Modeling of masonry structures strengthened by pre-tensioned glass-fiber-reinforced polymer laminates

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Abstract

Various aspects of rehabilitation of aging structural elements built from quasi-brittle materials such as masonry or concrete are investigated. In this paper a popular glass-fiber reinforced polymer matrix laminate bonded to masonry plate is used to add strength and provide for the lack of tensile, flexural and shear resistance of these materials. Here, the conventional application of such an external strengthener is further improved by initial pretension of the fiber mesh during fabrication of composite laminate. This modification reduces, for certain loading conditions, the undesirable tension in the masonry and prevents excessive deflection of the entire structure.

1 Introduction

The undoubtable benefits offered by composite materials such as a high strength, light weight, non-corrosive properties, etc., have recently attracted many design engineers in civil engineering industry primarily in conjunction with rehabilitation and repair of concrete and masonry structures. This paper advocates the use of advanced fiber reinforced epoxy composites in structural engineering as an external strengthener, particularly in applications where the conventional strengthening methods are less effective.

Here, the emphasis is put on improving the up-to-date use of composite laminates in above applications by pre- or post-stressing fibers during fabrication. Taking an advantage of the extremely high strength of the fiber phase provides an increase of flexural stiffness
of masonry - laminate structural elements subjected to bending and significantly reduces the lateral displacements. To accomplish this task we propose an energy based optimization procedure, developed within a framework of the modified Dvorak’s Transformation field analysis (TFA) [2], in which the initial fiber pre-stress is used as a control variable.

In this paper a comparable attention is given to experimental investigation of the behavior of actual masonry-laminate composite structures and to corresponding theoretical studies presented on basis of the refined laminated plate theory [2].

2 Material properties from experiments

Experimental results derived in the course of our investigation are discussed in this section to reveal a certain new aspects of behavior of the masonry structures strengthened by fiber reinforced polymers and provide an important data for numerical studies.

In the present paper, we limit our attention to a masonry plate reinforced by a glass-fiber/epoxy composite intended for applications, where bending and shear loading conditions are decisive. Henceforth, this structural element is sought in terms of two layered composite laminate.

Elastic behavior of the masonry-mortar system is generally well understood and does not require special attention, [1]. The overall material properties of glass-fiber/epoxy system, however, is strongly influenced by both the material properties of individual phases and the manufacturing process. These considerations are the basis for a selected experimental program discussed in the sequel.

2.1 Material properties of epoxy matrix

Our objective was to determine the elastic modulus and the specific strength of an epoxy matrix in tension and compression. Experiments were conducted on several specimens made of CHS-EPOXY531 resin mixed with curing agent CHS-P11 at room temperature (23°C).

Two specimens, 120 mm long, with 15x10 mm cross-sectional dimensions, were made for tension tests. For compression tests the third specimen was cut into four 27 mm long pieces. Each specimen was instrumented with symmetrically arranged strain gages (Type 6JP 120A) manufactured by Mikrotechna company. These strain
gages permit reliable strain measurements up to $10^0/00$. Temperature influence was eliminated by an additional strain gage attached to an untested specimen made of the same material. Testing was performed in FPZ 100 mechanical testing machine with displacement increments applied to the specimen. These were automatically transformed by the machine to corresponding tensile or compressive load increments. A resistance of the strain gages was evaluated by a UPM devise, Fig. 1, which provided the actual strain readout in units $[\mu\text{m m}^{-1}]$ to be stored in PS machine. All tests were performed at room temperature.

First, the set of shorter specimens were tested in compression to determine the compressive strength of the epoxy matrix. The prescribed load increment was set equal to 1kN. The typical machine readout appears in Fig. 1. Since the experiment was actually displacement controlled, the loading path contains also the descent branch. An influence of relaxation is evident. A significant increase in cross-sectional area during plastic flow is manifested by hardening of the material before collapse. The magnitudes of compressive strength limits attained for tested specimens range from $R^{-} = 104 \text{ MPa}$ to 106 MPa.

The tensile loading tests were performed similarly. The objective was to determine both the modulus of elasticity and the tensile strength of the epoxy matrix. In this case the loading program was characterized by an increment of tensile force equal to 0.5 kN. After each surcharge the specimen was unloaded to initial load level of 0.5 kN. Strain readout was picked up twice during each loading/unloading step in 120 s interval. The stress-strain diagram for the first specimen is shown in Fig. 2a. As oppose to compressive tests the specimens loaded in tension exhibited quite different behavior. An unexpected growth of the elastic modulus of the first specimen,
Figure 2: Stress - Strain relationship: (a) - epoxy matrix, (b) - glass fiber/epoxy laminate

Fig. 2a, and its rather low tensile strength ($R^+ = 24$ MPa) can be attributed to presence of pores in epoxy resin during curing. Manufacturing of the second specimen was done more carefully resulting in increase of tensile strength up to $R^+ = 33.1$ MPa. Derived magnitudes of the elastic moduli range from $E = 3200$ MPa to 3900 MPa in tension and $E = 3400$ MPa to 4000 MPa in compression.

2.2 Material properties of glass/epoxy laminate

Computational modeling discussed in Section 4 requires knowledge of the material properties of composite strengtheners. This section describes an experimental evaluation of the elastic moduli and strength limits of a hand-made two layer glass/epoxy laminate selected for an external reinforcement of the masonry plate.

The $[0/90]$ laminate was manufactured such that a 0.5 mm thick layer of the epoxy resin was swabbed on a PE foil. Then the fiberglass fabric was placed in the resin and coated with another layer of epoxy matrix. The process was repeated, but in this case the fiberglass mesh was arranged in $90^\circ$ angle with respect to warp fibers. The entire plate with in-plane dimensions $500 \times 400$ mm and 3 mm thickness was then cut into 12 pieces for testing purposes. Both ends of each 160 mm long specimen were adhesively bonded to plate attachments leaving about 90 mm of free specimen length. Two specimens were used to verify reliability of the attachments.

The effective elastic moduli of the laminate were determined from three separate tests. Due to significant unevenness of the plate surface it was necessary to substitute for previously used strain gages by an
inductive extensometer Hottinger, type D1, capable of measuring an extension of the specimen up to $\Delta l = \pm 0.25$ mm. As in experiments with epoxy matrix the tests were carried out on FPZ 100 testing machine with applied displacement increments corresponding to 1 kN at the rate of 0.5 mm min$^{-1}$, (4 kN min$^{-1}$). The extension readings were provided twice at every loading/unloading step in 60s intervals.

The stress-strain diagrams appear in Fig. 2b. It is evident that during experiments the epoxy matrix was overloaded resulting in initiation of the microcracks manifested by whitening of surface of the plate. This provides an explanation of the decrease of elastic slope shown in Fig. 2b. Above observation was even more pronounced when loading until failure. After fracture, however, an elastic shrinkage took place causing the specimen, except for fracture surface, to attain its original appearance.

To determine a compressive strength of the composite, an eight layer plate was tested. The loading conditions were similar to tensile tests. Until 90% of the compressive strength limit the specimens exhibited no external sign of the plastic flow of the matrix phase. An additional increase of external load, however, led to a significant deformation of the laminate cross-section, which caused a subsequent delamination of surface plies of the composite structure.

Experimentally determined values of mechanical properties of the hand-made composite laminate together with fiber volume fraction $c_f$ are listed in Table 1.

<table>
<thead>
<tr>
<th>properties</th>
<th>in tension</th>
<th>in compression</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elastic modulus (MPa)</td>
<td>11200 - 12500</td>
<td>---</td>
</tr>
<tr>
<td>Strength limit (MPa)</td>
<td>130 - 170</td>
<td>110 - 120</td>
</tr>
<tr>
<td>$c_f$</td>
<td>0.42</td>
<td>0.42</td>
</tr>
</tbody>
</table>

Table 1 shows that the compressive strength of the laminate is about 25% lower than corresponding tensile strength limit. Rather low magnitudes of the effective elastic moduli of the laminate can be attributed to selected manufacturing process, which could not, in no account, prevent an evolution of internal defects during fabrication.

### 2.3 Masonry plate reinforced by glass/epoxy laminate

The principal part of the experimental work was concerned with the behavior of the masonry plate strengthened by the fiber/glass epoxy laminate. The prime purpose of this investigation was to collect the
Reinforced masonry plate in bending

Figure 3: (a) - force-displacement record of the masonry/composite laminate in bending, (b) - relationship between the relative bending moment $\eta$ and the relative curvature $\kappa$.

The masonry plate was manufactured from solid bricks CP25 placed within wooden frame having inner dimensions equal to 875x440 mm. The mortar joints in 10 mm thickness were filled with the cement mortar MC10 and the entire plate specimen was left in rest for 14 days. The actual lamination was then performed directly on the mortar plate in the same way as described in Section 2.2.

The actual experiment, simply supported plate loaded in three point bending, was conducted on EUS 40 testing machine in stress control environment. The machine assured application of selected force increment of magnitude 1 kN at rate of 4 kN min$^{-1}$. In each incremental loading the load was kept constant for 15 s prior to displacement reading. The vertical deflection was measured twice in each loading step (right after load increment and then after 15 s).

The entire experiment was characterized by a creep deformation of the masonry manifested by an increase of the vertical deflection during constant load step, Fig. 3a. Audible cracking during experiment provided the solid evidence of loading the masonry structure beyond the elastic limit. The typical quasi-brittle behavior of the masonry material is also displayed in Fig. 3a. The onset of damage was caused by an initiation of microcracks in the tensile region under the applied load. Gradual accumulation of damage, localized within this region, then led to overloading the masonry in compression and eventually to the fatal failure at load level equal to 23 kN.
3 Estimates of bounds on bending stiffness

Fig. 3a shows that the bending stiffness of the reinforced masonry plate loaded in cyclic bending decreases. To estimate the bending stiffness of the plate at the beginning and then at the end of the loading program we propose two theoretical models developed within the context of the theory of plasticity, Fig. 4.

Figure 4: Models of elastic-plastic cross-section: (a) - tension included, (b) tension excluded

Fig. 4a provides through thickness distribution of the elastic-plastic normal stress including tension in the masonry, while Fig. 4b assumes no tension in the masonry. Relationships between the relative bending moment $\eta$ and the relative curvature $\kappa$ derived from both models appear in Fig. 3b.

Either curve in Fig. 3b was constructed for ratio of the elastic moduli of the composite laminate and the masonry plate $E_c/E_m = 2.08$ and the limit compressive strength of the masonry material $f_z = 5.3$ MPa. Fig. 3b reveals that the model in Fig. 4a provides significantly stiff response of the masonry/composite system as oppose to model in Fig. 4b, used to simulate an evolution of microcraks in tensile region. The shaded region in Fig. 3b represents the actual behavior of the laminated structure.

It is worth to note that the curve (a) in Fig. 3b approaches a yield curve obtained by dividing the cross-section of the plate into several layers with varying secant stiffnesses depending on the specific thru thickness distribution of the axial stress. The corresponding axial strain then follows from the constitutive equations. In addition, the relation between the bending moment and the curvature in Fig. 3b provides useful information for deriving the distribution of the curvature along the critical cross-section of the plate ($@x_1 = l/2$) for various magnitudes of the applied load. Then, by integrating the curvature curve using for example the Mohr theorem we arrive at the relation between the applied load and respective vertical displace-
ment evaluated at \( x_1 = x_2 = 0 \). The curves (a) and (b) in Fig. 3a correspond to two theoretical models (a) and (b) discussed in this section. As shown in Fig. 3a they provide a reliable estimate of the lower and upper bounds on the actual behavior of the masonry-composite material system observed during the experimental program.

4 Numerical modeling

The principle reason for using composites and the composite laminates in particular as an external strengthener for rehabilitation of aging structures is the high fiber strength. As described in our recent work [2], this benefit can be further exploited by initial fiber pre-stress prescribed during fabrication.

4.1 Theoretical formulation

We limit our attention to elastic analysis. A refined laminated plate theory is used for accurate representation of the local ply stresses [2]. The macroscopic response of each ply is that of a homogeneous layer with certain effective properties. In the local coordinates, \( x_1 \) is aligned with the fiber direction, we write the ply constitutive relations for the ply \((i)\) as

\[
\sigma^i_m = L^i_m \epsilon^i_m + \lambda^i_m, \quad \sigma^i_s = L^i_s \epsilon^i_s, \quad (1)
\]

where \( L^i_m \) is the \((3\times3)\) in-plane stiffness matrix of the ply; \( L^i_s \) is the \((2\times2)\) matrix which contains the out-of-plane shear moduli; \( \lambda^i_m \) is the uniform ply eigenstress and \( \sigma^i_m, \sigma^i_s \) are the in-plane and out-of-plane stress vectors respectively.

After substituting for ply strains and stresses from eqs. (1) into modified variational principle given by

\[
\Pi_0(u, \beta, \lambda_m) = U_{int} + \int_{S_m} \sum_{i=1}^{N-1} (\beta_i)^T g_i dS - U_{ext}, \quad (2)
\]

and taking the variation of (2) with respect to \( u \) and \( \beta \) we arrive at the system of governing differential equations to be solved for unknown displacements \( u \) and Lagrange multipliers \( \beta \). The Lagrange multipliers are introduced in eq. (2) for incorporating the displacement continuity conditions [2].
4.2 Applications

First step was to compare the experimental measurements discussed in former sections with results derived from numerical calculations. To this end, we selected a masonry-laminate composite system loaded in three point bending with the center-point force $F = 10$ kN. The effective properties of the laminate determined in Section 2.2, Table 1, were used in the analysis. The actual difference was found to be less than 3% ($u_3^{exp} = 2.16$ mm, $u_3^{plt} = 2.22$ mm). The through thickness variation of the tensile stress $\sigma_{11}$ evaluated in the center of the plate appears in Fig. 5a. It is evident that the resultant stresses in the masonry plate exceeded the allowable strength limits.

![Figure 5: Laminate stresses in MPa at $x_1 = x_2 = 0$, (a) - before optimization, (b) - after optimization](image)

The following section provides a remedy of such undesirable results by pre-tensioning fibers during manufacturing process. Here, the piecewise uniform thru thickness distribution of the initial fiber pre-stress follows from the energy based optimization procedure. In the present approach the fiber pre-stress or eigenstress [2] is used as control variable. The objective functional assumes the form

$$\Pi_{obj}(u, \beta, \lambda_m) = \bar{U}_{int} + \int_{S_m} \sum_{i=1}^{N-1} (\beta_i)^T g_i dS - U_{ext} + \sum_{i=1}^{N} f_i,$$

where $\bar{U}_{int}$ represent a modification to the internal energy of the system [2]. Last term in (3) provides certain constraints on the local phase stresses to guarantee that the desired components of the local stresses do not exceed certain allowable magnitudes. Based on the experimental observation, Section 2.1, we propose the penalty functions $f_i$ in the form

$$f^i = \left( \frac{\sigma_{f_{11}}^i}{R_f^+ + A_1} \right)^{B_1} + \left( \frac{\rho \sigma_{m}^i}{R_m^+ + A_2} \right)^{B_2} + \left( \frac{\sigma_{m_{eq}}^i}{Y_m + A_3} \right)^{B_3},$$
where $\sigma_{j_{11}}^f$ is the actual tensile stress in the fiber and $R_f^+$ is the corresponding strength limit; $\sigma_{m}^+$ stands for the maximum principal tensile stress found in the matrix and $R_m^+$ is the allowable tensile strength limit. Finally, $\sigma_{m_{eq}}^t$ represents the equivalent Mises stress in the matrix, which should not exceed the prescribed yield stress $Y_m$. The third term in eq. (4) applies when loading the epoxy matrix in compression, whereas the second term is decisive when tensile loading takes place.

Fig. 5b shows the distribution of $\sigma_{11}$ after optimization. It is evident that the pre-tensioned composite laminate not only reduced the maximum vertical deflection by almost 50%, $u_{3\text{opt}} = 1.13$ mm, but also annihilated undesirable tensile stresses in the masonry plate.

5 Conclusions

In the present paper the behavior of the masonry plate reinforced by glass-fiber epoxy-matrix laminate was studied. Initial experiments, conducted on the epoxy specimens revealed the characteristic plastic-fracturing response of the epoxy matrix. Additional experiments carried out on the reinforced masonry plate proved the capability of the fiberglass/epoxy composite to serve as an external strengthener.

The second part of our research was devoted to numerical modeling in order to bridge the common gap between the experimental work and the theory. In this part of our investigation we focused the possibility of pre-stressing the fiber mesh when manufacturing the composite strengthener. As shown in Fig. 5b the obtained results are quite encouraging.

Acknowledgment

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References
