Transactions on Modelling and Simulation vol 21, © 1999 WIT Press, www.witpress.com, ISSN 1743-355X

A strategy for benchmarking finite element analysis of nitrile rubbers

S.J. Jerrams & S.J. Harvey
Rubber Research and Technology Unit, Coventry University, Priory St., Coventry CV1 5FB, England
Email: cex076@coventry.ac.uk

Abstract

The confidence given to linear and nonlinear finite element analyses of metal components is unfortunately lacking for similar analyses of elastomers. A conventional Hookean material can be modelled in some standard test for which a classical solution exists. There are no such solutions for rubber compounds. Different structures and filler contents mean that certain rubbers are reasonably well represented by one of the phenomenological material models, whilst others are characterised by an alternative. Software suppliers do not provide a data-base of material properties for rubbers, nor should they be expected to. There is no agreed method for determining material constants; no definitive statement on the strain range required, feed rates, deformation modes or test-piece conditioning. Gent suggested that Boussinesq equations for axisymmetric indentation could be used for benchmarking elastomeric FEA. He argued that rubber has a constant rigidity modulus over a large strain range and such equations only included this one material constant. However, commercial rubber compounds comprise large amounts of filler, particularly carbon-black, for stiffness and cheapness. These materials exhibit a constant shear modulus for small initial strains only. This paper suggests that elastomeric FEA can be benchmarked by comparing load / displacement curves with equations deduced from plane strain indentation tests. Initially, tests on a range of hardness' of nitrile rubbers were conducted using indentors of different surface finish, at a selection of feed rates. Short-term load relaxation was also recorded. The indentation phases of the tests were markedly more linear than anticipated and could be represented by a simple formula. The popular Yeoh and Ogden material models were used to define the material properties in hyperelastic FEA and the analyses were compared with physical tests and the formula. Stress relaxation was defined by a logarithmic formula which offers the possibility of benchmarking viscoelastic FEA.

1 Introduction

Finite element analysis (FEA) of rubbers is complex for a number of reasons. These are:- i) elastomers are subject to large deformations and rotations when loaded and there are associated changes in boundary conditions, ii) the material is assumed isochoric, iii) instabilities result from the material description and iv) multiple solutions may exist. Even an experienced FE analyst will not always foresee the problems that ensue from these complexities. Some are due to the material description, some to the discretisation of the model and others to a combination of both. Hence, there is a need in all FEA of rubber components, to consider results carefully. A feasible procedure for benchmarking elastomeric finite element analysis is described. It allows confidence in results obtained for the high deformations experienced in rubber/rigid surface contact problems. Rubber components are considered to fail, not as a result of fracture, but when they cease to fullfil the function for which they were designed. Invariably a rubber component is deformed by contacting a rigid body or by rubber to rubber contact. Such a situation is shown in figure 1. The figure depicts an X-ray of a drive-shaft, constant velocity joint rubber boot where adjacent flanks contact during plunge, twist and rotation, resulting in the failure shown in figure 2. The X-ray illustrates the high level of deformation that a rubber component can experience. Attempts to model this situation using a function incapable of simulating high nonlinear strains or using an inappropriate mesh density are bound to lead to under-predictions of component deformation. Large nodal displacements and rotations could not be simulated. FEA that models hyperelastic behaviour is consequently more difficult to verify than modelling the behaviour of linear elastic solids. Previous approaches to benchmarking modal and stress analyses of elastomeric components have often relied on simple models that do not induce high, localised contact strains[1,2]. Gent[3] employed Boussinesq punch indentation using formulae advanced by Sneddon[4] to benchmark hyperelastic FEA. This approach simulated contact problems, but assumed that rubber complies with the Statistical (Kinetic) Theory[5] and exhibits a linear stress-strain relationship in shear. Some rubbers have a constant shear modulus (G) for shear strains (γ) up to 100%, but most, particularly those reinforced with carbon-black, display linearity for γ of below 10%[6,7] only. Observing surface deformations in the punch vicinity and any adhesion occurring between punch and indentor, requires complicated solutions[8,9]. 'Plane strain' indentation employing video microscopy to observe the indentation process offers an alternative to this approach. The recording of load-displacement data allows a relationship between force and indentation depth to be expressed as an empirical equation.

2 Plain strain tests

A range of acrylonitrile butadiene and hydrogenated acrylonitrile butadiene rubbers (NBRs and HNBRs) of different hardness were indented with three uniform plate indentors with semi-cylindrical edge forms as shown in figure 3. To achieve a satisfactory representation of plane strain indentation it was necessary to indent a block of material which, relative to the indentor width, approximated to a semi-infinite half space. Test-pieces, 25 x 50 x 50 mm, were manufactured from 40 Shore A hardness NBR and 50, 60 and 70 Shore A hardness HNBR. The test grade blocks were indented with a series of 2 mm wide (b) cylindrical ended indentors at differing feed rates, to a depth of 4 mm (d), using an Instron 8501 Dynamic Testing System. Strain in the 'z' direction was prevented by holding test-pieces in a fixture, having a Perspex insert in the front face that allowed indentation to be recorded. Tests were carried out at five different feed rates:- 5, 50, 100, 250 and 500 mm/min. This allowed the influence of variation in simultaneous stress relaxation and internal friction to be observed. The 500 mm/min feed rate is considered to give quasi-static loading. Hence virtually no simultaneous stress relaxation is assumed to have occurred at this rate. The indentor was held at the maximum indentation depth for ten minutes in each test to study the short-term stress relaxation behaviour of the materials. All the HNBR tests were repeated with the presence of a lubricant between the indentors and blocks. The lubricant used was a grease: Isoflex Topas L32 (Kluber Lubrication IR 206903; Art-Nr. 004 130). This is a rolling bearing grease used for low and high temperatures and for long term lubrication at high speeds. The surface finishes of the three indentors are given in table 1.The radiused end form of the indentors was chosen to allow a rigid surface to be represented by a continuum when finite element modelling. No plane strain indentor between the extremes of flat and infinitely sharp can negate the influence of edge form during initial ingress. Consequently it is reasonable to study load-displacement characteristics beyond an indentation depth where the indentor flanks come into full contact with the rubber. The influence of different edge forms on initial indentation can be studied separately. Hence the formulae for indentation of the NBR's and HNBR's discussed in section 4 are determined for displacements between 1 and 4 mm.

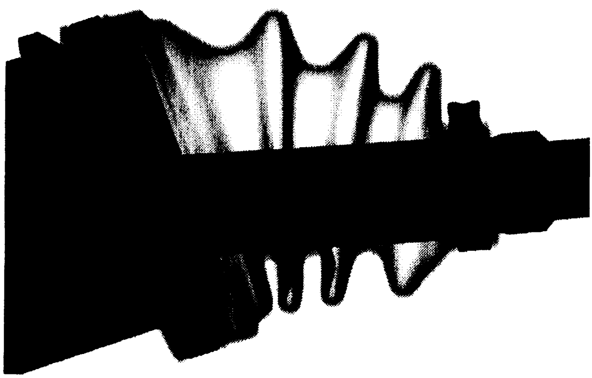

Figure 1. X-Ray showing high deformations in rubber boot

Figure 2. The CV joint rubber boot shown before assembly and after failure

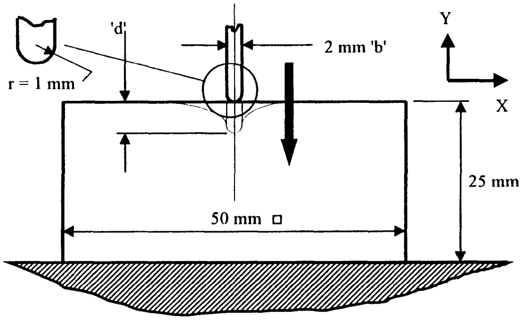

Figure 3. 'Plane strain' indentation of rubber test-pieces

Table 1 Plane strain indentor surface finishes

Finishing process	Surface Finish, R_a (μm)
Shot blown	1.88
Vapour blasted	1.06
Polished	0.30

3 Determination of material constants for FEA

The material constants for the HNBRs used in FEA simulations of the plane strain tests were determined from uniaxial tensile tests on type 2 dumbbell test-pieces[10]. The tests were carried out at the same five feed rates as the indentation tests. The varying speeds were employed to study the influence of feed rate on material models[11]. The models used were those postulated by Yeoh[12] and Ogden[13] and are plausible strain energy functions respectively using strain invariants and stretch ratios. Constants were calculated using the curve-fitting programme employed by MARC software, where different expressions are used at low and high strains to determine errors in the least squares fit procedure. They allow a sensible curve to be fitted to both the Gaussian and non-Gaussian regions of the test curve. For moderate strains up to stretch ratios (λ) of approximately 4, the distances between the ends of long-chain molecules are assumed to comply with Gaussian error function theory. Beyond this value the cross-links in the long chain molecules begin to break down and the material behaviour is said to be non-Gaussian. The formulae used for determining error in each part of the curve are respectively $(1 - \sigma_c/\sigma_m)^2$ and $(\sigma_m - \sigma_c)^2$, where σ_m is the measured stress and σ_c is the calculated stress. Consequently large changes in stress for smaller strain increments at high strains are assimilated. Johannknecht *et al* have suggested a curve-fitting procedure including the options of data weighting and plausibility analysis[14] that gives improved confidence in the material models. The data from the uniaxial tests on the nitrile rubbers, shown in table 2, did not include this refinement.

4 Formulae for plane strain indentation and stress relaxation

The influences of lubricant, feed rate and surface finish on the plane strain behaviour of nitrile rubbers are documented elsewhere[15]. The plane strain indentation can be represented by a formula having the form shown in eqn 1 and that for stress relaxation by eqn 2. Kaya[16] is investigating the FE modelling of plane strain stress relaxation by applying a Prony series to the viscoelastic data, in accordance with the method suggested by Simo[17]. The formulae were determined from inspection and analysis of 105 physical tests.

$$F = (\alpha \ln R + \beta)\delta \qquad (1)$$

$$F' = (\varsigma \ln R + \psi)F_{max}e^{-ct'} \qquad (2)$$

Table 2 Mean constants for Yeoh and Ogden models of nitrile rubbers

Hardness	40 Shore A	50 Shore A	60 Shore A	70 Shore A
	NBR	HNBR	HNBR	HNBR
		Yeoh		
C_{10}	0.19611	0.44129	0.61300	1.17881
C_{20}	0	0.01061	0.01537	0.02604
C_{30}	0.000043	0.00022	0.00234	0.01251
		Ogden		
μ_1	0.1204E–6	0.3268	0.2191	0.6502
α_1	9.7462	3.0815	3.6715	3.7845
μ_2	-0.1829	-0.1199	-0.1617	-0.1478
α_2	-5.1235	-6.0932	-7.3772	-7.6696
K	0.937E6	1.531E6	1.992E6	3.570E6

5 Finite element analysis of plane strain indentation

There are numerous strain energy density functions based on strain invariants, for use in FEA. They are generally determined from the James, Green and Simpson[18] function (eqn 3) Some are applicable to particular rubber compounds but give inaccurate stress strain relations for others. The Yeoh formulation (eqn 4) seeks to address two predominant concerns when using strain energy functions based on polynomial material models. These are:- i) they do not model the upturn at high strains on a Mooney-Rivlin plot, associated with the finite extensibility of the macromolecular network of rubbers, particularly those containing carbon black fillers and ii) that fitting a curve to tensile data for a rubber does not produce constants that are appropriate for predicting behaviour in other deformations modes. Gregory[19] had recognised that a simple relationship existed between uniaxial tension, uniaxial compression and simple shear for different compounds containing carbon-black. A single curve was obtained when $\sigma/(\lambda-\lambda^{-2})$ and $^{\tau}/\gamma$ were plotted against I_1-3. This situation can only exist when two conditions are satisfied:- i) $\delta W/\delta I_1$ must be much greater than $\delta W/\delta I_2$ and ii) $\delta W/\delta I_1$ must be independent of I_2 Though these two conditions are not applicable for unfilled rubbers, with the exception of small strains, they are satisfied to a first approximation for filled rubbers. Seki *et al*[20] and Kawabata and Kawai[21] have suggested that for such rubbers $\delta W/\delta I_2$ is very close to zero. If $\delta W/\delta I_2$ is assumed to be zero, then it follows that the partial derivative with respect to I_2 is $\delta W/\delta I_2 = C_{01} + C_{11}(I_1-3)$ and so for these rubbers C_{01} and C_{11} can be considered to equal zero. Thus Yeoh suggests that eqn 3 can be modified to eqn 4 to model filled rubbers. This gives a simple cubic equation that can be obtained from a third order deformation approximation. Setting C_{01} and C_{11} to zero and using eqn 4 which is widely available in commercial finite element software allows the revised formula to be used. Yeoh supported this suggestion by conducting uniaxial tensile, uniaxial compressive and simple shear tests on rubbers containing different amounts of carbon black filler and subjected to two alternative conditioning procedures. Again physical results and theoretical curves were compared by plotting $\sigma/(\lambda-\lambda^{-2})$ and $^{\tau}/\gamma$ against I_1-3 and, provided specimens deformed in different modes reached similar values of I_1-3, data obtained in tension was capable of being used to describe deformation in compression and shear.

$$W = C_{10}(I_1-3)+C_{01}(I_2-3)+C_{11}(I_1-3)(I_2-3)+C_{20}(I_1-3)^2+C_{30}(I_1-3)^3 \tag{3}$$

$$W = C_{10}(I_1-3)+C_{20}(I_1-3)^2+C_{30}(I_1-3)^3 \tag{4}$$

The most common strain energy density function using stretch ratios is that advanced by Ogden (eqn 5) who argued that the introduction of strain invariants was an unnecessary complication and that a formulation based on stretch ratios would be mathematically simpler. He derived a strain energy function for an incompressible rubber in series form.

Transactions on Modelling and Simulation vol 21, © 1999 WIT Press, www.witpress.com, ISSN 1743-355X

$$W = \sum_{n} \frac{\mu_n}{\alpha_n} \left(\lambda_1^{\alpha_n} + \lambda_2^{\alpha_n} + \lambda_3^{\alpha_n} - 3 \right) \qquad (5)$$

The α_n term can have any values and be positive or negative. The μ_n terms are constants and can also be positive or negative but pairs should possess the same sign since the initial shear modulus, which clearly must have a positive value, is given by eqn 6.

$$G_0 = \frac{1}{2} \sum_{n=1}^{N} \mu_n \alpha_n \qquad (6)$$

The plane strain indentation of nitrile rubber was modelled using MARC/MENTAT software. The format summarised below was initially adopted for all NBR and HNBR material models.
• A single feed rate of 0.0833 mm/s (5 mm/min). • A relative sliding velocity between indentor and rubber of 1/100th of feed rate. • 'Full' contribution of initial stress to stiffness. • 80 incremental load steps. • A contact tolerance of 0.25. • A friction model using 'Coulomb for rolling friction'. Later analyses used a friction model employing a stick-slip option.
The choice of feed rate had no bearing on the prediction of indentor loads because no viscoelastic behaviour was assumed. A plot of deformation in the indentor vicinity depicting the adaptive meshing used is shown in figure 4.

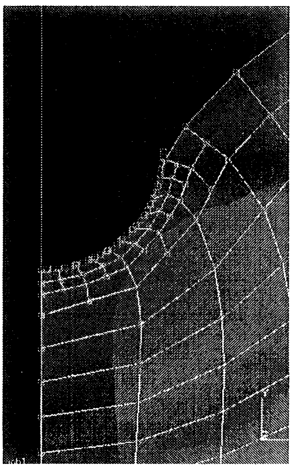

Figure 4. Adaptive meshing in the indentor vicinity

6 A comparison of tests, formulae and FEA

The correlation between FEA, physical testing and the empirical formulae are shown for an indentation depth of 4 mm in figure 5. The graph is a hybrid, as the softer rubber is an NBR and the others HNBRs. However the materials exhibit similar physical characteristics, so comparisons are valid. The 'plane strain' formula is a good predictor of indentation load and its accuracy is enhanced with increased testing. The Yeoh model underestimates contact forces for soft nitrile rubbers though its accuracy improves for harder compounds, whilst it overestimates contact force for the hardest rubber. Consequently it cannot be considered reliable for a range of filled rubbers. It is observed that the Ogden material models predict similar loads to those in the tests for the 40, 50 and 60 Shore A nitrile rubbers. The Yeoh model under-predicts these indentor forces. Surprisingly, both the Yeoh and Ogden models over-estimate the forces required to indent the 70 Shore A test blocks. It is conceivable that the Yeoh model improves with increase in filler content as the harder rubbers have very small values of $\partial W / \partial I_2$ by comparison with the softer rubbers. This would not explain the greater error for the 70 Shore A hardness rubber when compared with the 60 Shore A hardness. The large variation in material constants for the Ogden model of the hardest rubber cannot be used to explain the error in the Ogden analyses using them. The constants for each test at different feed rates were applied independently and predicted indentor loads in the range 16.00 N to 23.36 N; all of a similar value to the test loads which varied between 14.43N and 21.95N for the 70 Shore A HNBR. The analysis would be improved by using material data obtained from other deformation modes. In general the mean constants quoted for the Ogden model in table 2 can be used to predict stress-strain relations to a reasonable accuracy.

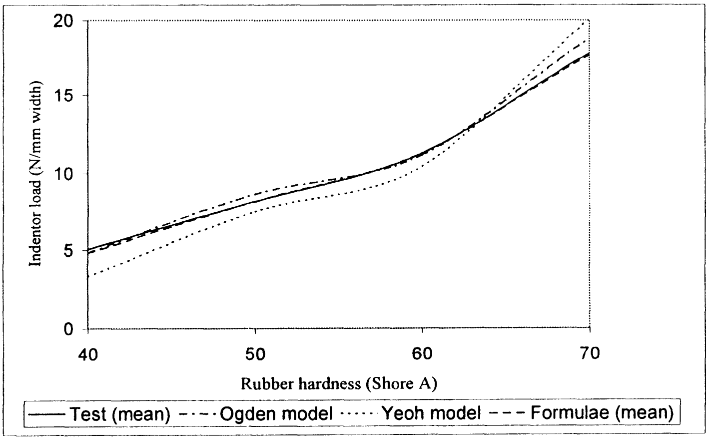

Figure 5. Comparisons of tests, FEA and formulae for plane strain loading

7 A benchmarking strategy for elastomeric FEA

Boussinesq formulae are inadequate to form the basis for benchmarking elastomeric FEA. A more rational approach would be to use an empirical formula for plane strain indentation of the form given in eqn 1. This necessitates a limited number of plane strain tests when benchmarking a particular compound. Similarly the plane strain formula for load relaxation (eqn 2) could be used to benchmark models requiring the simulation of short-term viscoelasticity, but the formula requires further development. Tests and analyses on rubbers other than nitrile rubber will substantiate if the benchmarking procedure could find universal application for elastomeric materials. Formulae relating force to indentation depth and feed rate should be rationalised to include a term for material hardness. Similarly, the formulae that express relaxed force in terms of feed rate and relaxation time could be improved by relating them to the rubber hardness and possibly indentor surface finish. The uniformity of the stress relaxation curves suggests that they could be depicted using a master curve and a shift function of the sort employed by MARC to show the relationship between relaxation moduli and time at different temperatures for polymers. They use two alternative shift functions; the well known semi-empirical Williams-Landel-Ferry equation and a polynomial expansion[22,23].

Symbols

C_{10}, C_{20} and C_{30} are Yeoh material constants (Mpa), F = Indentor force (N), F' = Indentor force during relaxation (N), G_0 = Initial rigidity modulus (Mpa), I_1 and I_2 are strain invariants, K = Bulk modulus (Mpa) which should be infinitely large for incompressibility, but must be equal to $G_0 \times 10^{-4}$ for input to MARC FEA, R = Feed rate (mm/s), t' = time (s), W = strain energy (J) and d, b and r are dimensions for plane strain testing (mm), α (kg mm^{-1} x 10^{-3}) and β (kgs $^{-1}$ x10^{-3}) are indentation constants, δ = displacement (mm), γ = shear strain, λ = stretch ratio, μ_n (Mpa) and α_n are Ogden constants, τ = shear stress (Mpa) and c (s^{-1}), ς (s mm^{-1}) and φ are relaxation constants.

References

1 Treloar R.L.G. "The Physics of Rubber Elasticity" 3rd Ed. p218 Clarendon Press, 1975
2 Flory P.J. "Statistical mechanics of chain molecules" Interscience, New York, 1969
3 Gent A.N. (Editor) "Engineering with Rubber. How to Design Rubber Components" pp 46-47 Hanser, 1992
4 Sneddon I.N. "Applications of Integral Transforms to the Theory of Elasticity" pp 70-97 Springer-Verlag, New York, 1975
5 Treloar R.L.G. "The Physics of Rubber Elasticity" 3rd Ed. pp 84-85 Clarendon Press, 1975
6 Jerrams S.J. and Reece-Pinchin N. "Benchmarking finite element simulation of rigid indentors in elastomers" CMEM 95, Capri, 1995
7 Jerrams S.J. and Bowen J. "Modelling the behaviour of rubber-like materials to obtain correlation with rigidity modulus tests" CMEM 95, Capri, 1995

8 Jerrams S.J., Johannknecht R. Hookes D.E. "Modelling Surface Deformations resulting from Rigid Punch Indentation" IASTD Con. Computer Applications in Industry. Cairo, 1995

9 Johannknecht R. "Behaviour of elastomers in physical tests and computer simulated finite element analysis" PhD thesis to be published in 1999

10 BS903 Part A2 "Physical testing of rubber. Method for determination of tensile stress-strain properties". 1995, ISO 37, 1994

11 Jerrams S.J., Kaya M and K.F. Soon "The effects of strain rate and hardness on the material constants of nitrile rubbers" The Journal of Materials and Design. Vol 19 No4, 1998

12 Yeoh O.H. "Characterization of Elastic Properties of Carbon-Black-Filled Rubber Vulcanizates" Rubber Chemistry and Technology, Vol 63, No. 5, pp 792-805, 1990

13 Ogden R.W. "Large Deformation Isotropic Elasticity: On the Correlation of Theory and Experiments for Incompressible Rubber-like Solids". Proceedings of the Royal Society Vol A (326) pp 556-585, 1992

14 Johannknecht R., Jerrams S.J. and Clauss G. "The uncertainty of implemented curve fitting procedures in finite element software" Finite Element Analysis of Elastomers. Ed. D. Boast and V. Coveny Professional Engineering Publishing (I Mech E) 1998

15 Jerrams S.J. "A study of the behaviour of isochoric materials in contact with rigid indentors", PhD thesis, 1998

16 Kaya M. "A study of rubber to metal contact problems in automotive assemblies" PhD Thesis to be submitted in 2000

17 Simo J.C. "On a fully three-dimensional finite-strain viscoelastic damage model: Formulation and computational aspects." Computer Methods in Applied Mechanics and Engineering, Vol 60, pp 105-122, 1980

18 James A.G., Green A. and Simpson G.M. "Strain energy functions of rubber, 1: Characterisation of gum vulcanisates". Journal of Applied Polymer Science, 19, pp 2033-2058, 1975

19 Gregory M.J. Plast. Rubber Materials Appl. 4., p 184, 1979

20 Seki W. Fukahori Y. Iseda Y. Matsunaga T. Rubber Chemistry and Technology. Vol 60, p 879, 1988

21 Kawabata A. and Kawai H. Advanced Polymer Science 24. p 89, 1997

22 MARC Analysis Research Corporation. "Analysis of visco-elastic and elastomeric materials" 4.12 (4.9/4.10) Course notes, 1998

23 MARC Analysis Research Corporation. "Analysis of visco-elastic and elastomeric materials" 4.12 (4.11) Course notes, 1998