Numerical simulation and experimental verification of Dam-Break flows with shocks
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Abstract
In this paper experimental results of 1D Dam-Break flows are compared with those obtained by means of a mathematical model based on the MacCormack shock-capturing scheme. Care has been devoted to the treatment of the source terms of the equations and to the resolution of shocks.
To verify the numerical model under severe test conditions, laboratory experiments have been carried out in which shock formation, reverse flow and wetting and drying conditions in the flow field were induced.
Comparison of experimental and numerical results shows a very good agreement, as a whole. The essential validity of the numerical model is then confirmed, even in situations in which St. Venant hypothesis are not completely verified.

Introduction
Floods resulting from the sudden collapse of a dam (Dam-Break) are often characterized by the formation of shock waves due to irregular bed topography and nonzero tailwater depth.
In the last decade many works have been carried out in the field of numerical solution of 1D and 2D St. Venant Equations, mainly devoted to the treatment of source-terms and to accurately capture discontinuities (e.g. Molinaro & Natale [1]).
Verification of the capabilities of the numerical schemes are often performed comparing the computed results with analytical solutions. Only few experiments in literature concern with formation and propagation of shock waves (Chervet & Dalléves [2], Bellos et al. [3]).

In this paper experimental results of 1D Dam-Break flows are compared with those obtained by means of a mathematical model based on the well known MacCormack shock-capturing scheme.

**Experimental setup and test conditions**

Tests were carried out in a tilting laboratory flume at Department of Civil Engineering of Parma University. The flume was rectangular in section, 1.0 m wide, 0.5 m high and 7.0 m long. The instantaneous dam failure was simulated by means of the sudden (less than 0.05 s) removal of a gate controlled by an oil pressurized circuit. To minimize disturbances in the flow field, the gate is mounted on a frame completely separated from the rest of the flume, without any guide rails on bottom and sides of the dam section.

Measurements of water depth versus time were made at four sections along the flume, including the dam site, based on video recordings of the flow. Velocity measurements were accomplished with an Acoustic Doppler Velocity meter (ADV Nortek). The control volume was placed on the symmetry axis near the bottom of the flume. Since the head of the ADV is 5 cm far from the control volume and must be submerged, there is a lack of data when the flow depth falls under 6.5 cm.

Experimental tests were designed to induce in the flow field shock formation and propagation, reverse flow and wetting and drying conditions. To obtain this behaviour, and in the meantime maintain the 1D characteristic of the flow field, an adverse slope was placed in the flume, starting nearly at half length of it.

The examined test cases are summarized in Table 1 and Figure 1.

![Figure 1. Channel geometry and main symbols.](image)
Table I. Test conditions.

<table>
<thead>
<tr>
<th>Test N.</th>
<th>Manning n</th>
<th>$i_1$ (%)</th>
<th>$i_2$ (%)</th>
<th>$x_c$ (m)</th>
<th>$h_1$ (m)</th>
<th>$h_0$ (m)</th>
<th>$x_{obs}$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.01</td>
<td>+1</td>
<td>-9</td>
<td>3.40</td>
<td>0.210</td>
<td>0</td>
<td>1.40, 3.40</td>
</tr>
<tr>
<td>2</td>
<td>0.01</td>
<td>0</td>
<td>-10</td>
<td>3.40</td>
<td>0.250</td>
<td>0</td>
<td>1.40, 2.25, 3.40, 4.50</td>
</tr>
<tr>
<td>3</td>
<td>0.01</td>
<td>0</td>
<td>-10</td>
<td>3.40</td>
<td>0.250</td>
<td>0.045</td>
<td>1.40, 2.25, 3.40, 4.50</td>
</tr>
<tr>
<td>4</td>
<td>0.025</td>
<td>0</td>
<td>-10</td>
<td>3.50</td>
<td>0.292</td>
<td>0</td>
<td>1.40, 2.25, 3.40, 4.50</td>
</tr>
<tr>
<td>5</td>
<td>0.025</td>
<td>+2</td>
<td>-8</td>
<td>3.50</td>
<td>0.250</td>
<td>0</td>
<td>1.40, 2.25, 3.40, 4.50</td>
</tr>
<tr>
<td>6</td>
<td>0.025</td>
<td>0</td>
<td>-10</td>
<td>3.50</td>
<td>0.292</td>
<td>0.050</td>
<td>1.40, 2.25, 3.40, 4.50</td>
</tr>
</tbody>
</table>

Smooth ($n = 0.01$) and rough ($n = 0.025$) conditions were considered; to obtain an almost isotropic roughness, the Plexiglas bottom of the flume was covered with nylon nails at a density of 200/m$^2$.

Tests were carried out both with zero and non-zero tailwater depths; in the wet cases the ratio $h_0/h_1$ was chosen close to 0.176, value that gives the maximum shock height in the Stoker analytical solution.

The main characteristics of the phenomenon in the smooth test cases are as follows: after the opening of the gate, the velocity of the wetting front progressively decreases, and a shock originates slightly downstream the beginning of the adverse slope. The shock wave moves upstream, reflects on the wall and starts to propagate downstream, vanishing nearly in the same section where it was initially formed. In the meantime on the adverse slope a wetting-drying front passes. The whole sequence is repeated but, after the reflection on the upstream wall, the shock is followed by a surface wave train. In the rough test case the behaviour is similar but surface waves already follow the first shock reflection.

**Numerical simulation**

The mathematical model is based on the $1D$ St. Venant equations written in conservation form:

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x} = \mathbf{S}, \quad \mathbf{U} = \begin{pmatrix} A \\ Q \end{pmatrix}, \quad \mathbf{F} = \begin{pmatrix} Q \\ \frac{Q^2}{A} + gI_1 \end{pmatrix}, \quad \mathbf{S} = \begin{pmatrix} 0 \\ gA(S_0 - S_f) + gI_2 \end{pmatrix}, \quad (1)$$

in which $Q =$ discharge, $A =$ wetted area, $x =$ distance along the channel (positive downstream), $t =$ time, $g =$ gravitational constant, $S_0 =$ bottom
slope, \( S_f \) = friction slope calculated according to Manning equation. The terms \( I_1 \) and \( I_2 \) are:

\[
I_1 = \int_0^h (h - \eta) \sigma(x, \eta) d\eta, \quad I_2 = \int_0^h (h - \eta) \frac{\partial \sigma}{\partial x} d\eta
\]

with \( h = \) water depth and \( \sigma(x, \eta) = \) width of the cross-section at height \( \eta \) above the bottom.

Numerical solution of eqn. (1) was based on the well known MacCormack [4] predictor-corrector finite difference scheme:

\[
U_i^n = U_i^n - \tau \left[ (1 - \theta) F_{i+1}^n - (1 - 2\theta) F_i^n - \Theta F_{i-1}^n \right] + \Delta t S_i^n
\]

\[
U_i^c = U_i^n - \tau \left[ \Theta F_{i+1}^p + (1 - 2\theta) F_i^p + (\theta - 1) F_{i-1}^p \right] + \Delta t S_i^p
\]

\[
U_i^{n+1} = \frac{1}{2} \left( U_i^p + U_i^c \right) + \tau \left( d_{i+1/2}^n - d_{i-1/2}^n \right) \quad (\theta = 0, 1),
\]

in which \( i \) and \( n \) are the spatial and temporal grid levels, \( \tau = \Delta t/\Delta x \) and the superscripts ‘\( p \)’ and ‘\( c \)’ denote the variables at predictor and corrector steps, respectively. \( \theta = 0 \) and \( \theta = 1 \) allow to interchange, also cyclically, the order of backward and forward differentiation in the scheme.

In eqns. (3) \( d \) is an artificial dissipation term that must be added to the original form of the MacCormack scheme in order to avoid spurious oscillations and non-physical discontinuities. Two versions of the model have been implemented: in the first \( d \) was calculated according to the classical theory (Jameson [5]), whereas in the second \( d \) is obtained directly from the discretization of the equations following the TVD approach and adopting Van Leer limiter function (Harten [6], Hirsh [7]). The artificial dissipation term \( d \) is enough to avoid the non-physical shock that would otherwise occur in the dam section due to the discontinuous initial conditions.

Eqns. (3) are solved at each time step only where at least one of the three nodes \( i-1, i, i+1 \) is wet. The computational domain then includes two more nodes at each side of the flow field (Fig.2a), (Bellos & Sakkas [8]). No boundary conditions are necessary except when a vertical wall is present at the upstream (and/or downstream) extreme grid point \( k \). In this case a fictitious node inside the solid wall is introduced by extending symmetrically the channel geometry (Fig.2b) and imposing a reflection boundary condition \( (A_{k-1} = A_{k+1}, Q_{k-1} = -Q_{k+1}) \).

Care was taken to properly treat source terms of the equations in order to avoid artificial sources where variations of channel cross-section
and/or bed slope (point \(i-2\) in Fig.2b) occur (Nujic [9], Zhang & Schmid [10]). This was achieved discretizing the spatial derivatives in the source terms in the same manner as \(\partial F/\partial x\) in the *predictor* and *corrector* steps.

![Figure 2. Definition sketch of Dam-Break problem (a) and boundary schematization (b).](image)

Spurious surface gradients at wet/dry boundaries (Fig.2b), that would act as a non-physical driving force in the momentum equation, have also to be avoided (Hervouet & Janin [11]). An horizontal water surface was then forced between \(i-1\) and \(i\) when the water elevation at \(i-1\) falls under the bottom elevation at \(i\); the water volume is consequently redefined. As \(h\to0\) friction becomes very large, causing numerical instability in the solution. To avoid this non-physical effect grid points are included in the computational domain only if calculated depths are greater than a threshold value \(h_c \approx 10^{-3} - 10^{-4} h_i\) (Bellos & Sakkas [8], Bechteler et al. [12]).

**Results and Concluding Remarks**

Figures 3 and 4 show numerical and experimental stage hydrographs for test cases N.2 and N.4 of Table 1, that could be considered representative of the whole set; results for the other tests can be found in Belicchi [13].

Solid lines refer to numerical solution obtained by the version of the model in which artificial dissipation \(\delta\) is introduced according to Jameson formulation. *TVD* version of the model provides almost identical results everywhere except around discontinuities (see details in Figure 3), that are more sharply captured.

Comparison of experimental and numerical results shows a very good agreement, as a whole. Shocks, reverse flows and wetting and drying fronts are well predicted. Of course, surface waves with non zero vertical velocity components cannot be handled by the model based on *1D* St.
Venant equations. Anyway, the numerical solution gives a satisfactory representation of the average depth.

Figure 5 shows calculated and experimental velocities for test case N.3 at sections $x=1.40\ m$ (inside the reservoir) and $x=3.40\ m$ (at the beginning of the adverse slope). Numerical results are very close to experiments, even if the first refer to mean velocity and the second to point velocity near the bottom. This also suggest that velocity distribution across the section is almost uniform, as proved by the observation of the suspended moving particles.

In conclusion, the agreement between experimental and computed results confirms the essential validity of the numerical model, even in situations in which St. Venant hypothesis are not completely verified.

Figure 3. Computed and measured stage hydrographs for test case N.2.
Figure 4. Computed and measured stage hydrographs for test case N.4.

Figure 5. Computed and measured velocities for test case N.3.
References


