Sound propagation from dipole and monopole sources in a stratified fluid above a layered poro-elastic solid
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Abstract

Sound propagation from multiple dipole and monopole sources in a system comprising of a horizontally stratified fluid above a horizontally stratified porous elastic solid is modelled using the global matrix method to solve the wave equation. A modified Biot-Stoll model is used for wave propagation in the porous elastic solid. Predictions of sound pressure level in the fluid are compared with other numerical routines to validate the program. Predicted radiation fields due to dipole and monopole sources are compared. Numerical techniques to ensure the accuracy and stability of the solution are described.

1 Introduction

The Fourier Transform method in the form of Fast Field Program has been used to solve the problem of wave propagation in range-independent stratified environments above an impedance or elastic solid\(^{[1,2]}\). The Global Matrix method solves the interface boundary conditions equations in both atmosphere and ground layers in transformed space. Recently, the sound fields due to a dipole and a quadrupole in a homogeneous atmosphere above an impedance ground have been calculated\(^{[3]}\). Here, we describe a system to calculate the field due to multiple sources (monopole and/or dipole) in a stratified fluid above a layered porous elastic ground in either media.

The details have been described elsewhere\(^{[4,5]}\). The program FFLAGS has been used to investigate the role of frame elasticity in
determining the sound field above a porous elastic foam layer. We show that very little extra computational effort is required to calculate the non-symmetric dipole field if certain techniques are used.

2 Theory

We assume a source, situated at (0,0,h), generating spherical waves in a stratified fluid (atmosphere) above a layered porous elastic solid (ground). Biot's hydrodynamic equations of poroelasticity, as modified by Stoll, are assumed to hold in the solid layers. These predict existence of three wave types in the porous elastic solid; two dilatational waves and one shear wave. The fast and slow compressional waves travel mainly in the solid and pore fluid respectively. The fast wave is very similar to the elastic P-wave with little attenuation, but the slow wave is highly attenuated.

The three displacement potentials corresponding to the fast, slow and shear waves, as well as acoustic waves in air, can be determined from the Hankel-transformed potentials:

\[ \Phi_i(r,z,k) = \int_0^\infty \varphi_i\left(z,k_r\right) \frac{J_0(k_r r)}{k_r} \, dk \quad l=0,1,2,3 \]  

where subscripts 1 and 2 refer to the two dilatational waves, 3 refers to the shear wave, and 0 refers to the displacement in the fluid above. \( \varphi_i \) are the range independent Green's functions. They are solutions of the transformed Helmholtz wave equations:

\[ \frac{d^2 \varphi_i}{dz^2} - \beta^2_i \varphi_i = S\delta(z-h) \]  

where \( \beta^2_i = k^2_i - k^2_r \), and \( k_i = \omega/c_i \)  

\( k_r \) is the horizontal component of wavenumber and \( c_i \) are the corresponding wave speeds which are determined from Biot's equations. \( b_i \) represent the vertical components of the complex wavenumbers and \( S \) is the source strength. Time dependence of \( e^{j\omega t} \) is suppressed.

2.1 Propagation in the fluid layers

The medium within each layer is assumed to be homogeneous, so that in the presence of boundaries at heights \( h_1 \) & \( h_2 \) \( (h_1<h_2) \) the Green's
function within the layer is thus the sum of upgoing and down going waves in the layer plus the direct source term if one exists there:

$$\varphi_0 = R \uparrow e^{j\beta_0(h_1-z)} + R \downarrow e^{j\beta_0(z-h_2)} + \Xi. \text{Source}$$  \hspace{1cm} (3)

\(\Xi\) is equal to 1 if \((h_1<h<h_2)\) and 0 otherwise. The effects of multiple sources can be added in a similar way. The source term depends on the type of the source (monopole or dipole etc.) and is discussed later.

2.2 Propagation in porous elastic layers

In the calculations reported here the viscosity correction function in the Biot-Stoll model \((F(\lambda))\) is determined using the Attenborough 4-parameter model\(^9\), using Flow Resistivity, Porosity, Tortuosity, and Pore Shape Factor Ratio and ensuring equivalence in the rigid porous limit. This has been superceded by an improved model including pore size distribution\(^10\), but this is not pursued at the present time. The model assumes the fluid filled pores in the solid are of arbitrary shape but identical dimensions. A further six parameters are needed to specify the medium in a ground layer completely. These are: Compressional and Shear wave speeds of the drained solid matrix, density, fused solid bulk modulus, layer thickness, and attenuation ratio. Biot theory determines the ratios of fluid to solid displacements for the three body waves as well as their complex velocities.

The three potentials within the poroelastic layers are then:

$$\varphi_1 = \begin{cases} A \downarrow e^{j\beta_2(z-d_1)} + A \uparrow e^{j\beta_2(d_2-z)} \\ B \downarrow e^{j\beta_2(z-d_1)} + B \uparrow e^{j\beta_2(d_2-z)} \end{cases}$$  \hspace{1cm} (4)

$$\varphi_2 = m_1 \begin{cases} A \downarrow e^{j\beta_1(z-d_1)} + A \uparrow e^{j\beta_1(d_2-z)} \\ m_2 \end{cases}$$  \hspace{1cm} (5)

\(m_1\) and \(m_2\) are constants that depend on the specific medium properties.
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\[ \varphi_3 = \left( C \downarrow e^{j\beta d_1(z - d_1)} + C \uparrow e^{j\beta d_2(z - d_2)} \right) \]  \hspace{1cm} (6)

d_1, d_2 and z are +ve distances from the air-ground interface \( (d_2 > d_1) \) and \( m_i \) are the above-mentioned ratios. One can include a variety of seismic sources within the ground layer as the source of excitation. This possibility is not covered here and the reader is referred to ref.[4]. The coefficients \( R, A, B \) and \( C \)'s for each layer are the amplitudes which are calculated from the boundary conditions.

2.3 Boundary conditions

Two continuity statements are required at an interface between two fluid layers:

1. continuity of pressure, and
2. continuity of vertical particle displacement.

There are four boundary conditions at the fluid-solid interface[9]:

1. continuity of total normal stress,
2. continuity of fluid pressure
3. continuity of vertical fluid particle displacement, and
4. zero solid tangential slip at the surface.

Finally, six equations are needed at solid-solid interfaces:

1. continuity of normal stress,
2. continuity of normal solid displacement,
3. continuity of tangential stress,
4. continuity of pore fluid pressure,
5. continuity of normal fluid displacement, and finally;
6. continuity of radial displacement.

We also assume that pores are open to one another at the interface and that the layers are 'welded' together. Care should also be taken to satisfy the radiation conditions in the upper and lower capping half-spaces.
2.4 Global matrix

Once all the boundary equations have been written down with the amplitudes as the unknowns, we form the matrix equation (Fig. 1):

\[ A \cdot X = B \]  

(7)

Here, \( X = (R_n^\wedge, R_{n-1}^\wedge, ..., R_1^\wedge, A_1^\wedge, A_2 \wedge, ..., C_m) \), \( A \) is the coefficient matrix, and \( B \) is the source term vector. This matrix equation is solved numerically; the required function in the receiver depth is reconstructed from the potentials by adding the necessary terms. More than one receiver can be specified once the amplitudes are known.

![Fig. 1 Schematic diagram of the global matrix](image)

The inverse Hankel transform is performed by substituting the large argument approximation for the Bessel function and replacing the integral by a finite sum:

\[ P(r_m, z) \approx \frac{\delta k\sqrt{N}}{2\pi \sqrt{m}} \left[ \sum_{n=0}^{N-1} \phi(k_n, z)e^{j2\pi m/n} \frac{\chi_n}{\chi_n^*} + \sum_{n=0}^{N-1} \phi(k_n, z)e^{-j2\pi m/n} \frac{\chi_n}{\chi_n^*} \right] \]  

(8)

The summation is performed using an efficient FFT method. This is the essence of the Fast Field Program technique. To ensure convergence of the sums and to avoid superficial oscillations in the result (Gibb’s oscillations) The horizontal component of the wavenumber is made complex by adding a small imaginary part to \( k_r \). This in effect alters the path of integration away from the real axis. Further corrections to the summation, due to Richards and Attenborough \(^{[11]}\) are required to allow for this change of path. Figure 2 shows a plot of ground particle velocity...
due to an airbourne source in the elastic limit compared with the output from the well-known SAFARI program.

![Soil particle velocity in an elastic ground layer calculated by SAFARI and FFLAGS](image)

**Fig.2** Soil particle velocity in an elastic ground layer calculated by SAFARI and FFLAGS

### 2.5 Source terms

The transformed potential for a monopole in a fluid is:

\[
\varphi_s = 5e^{j\beta_0|\nu-z|}
\]

Performing an inverse transform on \( \varphi \) will give the field of spherical waves in the absence of boundaries.

The field due to a vertical array of monopoles is, of course, cylindrically symmetric so that the problem is essentially a two-dimensional one. The same cannot be said of a multipole source. The general solution to the wave equation in three dimensions is:

\[
P(r,\nu,\psi) = \sum_{n=0}^{\infty} \left[ Q_{1n}(r,\nu)\cos n(\psi - \nu) + Q_{2n}(r,\nu)\sin n(\psi - \nu) \right]
\]

and in the transformed space:

\[
P(k,\nu,\psi) = \sum_{n=0}^{\infty} \frac{(-j)^n}{2\pi} \left[ q_{1n}(k,\nu)\cos n(\psi - \nu) + q_{2n}(k,\nu)\sin n(\psi - \nu) \right]
\]

where

\[
Q_{in} = \int_0^\infty q_{in}(\nu)J_n(kr)kdk, \quad i=1,2; \quad n=0,1,2,...
\]
Qₐ are determined in the transformed space and the infinite sum evaluated. This evaluation is very time consuming in general, but for a dipole or a quadrupole there are only two or three terms in the sum and these can be evaluated quite efficiently.

The dipole can be thought of as the sum of two out-of-phase monopoles distance 2d apart. Then we have [3]:

\[
\Phi_d = S(e^{jkR_1} - e^{jkR_2})
\]

where

\[
R_1 = \sqrt{(x-d \sin \gamma \cos \epsilon)^2 + (y-d \sin \gamma \sin \epsilon)^2 + (h-z-d \cos \gamma)^2}
\]

and

\[
R_2 = \sqrt{(x+d \sin \gamma \cos \epsilon)^2 + (y+d \sin \gamma \sin \epsilon)^2 + (h-z+d \cos \gamma)^2}
\]

where \(\gamma\) and \(\epsilon\) are the polar and azimuth inclination of the line joining two sources (dipole axis).

If we transform the potential to wavenumber domain and take the limit of small d (\(d \ll |h-z|\) and \(d \ll \lambda\)) we arrive at the following:

\[
\Phi_d = -jS_d e^{j\beta_0 |h-z|} \left\{ \text{sign}(h-z) \cos \gamma + \frac{k}{\beta_0} \sin \gamma \cos(\epsilon - \psi) \right\}
\]

where \(S_d = 2dS\) and \(\psi\) is the receiver azimuth. Note that the expression derived by Hu & Bolton [3] is valid only for \(h>z\), but our expression is for the general case.

Comparing the above expression with the general solution (eq. 11) indicates that the first term corresponds to \(q_{10}\) and the second term to \(q_{11}\). We can also calculate the field due to the two components at the same time if we use LU Decomposition techniques to solve the matrix equation (Crout’s method) since the main effort is taken up by the decomposition. We then use different source vectors to arrive at the two Green’s functions. The contour plot of the sound field is evaluated by adding the two terms of the Fourier sum with the varying receiver azimuth.

A vertical array of arbitrarily oriented dipoles can be treated in a similar fashion. For example a quadrupole can be treated as two out of phase dipoles. Fig. 3 shows a plot of SPL due to a vertical dipole and that
of a pair of out-of-phase, vertically separated monopoles as function of range at 500 Hz. This can act as the validation of the stated expressions.

![Graph showing SPL due to a vertical dipole using FFLAGS & DPOLE](image)

Fig. 3. Field due to a dipole and sum of two out-of-phase monopoles. The curves are coincident dB values are re an arbitrary value.

### 3 Results

Program DPOLE based on the FFP method was used to compare and contrast numerically, the characteristics of sound field above a poroelastic ground due to a dipole and a monopole. The ground elasticity does not have a significant influence on the propagation above ground.

![Graph showing plots of transmission loss at 500 Hz due to a dipole and a monopole](image)

Fig. 4. Show plots of transmission loss at 500 Hz due to a dipole and a monopole. dB values are re an arbitrary value.
4 Conclusion

A program to calculate sound field due to dipole as well as monopole sources in a stratified fluid above a layered porous elastic ground using the Global matrix method has been described. It was shown that numerically a dipole can be modelled as two out of phase monopoles. Calculating non-symmetric pressure contour lines due to a dipole present little extra computation time compared to the symmetric field due to a monopole and can easily be implemented if required.
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References


