Finite element analysis of filling pressures on cylindrical silo walls

A. Khelil

Université Henri Poincaré Nancy 1 – IUT NB, Département Génie Civil
54600 Villers Les Nancy, France
Email: khelil@iutnd.u-nancy.fr

Abstract

The design of storage structures requires, above all, the calculation of the normal pressures on silos walls. Two methods of calculation are generally used, the analytical and the numerical method. The analytical method is generally used for practical and rapid calculations of the silos. However, if we consider the complex behaviour of the ensiled material, the effect of the wall flexibility and the behaviour of the material-wall interface, only a numerical finite element technique can be used. Earlier studies, using this method, often consider a complex behaviour of the ensiled material. Our aim is to study the pressure distributions in circular silos with flexible walls and to show that comparatively simple finite element techniques, which include appropriate wall friction characteristics, can accurately model the pressures exerted by stored solids on silos walls in filling conditions. Four different contact elements are applied the interface: two for the cylindrical section and two for the conical section. The performance of this method is very satisfactory compared to the experimental results for squat and slender silos. It can be noted that the use of a very sophisticated behaviour law for ensiled material is not necessary, whereas an exact modelling of the wall friction is vital.

1 Introduction

The recent progress in numerical calculation have allowed several interesting contributions for the calculation by elements finish actions of the ensiled materials on walls of silos [1-3-4-5-6-7-8-9-10], up till now efforts are carried essentially on the research of constitutive laws appropriate of the ensiled materials. Several authors propose very complex constitutive laws with many
parameters that allow a good concordance on experimental results in laboratory. Phenomena of interaction between the wall and the granular material is very little considered. Among contributions that have taken into account the interaction as well as the effect of the flexibility of the wall on pressures, one quotes works of Mahmoud and Abdel-Sayed [7] and Ooi-Rotter [9].

In this work, a technique of calculation by finite element method relatively simple is presented by using appropriate characteristics of the wall friction. This approach concerns essentially the analysis of the filling actions on cylindrical silo walls, equipped or not of hopper. It takes in consideration simultaneously the behaviour of the ensiled materials and the circular silo wall. Results obtained by this method are compared with theoretical and experimental studies. We show as well as the complex characterisation of the ensiled material (with non linear behaviour) is not indispensable to determine the wall filling pressures. This contribution brings important results but simple that can be used as reference to more complex models. Some preliminary parametric studies on the distribution of the wall pressures are presented and parameters that allocate results are identified. Effects of flexibility of the wall are equally examined in this work.

2 Finite element discretization

The wall and the granular materials are supposed supply each a linear elastic constitutive law. This simplified assumption is adopted because the granular material is confined as a solid block against the wall. Results that result from this simple theory, show that these hypotheses are valid. The wall and the ensiled material are considered as been solids axisymmetric in the finite element calculations. The quadratic eight-noded element (with four points of integration of Gauss) provides the best results of this problem. A typical mesh is shown to represent a radial material section of the ensiled material and of the silo. We consider a fine mesh near the bottom and at the transition cell - hopper (figure 1). The interface material - wall is modelled by two elements of contact (four and six-noded elements). The ensiled materials are processed as having a constant bulk density, and residual stresses that be develop during the progressive filling are neglected.
3 Modelling of the composite ensiled "material – wall"

The composite structure (ensiled material - wall) will be modelled as a solid axisymmetric. The figure 2 presents an element of volume of this solid.

The theoretical problem is analogous to that plane stresses or strain. For symmetry reasons, the two components of the displacement in all section...
containing the axis of symmetry define completely the fields of strains - stresses. Such section is represented on the figure 2.

![Figure 3. axisymmetric elements](image)

r and z respectively axial and radial components from a point, u and v corresponding displacements

4 Contact element between the wall structure and the ensiled material

A contact element is introduced between ensiled material and the wall structure (Goodman [5]). We test two type contact elements: linear and quadratic. From an equation of energy minimised by report to nodal displacement, the rigidity for the linear 4-noded elements is obtained. This element of typical contact is represented on the figure 4.

![Figure 4: 4-noded element of contact, width = zero](image)

This element is length \( L \) and zero width, that is to say couples from points (1,4) and (2,3) have initially the same co-ordinated.

5 Modelling of the tangential pressure

5.1 Normal rigidity
The contact element allows the relative displacement of the node between the two sides of this element. The characteristic essential of the normal module is that the contact element has to behave a more rigidity than the wall structure of the silo. The radial rigidity of the wall is characterised by using the shell theory:

\[ k_p = \frac{E_p t}{R^2} \quad [1] \]

Where \( E_p \) is the module of elasticity of the wall, \( t \) is the thickness of the wall, and \( R \) is the ray of the silo. The contact element is therefore chosen to have a normal module \( k_n \) of:

\[ k_n = m \cdot k_p = m \cdot \frac{E_p t}{R^2} \quad [2] \]

Where \( m \) is great, for example in the order 1000. The value of the factor \( m \) is obtained from errors of wall pressure prediction. The value of 1000 gives an error of approximation of 0.1%.

5.2 Shear rigidity

The shear modulus is initially chosen to have an equal value to that the normal module and it is then decreased gradually from slipping between ensiled materials and the wall. The calculation is iterative. Once displacement of contact is obtained, we determine shear and normal stresses. There where the found normal stress is traction, the shear modulus is replaced by zero for the iterative following. There where the found normal stress is compression, the maximal shear stress is determined as follows:

\[ \tau_m = \mu \cdot p_n \quad [3] \]

\( \mu \) is the wall friction coefficient and it is function of the surface characteristics of the wall. If \( p_s \) are not greater than \( \tau_m \), the shear modulus for this point is left unchanged. If \( p_s \) is greater than \( \tau_m \), the shear modulus is decreased with the result that existent displacement satisfy the plastic limit of the yield shear. This manner modulates it the iterative \( i+1 \) is determined from the value of the iterative \( i \):

\[ (k_s)_i +1 = \frac{\tau_m}{p_s} (k_s)_i \quad [4] \]

The equation of the system is iteratively solved until what the tangential stress \( p_s \) is to less of 5% of the maximal shear stress \( \tau_m \).
6 Application of the proposed method to different silos

In this paragraph we present results obtained by using our numerical program in the case of different structure silos (squat and slender silos with or without hopper). Comparisons are established with, standard draft code, several classic theories, numerical calculations and experimental measures. Different parameter effects; the wall friction coefficient, the report R/t, the module of elasticity and the global rigidity, the Poisson coefficient of the ensiled material, the ensilage height and the bulk density on results, are equally exploited.

6.1 Squat silos

The normal walls pressures distribution in a squat silos is different that throbbed in slender silo. In the case, thanks to the effect of the soil mechanics, the silo is sufficiently flexible in order that we assume a slipping to the material interface - wall.

We give here, the distribution of pressures on the wall of a silo of 6m of height and 6m of diameter, figure 6. The bottom of silo is supposed rugged and the wall is blocked down and free in high. The wall elastic modulus is equal to $20.10^5$MPa and to 50MPa for the ensiled material with an equal Poisson coefficient to 0.3 in the two cases. The wall friction coefficient is 0.5 and the angle of the cone of filling is $25^\circ$. Results thus obtained show a good concordance with classic theories.

The figure 7 shows results obtained for a silo of diameter 6m and 8m of height (the cell D of the Chartres experimental base [6]). The characteristic mechanics of the wall structure and of the ensiled material are:

- $E_p = 2.10^5$ MPA, elasticity wall modulus
- $E_m = 50$ MPA, module of elasticity of the ensiled material
- $\nu = 0.3$, Poisson coefficient of the wall and of the ensiled material
- $\gamma = 8$ kN /m$^3$, bulk density
- $\mu = 0.33$ wall friction coefficient
- $\phi = 22^\circ$, internal friction angle

Results are compared with the elastic plastic solution established by E. Ragueno [10]. The ensiled material used in the experimental basis is the wheat of Chartres with the following characteristic:

- $\gamma = 8.1$ kN/m$^3$, bulk density
- $\mu = 0.306$, wall friction coefficient
- $\phi = 30^\circ$, internal friction angle
Normal pressure $\frac{2\mu p_n}{\gamma R}$

**Figure 6:** Wall pressures distribution

**Figure 7:** Comparison of results obtained with experimental measures
6.2 Slender silos

In the figure 8, we have compared results obtained for the case of the wheat with results obtained by using the constitutive law proposed by Kolymbas (hypo-elastic) and with the French Norm (P22-630). The used silo is 28 m of height and 5.6 m of diameter, filled with wheat. The wall friction coefficient is equal to 0.5; the elasticity modulus and the Poisson coefficient are respectively equal to 50 MPa and to 0.3. The bulk density of wheat is equal to 8.35 kN/m³. The normal pressures obtained from our study are well concordant with results obtained by Eibl - Rombach using a constitutive law hypo-elastic or elastic-plastic. The French Norm shows a maximal value of approximately 15%. These results show although a constitutive law for granular materials more sophisticated can not influence important normal wall pressures.

![Figure 8: Normal pressure distribution](image)

6.3 Silos with a hopper

To calculate normal wall pressures of the silo and the hopper, we have used the four-noded elements of contact (CT4) and the six node element (CT6). The figure 9 shows results obtained for an analogous cell to the cell D of the Chartres experimental base, diameter 6 m and 8 m of height with a hopper to 45° with an opening of 0.50 m. The characteristic mechanics of the wall structure and of the ensiled material are:
Ep = 2.10^5 MPA, elasticity wall modulus
Em = 50 MPA, module of elasticity of the ensiled material
ν = 0.3, Poisson coefficient of the wall and of the ensiled material
γ = 8 kN/m3, bulk density
μ = 0.33 wall friction coefficient
ϕ = 22°, internal friction angle
t = 6 mm, the thickness of the wall

The normal pressure distribution in the hopper is well concordant with the solution of Ragneau [10]. The long of the cylindrical part our approach gives inferior results to the two other solutions (elastic not linear and elastic – plastic). In our model, the flexibility of the wall is taken in account, what decreases the wall pressures.

7 Conclusion

The aims of this work are to show that the utilisation of a simple behaviour law gives good results in the case of the filling of silos. Theories proposing the study from complex rupture criteria are not therefore necessarily indispensable. The exact modelling of the friction interface by taking into account the flexibility of the wall is important. For the ensiled material a high internal angle of friction, the
elastic behaviour assumption is more adapted. When the cohesion of the granular material is weak, its internal friction angle increases. The omission of effects of the plasticity behaviour of the granular material entails a weak increase of the normal pressure in the inferior part and a weak diminution of this pressure in the part superior of the silo. When the geometrical report between the height and the diameter of silo H/d is superior 5, the deformation of the wall is important, what declines the precision of the functioning of the element of contact. The utilisation of this program is economic the viewpoint of the time of calculation; the linear constitutive law is behind a very important diminution of the time calculation of the program.

References