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3-D Computed Subsurface Tomography

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Abstract

We demonstrate a method for 3-D subsurface tomography in which crosswell tomographic signals are inverted in a novel way to yield 3-D parameter estimates. The inversion is accomplished via an approximate extended Kalman filter, in which geostatistics are used to implicitly smooth and regularize the inversion. Cluster analysis is used to dynamically determine the parameter dimensionality and structure; the ultimate resolution of heterogeneity is controlled by a cluster tolerance criterion. The size of the domain is incrementally increased as the estimation progresses, which keeps the computational requirements manageable. The method ultimately estimates the number, geometry, values and covariance of parameters. Example applications inverting ground-penetrating radar experiments are shown.

1 Introduction

Characterization of spatially-varying subsurface parameters and states remains as one of the most challenging aspects of subsurface hydrological and geophysical modeling. While sophisticated three-dimensional (3-D) predictive models exist, the accuracy of forecasts by these models is highly dependent on the accuracy of the estimates of underlying parameters. However, the resolution of heterogeneous parameters is typically orders of magnitude lower than that of model states. In typical subsurface hydrological inversion, parameters such as hydraulic conductivity are estimated for a small number of zones (or pilot points) that average the effective conductivity over large spatial volumes whose number and geometry are predetermined based on scanty geologic information. Alternatively, in geophysical acoustic or electromagnetic tomography, high resolution (pixel based) subsurface images are usually obtained for two-dimensional planes that offer limited coverage of the truly 3-D domain. We introduce a new method for full three-dimensional geophysical tomography and suggest ways in which the geohydrologist may take advantage of the high quality and quantity data obtainable by geophysical means.

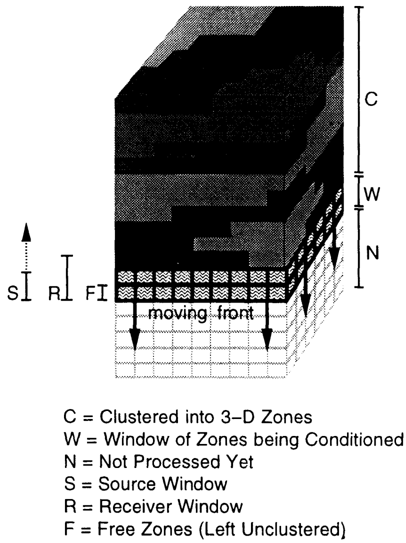

Figure 1: The model is constructed and conditioned incrementally, from top to bottom, in overlapping 3-D subdomains. (From Eppstein[1], with permission.)

2 Three-Dimensional Estimation Method

One-way crosshole traveltime measurements of waves (e.g., ground-penetrating radar or acoustic waves) are inverted to estimate the heterogeneous propagation velocity field in the subsurface. The inversion method is a 3-D extension to the method first introduced for 2-D hydraulic transmissivity estimation in Eppstein & Dougherty[2] and is fully described by Eppstein & Dougherty[4]. Bayesian data conditioning is performed on nodally distributed log slowness estimates (where slowness is the inverse of velocity) within a "sliding conditioning window" using the approximate extended Kalman filter (AEKF). The conditioning window is moved from the top of the domain to the bottom until an estimate for the entire domain has been achieved. Only measurements within or near to this window are used for conditioning. After conditioning, the number of velocity zones is reduced by combining adjacent velocity estimates using 3-D cluster analysis [Eppstein and Dougherty[3]] and random field union [Eppstein and Doughery[2]] in a process we call 3-D data-driven zonation. The estimation process is depicted in Figure 1.

The AEKF is an efficient approximation to the extended Kalman filter [Gelb[5]], which is a recursive, Bayesian, least-squares estimator for optimal (minimum-variance) filtering of time-series measurements. The AEKF procedure used for conditioning each 3-D subdomain is shown by the following pseudo-code:

FOR each source in S

$J = \frac{\partial \tau_R}{\partial ln(v^{-1})}$

$cov(\tau_R, \ln(v^{-1})) = J cov(\ln(v^{-1}))$

$cov(\tau_R) = cov(\tau_R, \ln(v^{-1}))J^T + Q_R$

$K_{\ln(v^{-1})} = cov(\tau_R, \ln(v^{-1}))^T(M_R + cov(\tau_R))^{-1}$

$\ln(v^{-1}) = \ln(v^{-1}) + K_{\ln(v^{-1})}(z_R - \tau_R)$

$cov(\ln(v^{-1})) = cov(\ln(v^{-1})) - K_{\ln(v^{-1})} cov(\tau_R, \ln(v^{-1}))$

END FOR

where

C = the previously conditioned portion of the domain
W = the sliding conditioning window just below C
S = the source window
R = the receiver window
v = GPR *velocities* for all zones in C and W
$\ln(v^{-1}) = \ln(slowness)$ for all zones in C and W
τ_R = estimated first arrival traveltimes in R
J = the Jacobian (sensitivity) matrix
Q_R = covariance matrix of system noise in R
M_R = covariance matrix of measurement error in R
z_R = measured traveltimes in R
$K_{\ln(v^{-1})}$ = the Kalman gain matrix for $ln(slowness)$
cov stands for covariance matrix

The method assumes that log slowness is normally distributed, as are measurement errors and system noise. We assume homogeneous initial estimates and a negative exponential correlation between log slowness values at nodes in each horizontal plane. The parameter covariance implicitly regularizes the system, interpolates, and smoothes the estimates. The user provides initial estimates of mean log slowness, variance of log slowness, and horizontal correlation length. Parameter structure is initially assumed to be fully distributed over all nodes in some discretization mesh; these nodes are merged into larger zones, as warranted by the current estimates, as the procedure progresses. Conditional estimates of the number and geometry of parameter zones, as well as the value and covariance of these zones, are evolved.

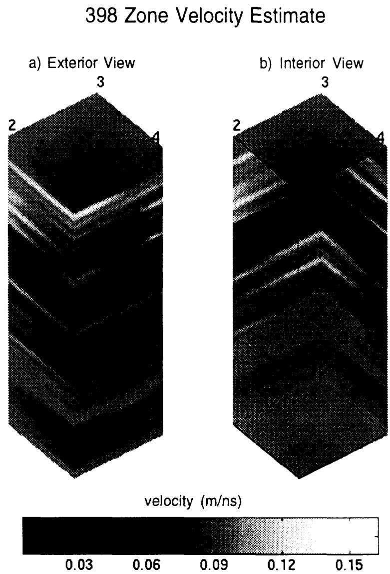

Figure 2: The six outside faces of the velocity estimate for the 3-D domain from depths $1.5m$ to $18m$ are shown. The location of the 4 geowells (numbered 1 through 4) and the the injection well (5) are shown. This estimate was produced using 138 crosswell measurements from well 1 to 2, well 1 to 3, well 4 to 1, and well 4 to 2, all taken 15 days prior to the injection of the salt water. A cluster tolerance of $0.003m/ns$ reduced the original 7260 zones to 398 in the final estimate. (From Eppstein[1], with permission.)

3 Sample Application of the Method

The method was applied to an incomplete data set of crosshole ground penetrating radar (GPR) first arrival traveltimes collected in February, 1997, as part of a pilot project by Applied Research Associates, Inc. to demonstrate the effectiveness of geowells in GPR and electrical resistance data collection. Four geowells were pushed into the corners of an almost level rectangular area ($7m \times 7.3m \times 18m$ deep) into unsaturated soils in terrace deposits above the White River near South Royalton, Vermont.

We used 138 measurements (each the average of 64 stacked transmissions) collected on February 3^{rd} to estimate velocities in three dimensions (Figure 2). The estimated velocity field reduces traveltime prediction error by 47%, in comparison to the initial homogeneous estimate. The structure estimate comprises subhorizontally stratified layers of velocities in ranges indicative of sands, silts, and clays that are consistent with independent cone penetrometer data collected at the four corner geowells. A complete description of the application and results can be found in Eppstein & Dougherty[4].

We also used 182 measurements collected on February 18^{th}, one day after the injection of 125 gallons of salt water roughly $4m$ deep near the center of the domain. A 4-D estimate was obtained by subtracting the 3-D velocity estimate post-injection from that obtained pre-injection. Velocity differences are interpreted as changes in the dielectric caused by changes in soil moisture; high velocity differences are presumed to imply that the soil became wetter. The results indicate that the location of the salt water "plume" was discontiguous and was highly influenced by small scale soil heterogeneity; wetter areas occured in regions which had high velocities (presumably indicative of dry sand) before the salt water was injected (Figure 3).

The number of zones that are estimated depends, in part, on the user-controlled "cluster tolerance criterion", which determines the required similarity of estimates of adjacent zones for them to be merged together into larger zones. The higher the cluster tolerance, the fewer the resulting number of zones and the higher the final prediction error (Figure 4). However, the overall geometry of the resulting parameter structure estimate remains consistent over a wide range of cluster tolerance criteria (Figure 5). Three-dimensional random fields may therefore be upscaled in a meaningful way by applying data-driven zonation in a user controlled fashion. For example, high-quality geo-

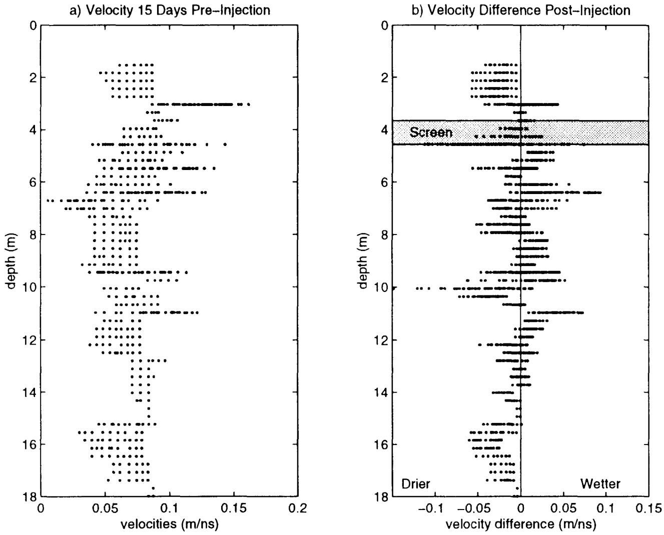

Figure 3: a) Pre-injection velocity estimates for all zones in the entire 3-D domain are shown as a function of depth; the number of dots per depth level is thus indicative of the heterogeneity at that level. b) Post-injection velocity differences for all zones in the 3-D domain are shown as a function of depth. (From Eppstein[1], with permission.)

physical measurements may be used to create high resolution tomographic estimates of parameter structure and correlation, which are then upscaled to a smaller number of geologically meaningful zones for which subsequent hydrologic data inversion may be feasible.

4 Conclusions

We have demonstrated a new method of three-dimensional tomographic inversion of crosswell traveltime data which is computationally feasible even on large 3-D domains. The Bayesian method uses parameter and measurement error covariances to regularize and interpolate the estimate and is capable of handling sparse and non-uniform data sets. Estimates of parameter number, geometry, value and covariance are evolved.

The method has several potential applications to subsurface hydrology, including moisture monitoring in the vadose zone, imaging the cone of depression during pumping, and estimating parameter

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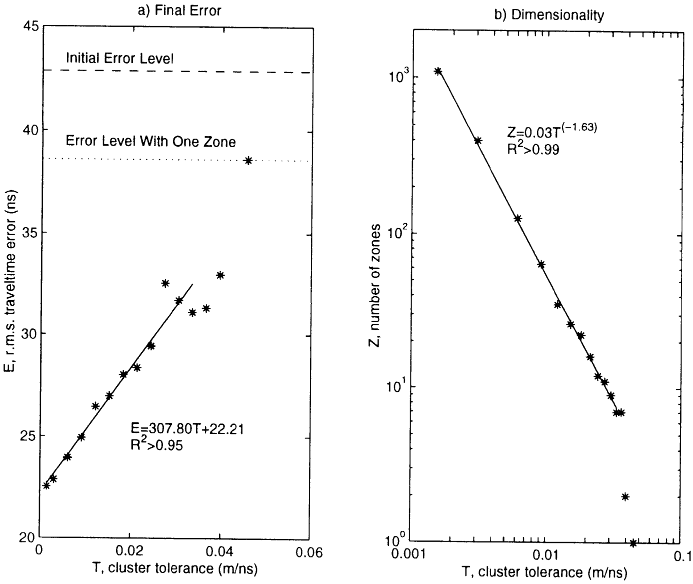

Figure 4: As cluster tolerance is increased, residual traveltime error after conditioning increases linearly (a), while the number of zones decreases with the -1.63 power of the cluster tolerance (b), until the tolerance becomes so high that the entire 3-D domain is clustered into 1 or 2 zones. (From Eppstein[1], with permission.)

structure and correlation for subsequent hydrological inversion. The 3-D data-driven zonation procedure provides a means for systematic upscaling of small-scale heterogeneities to larger scales of the user's choice.

Acknowledgements

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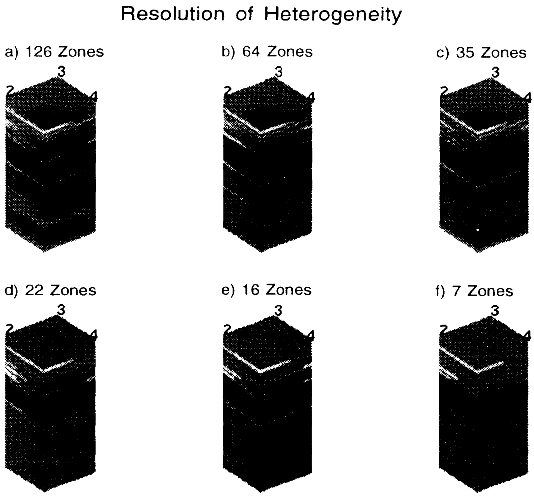

Figure 5: Exterior views of selected pre-injection velocity estimates for various cluster tolerance levels. Compare to the 398 zone estimate shown in Figure 2. (From Eppstein[1], with permission.)

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