Buckling of circumferentially corrugated cylinders under uniform external pressure

C.T.F. Ross,\textsuperscript{a} G. Apor,\textsuperscript{b} S.P. Claridge\textsuperscript{a}

\textsuperscript{a}Department of Mechanical and Manufacturing Engineering, University of Portsmouth, Portsmouth, Hampshire PO1 3DJ, UK
\textsuperscript{b}Xavier University, Cagayan de Oro City, Philippines

Abstract

The paper describes a theoretical and an experimental investigation into the buckling of three circumferentially corrugated circular cylinders under uniform external pressure. The experimental results showed that the vessels buckled through general instability.

The theoretical analysis was of a semi-empirical nature, which involved the finite element method and an elastic knockdown factor to cater for inelastic instability.

1 Introduction

The pressure hulls of submarines and the legs of off-shore drilling rigs are often constructed from thin-walled circular cylinders. Now under external water pressure, a thin-walled circular cylinder can buckle at a small fraction of the pressure to cause axisymmetric yield. This mode of failure is known as shell instability, and it is very structurally inefficient. One method of improving the structural efficiency of these vessels is to stiffen them with circumferential ring-stiffeners, as shown in Figure 1.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{Ring-stiffened circular cylinder}
\end{figure}
In Figure 1, it can be seen that the ring-stiffened pressure hull is surrounded by a hydrodynamic hull, where the latter has water pressure on both its internal and external surfaces. Thus, as the hydro-dynamic hull is in a state of hydrostatic stress, it is unlikely to fail due to the effects of hydrostatic pressure alone.

If the ring stiffeners are not strong enough, the entire ring-shell combination can buckle bodily, through general instability. In 1987, Ross [1] presented an alternative design to the ring-stiffened vessel, as shown in Figure 2.

![Figure 2 - Circumferentially Corrugated Pressure Hull](image1)

From Figure 2, it can be seen that the pressure hull was a circumferentially corrugated vessel. Using two mathematical models, Ross showed that the corrugated pressure hull was structurally more efficient than a ring-stiffened vessel of the same weight and volume. In one case, the corrugated vessel had nearly twice the strength of its ring-stiffened equivalent.

In fact, Kuan-Ya et al [2] showed that the corrugated cylinders could be made even more structurally efficient by increasing the cone angles to certain optimum values. The work reported so far is of a theoretical nature, and experimental work to verify the theoretical mode of failure was carried out by Ross et al [3 to 5], see Figure 3.

![Figure 3 - General instability of corrugated circular cylinders](image2)
These experimental investigations showed that like unstiffened and stiffened circular cylinders, the vessels were prone to suffer elastic knockdown due to initial out-of-circularity. To cater for this effect, Ross and Palmer [3] presented a thinness ratio $\lambda'$, which was based on the thinness ratio of Windenburg and Trilling [6]. The thinness ratio $\lambda'$ is defined as follows:

$$\lambda' = \left[\frac{(L_b/D')^2/(t'/D')}\right]^{0.25} \times \sqrt{\frac{\sigma_{yp}}{E}}$$  \hspace{1cm} (1)

where

- $D' = R_i + R_o$
- $t'$ = equivalent wall thickness
- $\sigma_{yp}$ = yield stress
- $E$ = Young's modulus
- $R_i$, $R_o$ and $L_b$ are defined in Figures 4 and 5

Figure 4 - Geometry of DML1

Figure 5 - Geometry of DMM3 & DMS3

In this paper a theoretical and an experimental investigation will be made on three corrugated cylinders of different geometries.
2 Experimental Apparatus

Three steel models were tested until failure. These models were named DML1, DMM3 and DMS3. The models are shown in Figures 6 to 8, and their geometrical details are given in Table 1, where

\[ N = \text{number of corrugations} \]
\[ t = \text{wall thickness} \]

Table 1 - Geometrical details (mm)

<table>
<thead>
<tr>
<th></th>
<th>N</th>
<th>( L_b )</th>
<th>( L_r )</th>
<th>( L_{12} )</th>
<th>( L_1 )</th>
<th>( R )</th>
<th>( R_a )</th>
<th>( R_i )</th>
<th>t</th>
</tr>
</thead>
<tbody>
<tr>
<td>DML1</td>
<td>8</td>
<td>173.0</td>
<td>25.10</td>
<td>17.84</td>
<td>12.48</td>
<td>76.53</td>
<td>76.33</td>
<td>75.23</td>
<td>0.310</td>
</tr>
<tr>
<td>DMM3</td>
<td>15</td>
<td>111.6</td>
<td>23.30</td>
<td>-</td>
<td>4.333</td>
<td>49.81</td>
<td>49.97</td>
<td>49.48</td>
<td>0.222</td>
</tr>
<tr>
<td>DMS3</td>
<td>15</td>
<td>101.3</td>
<td>18.23</td>
<td>-</td>
<td>4.373</td>
<td>41.99</td>
<td>42.42</td>
<td>41.85</td>
<td>0.208</td>
</tr>
</tbody>
</table>

Figure 6 - Model DML1

Figure 7 - Model DMM3

Figure 8 - Model DMS3
All models were manufactured by Del Monte Phillipines Incorporated; their material properties were as follows:

- Modulus (E) = 190 GPa
- Density (ρ) = 7800 kg/m³
- Poisson's ratio (ν) = 0.3 (assumed)

\[ \sigma_{yp} (DML1) = 344 \text{MPa} \]
\[ \sigma_{yp} (DMM3) = 453 \text{MPa} \]
\[ \sigma_{yp} (DMS3) = 475 \text{MPa} \]

The longitudinal profiles of the vessels were measured by a Mitutoyo coordinate measuring machine. The profiles were of a sinusoidal nature, but for the theoretical analysis, it was convenient to represent each corrugation with two truncated finite element conical shell elements; previous work [7] has found this to be satisfactory. The ends of the vessels were blocked off by end disc caps, sealed by a household silicone sealant, as shown in Figures 6 to 8.

The out-of-circularity of the vessels is given in Table 2. Out-of-circularity is defined as maximum difference between the maximum inward and outward deviations of the external surface from a mean circle (using a least squares fit).

### Table 2 - Out-of-circularity measurements (mm)

<table>
<thead>
<tr>
<th></th>
<th>DML1</th>
<th>DMM3</th>
<th>DMS3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Middle</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Top</td>
<td>1.7752</td>
<td>1.6511</td>
<td>0.6550</td>
</tr>
<tr>
<td>Bottom</td>
<td>1.6458</td>
<td>1.2767</td>
<td></td>
</tr>
</tbody>
</table>

### 3 Experimental Procedure

The models were tested in a test tank shown in Figure 9. Water was used as the pressure raising fluid, with the aid of a manually-operated hydraulic pump.
Line losses were negligible, as the hose was only 2m in length, and the pump was manually operated. Prior to raising the pressure, a bleed screw was left open, so that any trapped air could be pumped out. When the trapped air was removed, the bleed screw was sealed and the pressure tests commenced. The experimental boundary conditions for each vessel was probably in-between clamped and simply-supported.

3.1 Testing the Models

Each one of the three models had attached to it, several linear strain gauges placed in the circumferential direction. The purpose of these strain gauges was to observe the circumferential buckling patterns, throughout each test. The strain gauges were spaced at equidistances apart. Ten strain gauges were attached to DML1, the largest of the three models, eight strain gauges to DMM3 and six strain gauges to DMS3, the smallest of the three models.

Each model was tested to destruction. Model DML1 collapsed at a pressure of 2.55 bar with the formation of four lobes. Model DMM3 collapsed to a pressure of 1.931 bar and model DMS3 collapsed at a pressure of 3.172 bar, the number of lobes corresponding to each of these two buckling pressure was not as easily determined as for model DML1.

Plots of strain recording for various values of pressure are shown in Figures 10 to 12, and pictures of the collapsed models are shown in Figures 13 to 15.

![Figure 10 - Strain recordings for DML1 Buckling Test (micro-strain)](image-url)
Figures 11 - Strain recordings for DMM3 Buckling Test (micro-strain)

Figure 12 - Strain recordings for DMS3 Buckling Test (micro-strain)

Figure 13 - Collapsed model DMM3
As the strain recordings for DML1 were well below the yield strain, it is likely that these vessels buckled elastically. In the case of DMM3 and DMS3, the strain recordings were well past the yield strain, hence, these vessels buckled inelastically, and were more likely to suffer elastic knockdown.

4 Theoretical Analysis

The theoretical buckling pressures were calculated by a computer program called CONEBUCK [8], which is based on the finite element method. The thin-walled truncated conical element was used and is reported elsewhere (Ross [9]). The theoretical buckling pressures \( P_1 \) were found to be 3.267 bar, 3.113 bar and 4.0 bar for DML1, DMM3 and DMS3 respectively.

4.1 Design procedure

The design procedure, therefore, is to calculate the theoretical buckling pressure \( P_\text{cr} \) and the thinness ratio \( \lambda' \), and to divide \( P_\text{cr} \) by the elastic knockdown factor, namely \( (P_\text{cr}/P_\text{exp}) \), to obtain the actual collapse pressure. The elastic knockdown factor will be obtained from a design curve, when sufficient experimental results are available, where \( \lambda' \) will be plotted against \( P_\text{cr}/P_\text{exp} \) for each model, where, \( P_\text{exp} = \) experimental buckling pressure.

To obtain the design pressure, it will be necessary to divide the collapse pressure by a large safety factor.
4.2 Calculation of thinness ratio \( \lambda' \)

The thinness ratio \( \lambda' \) is based on representing the corrugated cylinder by an equivalent unstiffened circular cylinder, with equivalent thickness, namely \( t' \), such that the flexural stiffness of the equivalent cylinder is the same as that of the corrugated vessel; that is:

\[
\frac{L_b \times t'^3}{12} = (I_{NA} \times N) + \sum [\frac{(L_f \times t'^3)}{12} \times 2]
\]

where,

\[
I_{NA} = ((t_w \times D_s^3)/12) \times 2
\]

\( N = \) number of corrugations, \( t = \) wall thickness

\( t_w = \) width of a leg of the corrugation (see Figure 24)

\( D_s = \) depth of a leg of the corrugation

\( \alpha = \tan^{-1} \left[ (2 \times D_s)/ L_c \right] \)

\( L_s = \) length of an individual corrugation

\( L_b = \) length of model

\( L_f = \) length of flat or non-corrugated section

\( t' = \) thickness of equivalent unstiffened circular cylinder

![Diagram of a corrugated cylinder](image)

Figure 16 - leg of the corrugation (assumed to be a parallelogram)

Table 3 gives the values for \( \lambda' \) and \( P_{cr}/P_{exp} \), together with \( e/t' \), where, \( e = \) out-of-circularity measurements (mm)

<table>
<thead>
<tr>
<th>Model</th>
<th>( \lambda' )</th>
<th>( e/t )</th>
<th>( e/t' )</th>
<th>( P_{exp} )</th>
<th>( P_{cr} )</th>
<th>( P_{cr}/P_{exp} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>DML1</td>
<td>2.82</td>
<td>5.726</td>
<td>2.877</td>
<td>2.55</td>
<td>3.267</td>
<td>1.281</td>
</tr>
<tr>
<td>DMM3</td>
<td>3.71</td>
<td>7.425</td>
<td>4.988</td>
<td>1.931</td>
<td>3.113</td>
<td>1.612</td>
</tr>
<tr>
<td>DMS3</td>
<td>3.28</td>
<td>4.644</td>
<td>2.683</td>
<td>3.172</td>
<td>4.0</td>
<td>1.333</td>
</tr>
</tbody>
</table>
From Table 3, it can be seen that the model DMM3 had the largest value of elastic knockdown, and this was due to the fact that it also had the largest value of $e/t'$. Similarly, as both $\lambda'$ and $e/t'$ were similar for DML1 and DMS3, their elastic knockdown factors were similar. Table 4 compares these results with results obtained from other recent tests.

Table 4 - Comparisons of $\lambda'$, $e/t'$ and $P_{cr}/P_{exp}$

<table>
<thead>
<tr>
<th>Model</th>
<th>$e/t'$</th>
<th>$\lambda'$</th>
<th>$P_{cr}/P_{exp}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>DML1</td>
<td>2.877</td>
<td>2.82</td>
<td>1.28</td>
</tr>
<tr>
<td>DMM3</td>
<td>4.988</td>
<td>3.74</td>
<td>1.61</td>
</tr>
<tr>
<td>DMSB</td>
<td>2.683</td>
<td>3.28</td>
<td>1.33</td>
</tr>
<tr>
<td>DF</td>
<td>0.185</td>
<td>3.57</td>
<td>1.15</td>
</tr>
<tr>
<td>MBS</td>
<td>0.185</td>
<td>2.67</td>
<td>1.61</td>
</tr>
<tr>
<td>MBL</td>
<td>0.365</td>
<td>3.53</td>
<td>1.69</td>
</tr>
<tr>
<td>CA</td>
<td>0.46</td>
<td>2.73</td>
<td>1.45</td>
</tr>
</tbody>
</table>

5 Conclusions

The results have shown that it is possible to replace a ring stiffened pressure hull of a submarine with a similar corrugated vessel. This should be particularly attractive if the hull is to be made from carbon or glass fibre composite material, as it should be easier to construct such a vessel in corrugated form, rather than as a ring stiffened vessel. Further research is required to determine the loss of buckling resistance due to initial out-of-circularity of the cylinders.

References


