Experimental and theoretical analysis of plates stiffened by two girders

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Abstract

The reinforced concrete plates with two stiffening girders have been tested by both the static and dynamic loads. The finite element method was applied to both the static and dynamic analysis and the hammer blow method together with the modal analysis show the degradation of specimens as a function of the static load. The degradation is demonstrated in the form of a decreasing tendency of the natural frequency if the applied static load grows.

1 Introduction

Plates and girders are two main elements of reinforced concrete bridges. Therefore, the European Rail Research Institute (ERRI) established a committee of experts (under the chairmanship of the first of the authors) which has investigated the interaction of plates with stiffening girders as a particular problem, [1].

2 Test specimens

A specimen was designed with a plate 2000 x 1000 x 100 mm stiffened by two girders 2000 x 150 x 400 mm at its longer edges, see Figure 1. The span in the longitudinal direction is 1850 mm while in the transverse direction...
850 mm. The simple supports were applied enabling the movements in appropriate directions and, thus, reducing the temperature and other effects. The reaction forces are transmitted by steel plates 100 x 100 x 10 mm in the specimen.

Two series of specimens were produced: A with rebar reinforcement and stirrups (9 pieces) and B with rebar net without stirrups (8 pieces), both of the concrete C 35/45. The specimen was designed to a load $F = 50$ kN which is applied to the centre of the plate, see Figure 1. The test load acts at the same point and is transmitted by a steel plate 200 x 100 x 10 mm.

![Figure 1. Test specimen (in mm) subjected to a centred force $F$.](image)

### 3 Static tests

The basic static tests checked the quality of the concrete according to the standards, i.e. after 28 days after casting and at the time of testing. Some results are presented in the Figures 2, 3 and 4. They show the modulus of elasticity of concrete $E_c$, the compressive strength of concrete $R_c$ measured on cubes 150 x 150 x 150 mm and the tensile strength of concrete $R_t$ measured on bars 100 x 100 x 400 mm as functions of the time (in days) after casting.

The Figures 2, 3 and 4 represent slowly increasing functions which confirm the well known property of concrete: it improves its quality with time.

The specimens were tested by the following load steps: 0, 25, 0, 50, 0, 75, 0, 100, 0, 125, 0, 150, 0, 175, 0, 200, 0, 225, ... kN up to the total collapse of the specimen. The vertical deflection at the centre of the plate together
with the crack propagation were measured at each step. The experimental results of the total, elastic and permanent deformations are summarized in the Table 1 and compared with the theoretical values received by the finite element method (ANSYS software). The experimental and theoretical values correspond to the design load $F = 50$ kN.

The Table 1 shows also the ultimate load under which the test specimen collapsed together with the number of days $D$ after which the test was performed.

Table 1. Static tests of specimens

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Deflections (in mm) under the load 50 kN</th>
<th>Ultimate load (kN)</th>
<th>$D$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>total</td>
<td>elastic</td>
<td>permanent</td>
</tr>
<tr>
<td>A 0</td>
<td>0.380</td>
<td>0.357</td>
<td>0.023</td>
</tr>
<tr>
<td>A 1</td>
<td>0.360</td>
<td>0.310</td>
<td>0.050</td>
</tr>
<tr>
<td>A 2</td>
<td>0.265</td>
<td>0.250</td>
<td>0.015</td>
</tr>
<tr>
<td>B 0</td>
<td>0.492</td>
<td>0.370</td>
<td>0.122</td>
</tr>
<tr>
<td>B 1</td>
<td>0.427</td>
<td>0.245</td>
<td>0.182</td>
</tr>
<tr>
<td>Mean</td>
<td>0.385</td>
<td>0.307</td>
<td>0.078</td>
</tr>
</tbody>
</table>

The test results in the Figures 2 to 4 and Table 1 show a satisfactory quality of concrete, however, even a dispersion which is typical for concrete structures.

Figure 2. Modulus of elasticity of the concrete $E_c$ as a function of the time (in days) after casting.
Figure 3. Compression strength $R_c$ of concrete as a function of the time (in days) after casting.

Figure 4. Tensile strength $R_t$ of concrete as a function of the time (in days) after casting.

4 Dynamic tests

The dynamic tests ought to show the state of damage of a structure by means of its dynamic response. The vibration is excited by a harmonic force or by an impact and the response is analysed using the modal analysis. The dynamic test results in the measured natural frequencies and modes of natural vibration, [2], [3]. The laboratory tests enable to define the crack characteristics and their propagation, the effect of damping etc. The degree of damage manifests itself by a change of natural frequencies and natural modes.

A special method COMAC (Coordinate Modal Assurance Criterion), [3], was developed for the evaluation of natural frequencies. It sufficiently
indicates a crack as far as it remains normal to the plane where the shortest radius exists on the surface of the normal mode.

The COMAC method is intended for a comparison of vibration modes and is suitable in the cases when the natural frequencies of various vibration modes are very close each other. The method informs about the conformity of natural vibration modes between two states or between undamaged and damaged structure. It is expressed by the relation:

$$COMAC(j) = \frac{\left[\sum_{i=1}^{N} v_{iv}(j) \cdot v_{id}(j)\right]^2}{\sum_{i=1}^{N} v_{iv}(j) \cdot v_{iv}(j) \cdot \sum_{i=1}^{N} v_{id}(j) \cdot v_{id}(j)}$$  \hspace{1cm} (1)$$

where:

- $v_{iv}(j)$ is the displacement of the $i$-th vibration mode of the first state of the structure at point $j$,
- $v_{id}(j)$ is the displacement of the $i$-th vibration mode of the second state of the structure at point $j$,
- $N$ is the number of excited vibration modes.

If the modal analysis is used for the damage identification it is necessary to eliminate fully the influence of the changes of boundary conditions, temperature and other imperfections. Nevertheless, the monitoring of dynamic response remains an ideal method of observation of increase of the damage in structures.

Therefore, a hammer impact was applied to the excitation of the test specimens after each static load steps 0, 50, 100, 150, 200, 250 kN. The results of the modal analysis is represented in the Figure 5 which shows how the damages in specimens reduce their natural frequencies. The Figure 5 indicates also that the natural frequency remains unchanged up to the design load $F = 50$ kN and then it falls down.

The theoretical first natural frequency [4] of the specimen A 3 according to the ANSYS software is

$$f_{(1)} = 170.03 \text{ Hz}$$  \hspace{1cm} (2)$$

while the experimental value of the undamaged specimen

$$f_{(1)} = 176 \text{ to } 177 \text{ Hz}$$  \hspace{1cm} (3)$$

was measured after the hammer blow test and evaluated by the modal analysis. Their exactness is about 3 to 5 %.

The research [1] follows the fatigue properties of the specimens as well and the hammer blow test together with the modal analysis are intended to be performed after each 500 000 stress cycles. The similar results as in the Figure 5 but the natural frequency plotted as the function of the number of stress cycles are expected and will be shown at the conference.
5 Conclusions

The theoretical and experimental research of plates stiffened by two girders was performed using the finite element method, hammer blow test and modal analysis. The design load of the specimen was 50 kN while the ultimate load at its collapse reached 170 to 240 kN. The specimens provide rather high natural frequencies. The comparison of the measured static deflections and the first natural frequency with the theoretical values is satisfactory, however, a dispersion occurs.

The dynamic tests show a decreasing tendency of the natural frequency if the increasing static load is applied. However, no changes were observed as far as the design load was not reached. The fatigue tests have been running and they should prove the endurance of reinforced concrete slabs and their fatigue life. It has a far reaching consequences on the maintenance and inspection intervals of bridges.
Acknowledgement

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References


