Design of partially prestressed concrete beam-column connection with singly prestressed sections under cyclic loadings

Ir.B. Budiono
Civil Engineering Department, Institut Teknologi Bandung, Jl. Ganesha 10, Bandung 40132, Indonesia

Abstract

Prestressed Concrete framed structure are commonly designed to provide prestressing steel to resist gravity loads. Therefore most of the prestress tendons are located at the top of the beam at beam-column connections. Experimental and numerical non-linear finite element studies developed by the author show that these types of structures are capable to resist severe cyclic loading provided sufficient care is given. Special attention must be considered in the limitation of ductility demand, partial prestressing ratio and detailings in the potential plastic hinge regions. In this paper a simple and semi-rational design method for partially-prestressed interior beam-column connection in proposed. As the behavior of connections are non-flexural, the design method developed is based on major and minor strut modeling using plasticity theory.

1 Introduction

ACI 318M-89 requires that beam-column joints of frames in designated earthquake regions must the designed for seismic conditions. The code specifies that the shear strength of the beam-column connections are to be designed for inelastic response to severe seismic loadings with flexural plastic hinges to be developed in the beams adjacent to the column faces. Flexural actions in the beams generate joint shear forces. In addition, the joint forces are multiplied by an overstrength factor (α) to allow for the higher mean yield strength of the flexural reinforcement than the specified characteristic strength, fy. ACI 318M-89 gives α is equal to 1.25. The requirements in the code indicate that the joint shear strength is a function of only the shear strength of the concrete. Thus, only minimum transverse reinforcement is required in the joint.

In generating shear forces acting on the beam-column connections, the New Zealand code of practice (NZS 3101-82) uses a similar approach to
that of ACI 318M-89. Flexural actions of the plastic hinges of the beams develop the horizontal shear forces in the joint, while the vertical shear forces are generated by flexural actions of the upper and lower columns of the joint. However, the shear resisting mechanisms specified by NZS 3101-82 are different than those developed in ACI 318M-89. Two major shear resisting mechanisms are considered; they are: (i) diagonal concrete strut and, (ii) panel truss actions. The diagonal strut action resists the forces resulting from the compressive and shear forces in the beam and the column. The shear forces are transmitted by the flexural reinforcement to the joint via bond. These bond forces produce diagonal tension force in the joint which are resisted by vertical and horizontal reinforcement to gave equilibrium in an equivalent panel truss action. For a low axial compressive force in the column (representing the worst condition in the beam-column connection under severe seismic actions) the portion of the joint shear forces resisted by the panel truss action is assumed to be no more than 50 % of the total shear force acting in the joint, as recommended by Paulay et al. (1978).

In this paper, a simple and semi-rational design method for partially-prestressed interior beam-column connection is proposed. As the behaviour of connections are non-flexural (ACI 318 RM-89 Commentary, Section R12.6.3), the design method developed is based on major and minor strut modeling using plasticity theory.

2 Forces Acting on Beam-Column Connections

Under large lateral loads the forces acting on an internal joint using a truss model are illustrated in Figure 1.

The diagonal compression force in the joint, Dj (as shown in Figure 1), is calculated from equilibrium. At point A (as shown in Figure 1), diagonal compressive force of the left beam and the upper column of the joint, Dv1 and Dv3, respectively, act on the joint with an angle θ. If the angle θ is taken as 45°, then the horizontal and vertical component of Dv1 are equal to Vj, and V1 can be taken as shear force of the left beam. A similar expression can be derived for Dv3 that yields V3 which is equal to upper column shear force. Therefore the horizontal and vertical joint shears, Vjh and Vjv, respectively, are

\[
\begin{align*}
V_{jh} &= C_1 + T_2 + V_1 - V_3 \\
V_{jv} &= C_3 + T_4 + V_3 - V_1
\end{align*}
\]

For the horizontal shear in the joint (Vjh), C1 combined with V1 form the flexural compressive force at the top of the left beam, T2 is tension force in the top of the joint, V1 and V3 are the shear forces acting on the left beam and the upper column of the joint, respectively. A similar derivation can be obtained for the vertical shear in the joint (Vjv).
Horizontal equilibrium of the left beam and vertical equilibrium of the upper column (shown in Figure 1) give

\[ T_1 = C_1 + V_1 \]
\[ T_3 = C_3 + V_3 \]  \hspace{1cm} \text{(A.2)}

Substituting Eq. A.2 into Eq. A.1 yields

\[ V_{jh} = T_1 + T_2 - V_3 \]
\[ V_{jv} = T_3 + T_4 - V_1 \]  \hspace{1cm} \text{(A.3)}

The tension forces \( T_i \) (\( i=1,2 \)) calculated when the beams reach their ultimate moments are given by

\[ T_i = \alpha \left[ A_{si} f_{yi} + A_{pi} f_{pui} \right] \quad (i = 1, 2) \]  \hspace{1cm} \text{(A.4)}

where \( A_{si} \) is the reinforcement area; \( A_{pi} \) is the area of the prestressing steel; \( f_{yi} \) is yield stress of the reinforcement; \( f_{pui} \) the ultimate stress of the prestressing steel; and \( \alpha \) is an overstrength factor.
As the column is proportional to be flexurally stronger than the beams, the elastic vertical shear of the joint \( V_{jv} \) becomes difficult to assess when plastic hinges develop in the beams adjacent to the column faces. However, the worst case occurs when the axial force acting on the column is small. In this case, \( V_{jv} \) can be taken as

\[
V_{jv} = V_{jv} \tan \beta \tag{A.5}
\]

\( V_{jh} \) and \( V_{jv} \) are the horizontal and vertical components of the diagonal force in the concrete compressive strut \( (D_j) \) in the joint as shown in Figure 2b, and

\[
D_j = \frac{V_{jh}}{\cos \beta} \tag{A.6}
\]

where the angle \( \beta \), for all practical purposes, can be taken as

\[
\beta = \tan^{-1} \left( \frac{h_b}{h_c} \right) \tag{A.7}
\]

where \( h_b \) and \( h_c \) are the total depth of the beam and the column sections, respectively.

3 Shear Resisting Mechanisms of Beam-Column Connections

Shear across a joint core is transferred by major and minor diagonal concrete strut actions. The major strut action is shown in Figure 2a. The major diagonal strut is capable of transmitting a significant fraction of shear forces across the joint. If all of the shear forces resisted by the major strut \( (D_c) \), then \( D_c \) is equal to \( D_j \), as given by Eq. A.6.

If the shear forces acting on the joint is greater than the major strut capacity then the excess forces \( (D_s) \) are transmitted by the minor strut. In this case, vertical and horizontal hanger reinforcement are required to carry the diagonal tension forces induced by bond forces. The bond forces are developed from the flexural reinforcement of the beams transferred across the joint core. The minor strut action is shown in Figure 2b.

In the general formulation, the shear resisting mechanism of a beam-column connection is given by

\[
D_j = D_c + D_s \tag{A.8}
\]

where \( D_c \) and \( D_s \) are the forces carried by the major and minor struts, respectively. Equilibrium in the horizontal and vertical directions are given by:

\[
\begin{align*}
V_{jh} &= V_{ch} + V_{sh} \tag{A.9} \\
V_{jv} &= V_{cv} + V_{sv}
\end{align*}
\]
where $V_c$ and $V_s$ are the components of the forces in the major and minor struts, respectively; the subscripts $h$ and $v$ are representing the horizontal and vertical directions, respectively.

![Diagram of Plastic Truss Model of An Interior Beam-Column Connection](image)

**Figure 2. Plastic Truss Model of An Interior Beam-Column Connection**

### 3.1 Major Diagonal Compressive Strut ($D_c$)

A significant portion of the shear forces acting on the joint are resisted by a diagonal compression strut. As the concrete does not behave as a perfectly plastic material, particularly when higher strength concrete is used, the effective concrete strength is lower than the characteristic compressive strength, $f_{c'}$. Some parts of the failure surface may be softening before other parts have mobilized their full strength (Foster, 1992). In addition, Vecchio and Collin (1986) showed that large transverse strains reduce the peak compressive strength. To accommodate these reductions of the peak strength, the effective concrete strength is given by

$$f_c^* = v f_c'$$

where $v$ is an efficiency factor ($v \leq 1$). Warwick and Foster (1993) proposed an efficiency factor for concrete with characteristic strength ranging from 20 to 100 MPa. Noting that the shear span to effective depth ratio ($a/d) = \tan \beta$, the Warwick and Foster relationship can be written in the form

$$v = \begin{cases} 1.25 - \frac{f_c'}{500} & \text{for } \tan \beta < 2 \\ 0.72 \tan \beta + 0.18 \tan^2 \beta & \text{for } \tan \beta \geq 2 \end{cases}$$

The maximum forces that can be taken by the compression strut is

$$D_c = \phi_c f_c^* d_c b$$
where $\phi_c$ is the material strength reduction factor for compressive failure, $f_c^*$ is the maximum average compressive stress in the compressive strut at failure (given by Eq. A.10), $d_c$ is the depth of the compression strut, and $b$ is the width of the joint. Foster (1994) recommended that the value of $\phi_c$ be taken as 0.6.

In order to obtain an estimate for $d_c$, a finite element experiment on a reinforced concrete beam-column connection was undertaken. The reinforcement of the beams and the column was proportioned to produce compression failure in the joint.

The idealized diagonal compression strut, together with the bearing support in each corner of the joint, is illustrated in Figure 3. The horizontal and vertical dimensions of the bearing supports are given by

$$
\begin{align*}
W_h &= d_c \cos \beta \\
W_v &= d_c \sin \beta
\end{align*}
$$

(A.13)

where $W_h$ and $W_v$ are the horizontal and vertical bearing areas of the joint, respectively, and $d_c$ is the width of the strut.

\[ \text{Figure 3. Bearing Support and the Width of the Major Diagonal Strut} \]

Based on the parameters discussed earlier, it is found that the width of horizontal bearing area is approximately 0.5 $h_b$. The equations for the horizontal and vertical components of the major strut are
where $V_{ch}$ and $V_{cv}$ are the horizontal and vertical components of the major strut, respectively; $\phi_c$ is the strength reduction factor for compression failure; $f_c^*$ is determined using Eq. A.10; $h_b$ and $h_c$ are the overall depths of the beam and column, respectively; and $b$ is the width of the joint.

### 3.2 Minor Compressive Struts

If the capacity of the major strut is exceeded then minor struts develop to balance the excess shear flow on the boundary of the joint. This shear flow is generated by bond stresses transferred from the flexural reinforcement of the beams across the joint zone (shown in Figure 2b). To maintain equilibrium, the forces in the minor compression struts must be balanced by additional reinforcement in the horizontal and vertical directions through the joint. The shear forces resisted by minor struts in the horizontal and vertical directions are

\[
V_{sh} = V_{jh} - V_{ch} \tag{A.15}
\]

\[
V_{sv} = V_{jv} - V_{cv} \tag{A.16}
\]

For practical purpose, $V_{sv}$ is given by

\[
V_{sv} = V_{sh} \tan \beta \tag{A.16}
\]

When both horizontal and vertical reinforcement are provided, the areas of the reinforcement required

\[
A_{sh} = \frac{1}{\phi_s} \frac{V_{sh}}{f_{jy}} \tag{A.17}
\]

\[
A_{sv} = A_{sh} \tan \beta \tag{A.17}
\]

where $A_{sh}$ and $A_{sv}$ are the areas of horizontal and vertical reinforcement in the joint, respectively, $\phi_s$ is the strength reduction factor for tension failure; $f_{jy}$ is the yield strength of the joint reinforcement.

$\phi_s$ can be taken equal to 0.7 as recommended by Foster (1992), and vertical joint reinforcement may be provided by the column reinforcement distributed around the perimeter of the column.

### 3.3 Diagonal Tension Failure

The finite element result, showed that the joints behaved elastically under the cyclic tests with non-uniform stress contours in the joint.
The non-uniform stress distribution in the strut develops bursting forces, $T_b$, and leads to the possibility of a diagonal splitting failure, as shown in Figure 4. It may be observed in Figure 4 that the bursting force, $T_b$ is given by

$$T_b = \frac{n}{4m} D_c$$  \hspace{1cm} (A.18)

Finite element experiments (Budiono, 1995) gave $n$ approximately equal to 0.5 m. Taking $n = 0.5$ m and substituting into Eq. A.18 gives

$$T_b = 0.125 D$$

where

$$D_c = \frac{V_{ch}}{\cos \beta}$$  \hspace{1cm} (A.19)

In addition, finite element studies undertaken by Warwick and Foster (1993) suggest that the percentage of web reinforcement in the vertical and horizontal directions should not be less than 0.2 % when $f_{c'} \leq 50$ MPa, and 0.4 % when $f_{c'} > 50$ MPa.

To allow the full strength of the compression strut to be develop, diagonal splitting failure should be avoided. Vertical and horizontal reinforcement may be added to the joint to increase the capacity of the joint against the diagonal splitting mode of failure.

$$A_{wh} = \frac{1}{\phi_s} \frac{T_b \sin \beta}{f_{jy}}$$

$$A_{wv} = \frac{1}{\phi_s} \frac{T_b \sin \beta}{f_{jy}} = A_{wh} / \tan \beta$$  \hspace{1cm} (A.20)

This web reinforcement required is added to the shear reinforcement required by Eq. A.17.

![Figure 4](image_url)  \hspace{1cm} Figure 4 Result Forces in Compression Strut Due To the Changing Stress Trajectories in the Web
3.4 Provisions for Ductile Beam-Column Connections

(a) Partial Prestressing Ratio and Ductility Factor

In order to obtain a suitably ductile response in a partially-prestressed concrete beam-column connection, the partial prestressing ratio (PPR) and the ductility factor should satisfy the limitations given as follow (Budiono, 1995):

(i) For PPR less than 0.2 the displacement ductility factor ($\mu$) of 4.0 should be applied.

(ii) For $0.2 \leq PPR < 0.6$ limited ductility approach should be considered ($\mu = 2.0$), while for $PPR \geq 0.6$ no inelastic ductility is permitted.

(b) Plastic Hinge Region

At failure, finite element analysis showed that the concrete in the hinge zones was crushing and the effectiveness of the prestress was greatly reduced. To ensure that shear failure is prevented during the flexural hinge formation, it is recommended that in the hinge zone the contribution of concrete in resisting the factored design shear force is to be neglected.

(c) Strong Column Weak Beam

To ensure that column is stronger than the beam during the formation of the beam plastic hinges, it is recommended that in the vicinity of the joint, the flexural strength of each column should be at least greater than 1.5 times the maximum beam flexural strength.

(d) Bond Stress in the Joint

To ensure the breakdown of bond between the longitudinal reinforcement across the joint and the concrete is prevented, the column depth ($h_c$) should be taken larger than 20 times the largest diameter of the longitudinal reinforcement of the beams passing through the joint, as recommended by ACI-ASCE Committee 352 (1985).

4 Summary and Conclusion

A design for detailing joints partially-prestressed, beam-column connection is proposed. The design considers the strength of joint when plastic hinges develop in the beams adjacent to the column faces. Two mechanisms in resisting shear force in the joint are considered; that is the shear strength contributed by the major and minor compressive struts.
Horizontal and vertical joint reinforcement are required to carry tension forces generated by bursting action of the major struts and for hanger reinforcement to carry tension force derived from minor strut action.

In the beam hinge regions, the contribution of concrete to the shear strength is neglected, as part of the concrete in these regions become ineffective due to crushing. This allows for the formation of ductile hinges by flexural yielding of the longitudinal reinforcement in the beams. To ensure that the columns are stronger than the beams and that the breakdown of the bond stress in the joint is prevented, the special provisions for column strength recommended by ACI-ASCE Committee 352 (1985) are maintained.

REFERENCES

2) ACI 318-89 (Revised 1992) and Commentary-ACI 318 R-89 (Revised 1992), American Concrete Institute, Detroit, Michigan.
3) Budiono, B., Hysteretic Behaviour of Partially-Prestressed Concrete Beam - Column Connections, Ph.D Thesis Department of Structural Engineering, School of Civil Engineering, University of New South Wales, 1995.