where the analysis time per frequency step is long. It requires small modifications to the software concerning postprocessing stage. This type of parallelism on a 12 node IBM SP2 machine was used for the FE

calculations reported in this paper. In our case it brought a one week job to a 12 hours one. The higher level of parallelism is to distribute a single frequency step over several processors. For this case, usually a parallel machine with several processors accessing the common memory is required. It is appropriate for very large jobs, where a single frequency step is too big to fit into the working space of a single workstation.

7. Conclusions

The numerical methods based on wave theoretical acoustics proved successful in modeling the experiments. All methods tested gave the same results provided the same underlying model was used. One interesting remark can be, that in principle the same partition was tested in both laboratories. Both the experimental results and the results of numerical simulations, however, differ substantially between the laboratories. It is yet another demonstration of the today already widely accepted fact, that at low frequencies in particular the rooms have significant influence on the results of sound reduction estimation.

Acknowledgments

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