New analytical model of bolted connections subjected to eccentric loading: The case of T-flange connection

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Abstract

The behaviour of bolted connections is illustrated by an example of an eccentrically loaded joint (T-flange joint) examined through a new analytical model called beam on elastic supports model. The development of this model is described. The additional load and the bending moment induced in the bolt are calculated analytically and the analytical results are in good agreement with the F.E ones that are justified experimentally. A key factor in the analysis is the area in compression of the joint. This model considers the compressed surface between the connected parts as an elastic foundation of constant stiffness but with variable contact zone.

Introduction

In many applications, fatigue life of a bolted joint may be increased through correct joint design and proper preload. The effect of initial preload is to induce a compressive stress in the clamped part. This reduces the bolt cyclic load caused by external loading. It further eliminates the chance of joint separation given that the external load does not induce a force in excess of the bolt preload. If the applied load is eccentric, bending stresses are also induced in the bolt due to the bending of the
clamped parts. It is necessary to discuss these eccentrically loaded bolted joints. From these view points investigations have been made by Sawa et al.\[1\], Hagiwara and Yoshimoto \[2\]. But the beam model proposed by Agatonovic \[3\] represents correctly the behaviour of the connection. This model is modified by Guillot \[4\] by introducing a new parameter \(\gamma\), Bakhiet and Guillot \[5\] proposed an equation for the determination of this parameter. However the effect of the edge distance has not been taken into consideration in these models, consequently if the edge distance is small \((u < hp)\) the analytical results will notably differ from that of the experimental ones. A new analytical model called "Beam on Elastic Supports Model" have been proposed by Bakhiet \[6\] This model takes into consideration all the design parameters in its definition.

In this paper, the principle of the Beam on Elastic Supports Model is presented. The additional load and the bending moment induced in the bolt in the case of T-flange connection, which is an important example of bolted joints, are calculated by using this model. The analytical results are in very good agreement with the finite elements ones that are validated experimentally. This model could be applied to all cases of connections including small eccentric loaded joints and that having small edge distances.

**Pressure distribution at the interface**

The behaviour of a T-flange bolted connection, as an example of an eccentric tensile loaded bolted connection, and the load distribution under working conditions was shown with the help of a F.E. model in figure 1. The pressure at the interface after introduction of the bolt preload is shown in figure 1a. This pressure is independent of the bolt position and symmetric relative to the bolt axis. The pressure distribution at the interface (in the disconnecting plane) is changed during the gradual growth of external load. It can be found that the pressure force on a joint migrates towards the external edge as shown in figures 1b, c and d. In this way the contact zone may be simulated as a beam on elastic foundation of constant stiffness but with variable contact zone.

**Beam on Elastic Supports Model**

**Stiffness of Contact Zone**

Taking into consideration the variation of the pressure at the contact zone, the analytical model is shown in figure 2 where the connection is transfered into a beam on elastic supports each of constant stiffness \(K_0\), and the reaction forces at the interface are assumed to distribute
uniformly along a width of 2 \((u - s)\) where the two parts come into contact with each other. In this case the stiffness of the contact zone will be,

\[ K_C = K_0 \times 2(u - s) \]  

(1)

**Basic equation**

Using the equilibrium condition of the beam with the compatibility of displacement at the bolt axis, a relation between the eccentricity of the reaction forces in the disconnecting plane \(F_C\) and its eccentricity \(s\) in function of the external load \(F_E\) is given by,

\[ F_C \left( \frac{s^3}{6E_p I_p S} - 1 \right) - \frac{(S_R + S^*_p)}{S} F_E + Q = 0 \]  

(2)

Eqn. (2) is valid for all types of bolted assembly. It is an equation in two unknowns \(F_C\) and \(s\) for a given external force \(F_E\). For this equation to be solved another equation is needed which could be obtained from the continuity of deformation at the cut boundaries between the bolted region and the rest of the structure as shown in Figure 3.

\[ \theta_E = \theta_s \]  

(3)

**Figure 1**: Pressure distribution at the interface.
Case of T-Flange connection

For the T-flange connection, loaded as shown in figure 4 the condition of continuity of deformation is given by,

\[ \theta_E = 0 \]  \hspace{1cm} (4)

Figure 2: Beam on elastic supports model.

Figure 3: Condition of continuity of deformation.

Figure 4: Deformation of T-Flange connection.
The bending moment at any point $x$ is given by,

$$M_x = F_C x - F_B (x - s)$$  \hspace{1cm} (5)

Starting from the differential equation

$$E_p I_p \frac{d^2 y}{dx^2} = -M_x$$  \hspace{1cm} (6)

And using the boundaries conditions at the ends of the beam. The second equation could be obtained by,

$$F_C = \frac{m^2}{s^2 + 2ms} F_E$$  \hspace{1cm} (7)

By using equations (2) and (7), after rearranging, the characteristic equation permits calculating $s$ is given by,

$$\frac{ms^3}{6 E_p I_p S} + \left( \frac{s_B + s_p^*}{F_E} - \frac{Q}{S} \right) s^2 + \left( \frac{Q}{F_E} - \frac{s_B + s_p^*}{S} \right) 2s - m = 0$$  \hspace{1cm} (8)

When $s$ is determined from eqn. (8) the axial force and the bending moment induced in the bolt could be calculated by,

$$F_B = \left( \frac{1}{1 - \left( \frac{m}{m + s} \right)^2} \right) F_E$$  \hspace{1cm} (9)

$$M_{FB} = \left( \frac{m^2}{2 h_p E_p I_p + 1} \right) \frac{F_E}{(s + 2m)}$$  \hspace{1cm} (10)

This model gives very good results until a critical length of $s$ defined by the distance between the center of theoretical contact zone and the bolt axis, $s_C$. This critical length depend on the value of the edge distance $u$. Beyond this value the behaviour of the joint changes and the additional load in the bolt and its representative curve will be simulated by a part of hyperbolic shape of the same asymptote as shown in figure 5. In this case the stiffness of the contact zone and the total bolt load $F_B$ can be obtained by,

$$K_C = K_0 \times 2(u - s_C) \times b$$  \hspace{1cm} (11)

$$F_B = \sqrt{\frac{a^2}{b^2} \left( F_E^2 + b^2 \right)}$$  \hspace{1cm} (12)
The bending moment is then determined by considering an angular rotation given by rotating the part as one unit around the critical point as shown in figure 6.

\[ M_{FB} = M_{FBC} + \left( \frac{F_B + F_{BC}}{K_B} \right) \times \left( \frac{E_B I_B \times E_p I_p}{h_p E_p I_p + s_C E_B I_B} \right) \]  

(13)

Where

\[ b^2 = \left( \frac{F_{CC} + F_{EC}}{\beta} \right)^2 - F_{EC}^2 \]

\[ \beta = \frac{1}{1 - \left( \frac{m}{m + u} \right)^2} \]

\[ M_{FBC} = \left( \frac{s_C}{2 h_p E_p I_p} \left( \frac{s_C}{E_B I_B} + 2 \right) \right) F_{EC} \]

\[ F_{BC} = F_{CC} + F_{EC} \]

\[ F_{CC} = \left( \frac{m^2}{s_C^2 + 2 m s_C} \right) F_{EC} \]

\[ S = S_B + S_p^* + S_C \]

\[ S_p^* = \frac{1}{K_p^*}, \quad S_B = \frac{1}{K_B}, \quad S_C = \frac{1}{K_C} \]

Figure 7 gives the excellent results obtained analytically by using this model compared to finite element ones that are validated experimentally.
Figure 6: Rotating of the part around the contact point.

a: Additional load in the bolt

b: Bending moment induced in the bolt

Figure 7: Analytical results compared to finite elements ones.
Conclusion

A new analytical model named "Beam on Elastic Supports Model" is proposed to calculate accurately the stresses induced in the bolt when the connection subjected to a high eccentric loading. This model takes into consideration all the design parameters. Accurate results are obtained in all geometric configurations including joints subjected to small eccentric loading and those have small edge distances.

References


Nomenclature

\[
\begin{align*}
E_B &: \text{modulus of elasticity of the bolt} \\
E_P &: \text{modulus of elasticity of the part} \\
F_B &: \text{total bolt load} \\
F_E &: \text{external applied load} \\
F_{BC} &: \text{total bolt load at } s = s_C \\
F_{EC} &: \text{external applied load at } s = s_C
\end{align*}
\]
\( F_C \) : clamping force  
\( K_0 \) : stiffness per unit area of the contact zone  
\( K_B \) : stiffness of the bolt  
\( K_C \) : stiffness of the contact zone  
\( Q \) : preload force  
\( S_B \) : bolt resilience of a symmetrical half of the connection  
\( S_p^* \) : resilience of the part under bolt head (half of the connection)  
\( S_C \) : resilience of the contact zone  
\( b \) : width of the pare  
\( h_p \) : thickness of the part  
\( m \) : distance from the load axis to the bolt axis  
\( s \) : eccentricity of the clamping force  
\( u \) : edge distance