

Mixing regimes and the scale up problem for multiphase flow in porous media

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Abstract

We examine fluid mixing dynamics for two-phase, immiscible flow in heterogeneous porous media. To assess the combined effect of nonlinear flow instability and heterogeneity, we solve the model problem numerically, using large suites of realizations of stochastically generated permeability fields. Different mixing regimes are identified, depending on the relative importance of nonlinearity in the flow equations and heterogeneity.

1 Introduction

Even the fine scale spatial variability of permeability can have a significant effect on a variety of flow processes in heterogeneous porous media. Because of this, highly detailed geostatistical realizations of rock properties are generated to be used as input to fluid flow simulation models. If this detailed geological information is included, the computational problem of integration of the flow equations may become prohibitively intensive. Thus some type of scale up or coarsening procedure is needed to assign suitable values of rock properties, mainly permeability, and other flow functions on a coarser scale suitable for fluid flow computations.

The scaling up is in general difficult. Difficulties arise both from the highly nonlinear fluid-fluid interactions in the flow of multiphase fluids in a heterogeneous medium and from the complex interactions between heterogeneities and nonlinearities. Indeed, preliminary results [9] indicate that several different flow regimes occur, depending

on the relative importance of flow nonlinearity and medium heterogeneity, as well as on the spatial structure of such heterogeneity. (See also [1, 10].)

Despite this, most approaches for scaling up multiphase flow systems operate within a fixed framework, based on the renormalization or ‘pseudoization’ of the fine scale relative permeability curves to account for the effects of heterogeneities [2, 4, 7]. All such approaches entail a hyperbolic renormalization of the fluid transport equations in which only the first order (hyperbolic) transport effects are modified. Thus it can be argued that this framework might be too limited to capture the multitude of multiphase, heterogeneous flow behavior.

As others have noted [10, 13], the form of the renormalized fluid transport equation should depend on the flow regime, convective, dispersive or more complex. We support this view and claim that an improved scientific understanding of the interplay between heterogeneity and nonlinearity is required for the formulation of proper mathematical models for a large-scale description of multiphase flow in heterogeneous media.

This paper is part of our effort to characterize the fluid mixing regimes in multiphase, immiscible, heterogeneous flow. We believe such a characterization is an essential step in the scientific program mentioned above. In section 2 we introduce the nonlinear model used to describe two-phase, immiscible flow in heterogeneous porous media together with a brief review of the numerical method that we employ. In section 3 we discuss a proper way to characterize fluid mixing regimes. Section 4 presents some numerical results.

2 Model Equations and Numerical Method

The global pressure p and total velocity $\mathbf{u}$ formulation of the incompressible flow of two immiscible fluids (which we shall think of as being oil and water) in a heterogeneous porous medium, in the absence of gravity, sources and sinks, is given by the following equations [3, 5]:

$$\phi \frac{\partial s}{\partial t} + \nabla \cdot (\lambda_w \mathbf{u} + k \lambda \lambda_w \lambda_o p'_c \nabla s) = 0, \quad (1)$$

$$\nabla \cdot \mathbf{u} = 0, \quad (2)$$

$$\mathbf{u} = -k \lambda \nabla p. \quad (3)$$

Here, the subscripts o and w refer to oil and water, respectively; $s(\mathbf{x}, t)$ is the water saturation; ϕ is the porosity; $k(\mathbf{x})$ is the absolute

permeability; and $p_c(s) = p_o - p_w$ is the capillary pressure. The global pressure is defined by

$$p = \frac{1}{2} \left\{ p_o + p_w + \int_0^{p_c} \left[\lambda_o(p_c^{-1}(\zeta)) - \lambda_w(p_c^{-1}(\zeta)) \right] d\zeta \right\},$$

with $\lambda(s) = k_{ro}(s)/\mu_o + k_{rw}(s)/\mu_w$ and $\lambda_i(s) = k_{ri}(s)/\mu_i \lambda(s)$, $i = o, w$, denoting the total mobility of the two-phase fluid and the fractional flow functions of the two phases, respectively; further, $k_{ri}(s)$ is the relative permeability and μ_i the viscosity, each with respect to the i^{th} fluid.

Define

$$\mathbf{v} = k \lambda \lambda_w \lambda_o p'_c \nabla s. \quad (4)$$

We shall be solving Eqs. (1)–(3) in a two-dimensional domain $\Omega = (0, X) \times (0, Y)$, subject to the boundary conditions

$$\begin{aligned} \mathbf{u} \cdot \mathbf{n} &= q, & \mathbf{v} \cdot \mathbf{n} &= (1 - \lambda_w)q & \text{on } x &= 0, \\ p &= 0, & \mathbf{v} \cdot \mathbf{n} &= 0 & \text{on } x &= X, \\ \mathbf{u} \cdot \mathbf{n} &= 0, & \mathbf{v} \cdot \mathbf{n} &= 0 & \text{on } y &= 0, Y, \end{aligned} \quad (5)$$

where $\mathbf{n}$ is the outward-pointing normal vector to $\partial\Omega$, and the initial condition

$$s(\mathbf{x}, 0) = s_0(\mathbf{x}). \quad (6)$$

The boundary conditions (5) result from assuming that only water is being injected (at a constant rate $-q$) through the left vertical boundary ($x = 0$) of Ω , that on the right vertical boundary ($x = X$) the total flow splits according to mobilities between water and oil, and that the horizontal boundaries ($y = 0, Y$) of Ω are impermeable.

A detailed description of the numerical method that we employ for the solution of Eqs. (1), (2), and (3) is given in [5]. This method has proved to be computationally efficient in producing accurate numerical solutions for transport-dominated diffusion processes such as those generated by nonlinear flows in porous media. The method utilizes a recently developed conservative version of the modified method of characteristics (MMOC) [6]) for the time discretization of the saturation equation (1). This technique, introduced in [5] where it was called the modified method of characteristics with adjusted advection (MMOCAA), inherits the computational efficiency of the standard MMOC and conserves mass (which is not the case with the standard MMOC). It is an operator-split version of the MMOC altered to conserve mass. The operator splitting separates the effect of transport from diffusion in the saturation calculation. This allows the use of distinct time steps for the transport and diffusion.

A second operator splitting separates the calculation of the saturation from the calculation of the global pressure (Eqs. (2) and (3)). For the latter, we use a hybridized mixed finite element discretization equivalent to cell-centered finite differences, which effectively treats the rapidly changing permeabilities that arise from stochastic geology and produces accurate velocity fields.

Important features of the simulation code described in [5] include mass conservation of fluid phases, absence of grid orientation bias, reduced numerical diffusion, and the possibility of taking large time steps for pressure and diffusion calculations.

3 Characterization of Mixing Regimes

Both nonlinearity of the flow equations and heterogeneity cause macroscopic mixing of the fluid transport in heterogeneous porous media. In certain parameter regimes, the nonlinear coupling of the transport equation (1) to Darcy's law (3) causes the unstable growth of small fluctuations in saturation fronts into highly fingered flow patterns (viscous fingering; see, e.g., [12]). Heterogeneities in permeability cause velocity variations which in turn introduce dispersion into the flow and the development of high flux flow channels.

To understand the combined effect of nonlinearity and heterogeneity on the mixing process, we analyze the scaling behavior of the mixing region as a function of travel distance. The mixing region scaling exponent is the slope in a log-log plot of these variables. A scaling exponent equal to 0.5 corresponds to the large time asymptotic behavior of classical (Fickian) diffusion. A scaling exponent equal to 1.0 corresponds to a 'convective' mixing regime, typical of viscous fingering driven by the nonlinear instability of the flow [8].

Our study of the mixing process proceeds with the numerical solution of the model problem (1)–(6), using suites of realizations of the permeability field. We generate these realizations using statistical ensembles having lognormally distributed values of $k(\mathbf{x})$ with prescribed spatial correlation structure. The saturation s is then averaged with respect to the ensemble as well as in the direction transverse to the mean velocity (the x -direction) to define $\bar{s}(x, t)$. Next, S_H , the saturation solution of the flow equations (1)–(6) with a constant permeability $\bar{k} \equiv \langle k(\mathbf{x}) \rangle$ (the angle brackets $\langle \cdot \rangle$ denote ensemble averaging), is computed and used to define the mixing length (c.f. Fig. (1)):

$$\ell(t) = \frac{1}{2} \int_0^X |\bar{s}(x, t) - S_H(x, t)| dx. \quad (7)$$

Note that (7) is essentially the volume of water displaced to the region ahead of the sharp front defined by the solution S_H by the macroscopic fluid mixing (fingering).

A detailed discussion of this procedure for the analysis of fluid mixing dynamics in multiphase flow is given in [8], where it was validated by comparison to previously obtained results [11] from perturbation theory and carefully resolved front-tracking calculations for the problem of linear flow (tracer flow) in heterogeneous porous media.

4 A Numerical Study of Mixing

The present study focuses on the role of heterogeneity in determining fluid mixing regimes. Thus all fluid parameters are kept fixed: viscosity ratio $M \equiv \mu_o/\mu_w = 5$; initial water saturation $s_0(\mathbf{x}) = 0.21$; injection rate $-q =$ one pore-volume every five years. The (fixed) capillary strength is chosen so that the flow is in the viscous dominated regime, with negligible capillary pressure effects, and quadratic relative permeability functions are used. For our choice of parameters the flow is weakly unstable to viscous fingering.

The porosity is also kept fixed, $\phi = 0.2$. The dimensions of the computational region are $X = 512$ meters, $Y = 128$ meters and the computational grid size is 512×128 ; the grid for the specification of the heterogeneous permeability has size 256×64 .

In the simulations presented here two sets of stochastically generated permeability fields were considered. In each set the fields are isotropic, spatially uncorrelated (with respect to the heterogeneity grid) and lognormally distributed. One of the sets involves weakly heterogeneous permeability fields with coefficient of variation, i.e., the ratio of the permeability standard deviation to its mean, $C_V = 0.54$. The other set involves more heterogeneous permeability fields with $C_V = 2.93$.

Figure 1 shows log-log plots of the mixing length ℓ as a function of travel distance for the two cases of heterogeneity considered, $C_V = 0.54$ (top) and $C_V = 2.93$ (bottom). Also plotted are two straight lines with vertical positions fixed to agree with the last plotted points on the mixing length curves. The slopes of these straight lines are 0.5, the scaling exponent characteristic of the large time asymptotic behavior of Fickian diffusion, and 1, the asymptotic scaling exponent of the fluid mixing driven solely by viscous fingering (homogeneous porous media) [8].

In the weakly heterogeneous case ($C_V = 0.54$), after a relatively

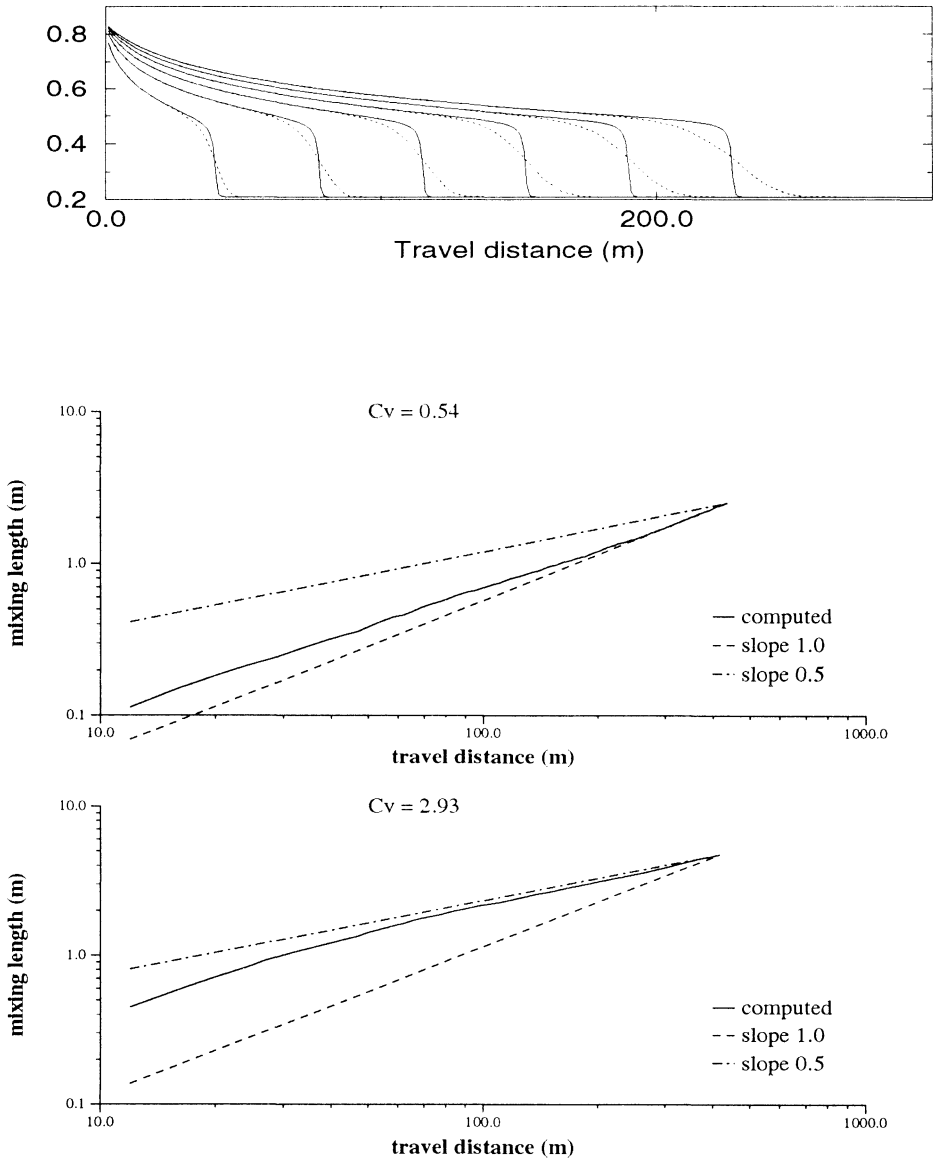


Figure 1: A. $\bar{s}$ profiles (dashed curves) at representative times of the fluid flow runs. Superimposed upon these are the S_H profiles (solid curves) for the corresponding homogeneous run at the same times. B. Log-log plot of mixing length, ℓ , as a function of travel distance for increasing values (top to bottom) of the heterogeneity strength ($C_V = 0.54, 2.93$).

short transient regime (of about 250 meters) the mixing length curve coincides with the straight line with slope 1, indicating that in this case the heterogeneity induced mixing region grows as a linear function of travel distance (or, equivalently, of time). In this case, the effects of heterogeneity can be captured by a proper modification of λ_w in Eq. (1), i.e., by a hyperbolic renormalization of the transport equation. The traditional way of doing this relies on the definition of pseudo relative permeabilities (see, e.g., [2, 4, 7] and the references cited therein).

As the strength of the heterogeneity is increased ($C_V = 2.93$), though, a different mixing regime is observed with the mixing region growth rate exponent approximately equal to 0.5, suggesting that in this case a Fickian, or dispersive, renormalization might be more appropriate (see also [10]).

5 Conclusions

Highly resolved numerical simulations were conducted to study the fluid mixing dynamics for two-phase, immiscible flow in heterogeneous porous media. Conceptually, we regard the entire computational domain as a single coarse-grid cell in some larger, highly detailed simulation problem. We performed a parameter study varying the strength of the heterogeneities relative to the strength of the non-linearity (fluid viscosity ratio) of the flow equations.

For weak heterogeneities the mixing process is 'convective', with the mixing region growing linearly as a function of time. This behavior is typical of mixing processes driven by the nonlinear instability of the flow and should be properly modeled with a hyperbolic renormalization of the flow equations. However, for sufficiently strong heterogeneities with short correlation length, the mixing process is Fickian and a dispersive renormalization, like the one developed recently for linear flow [11], should perhaps be more appropriate.

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