High propulsive efficiency by a system of oscillating wing tails

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Abstract

The analysis of harmonically oscillating wing tails, considered as an unsteady propulsion system, is extended to the case of large amplitudes of oscillatory motion. Each wing undergoes a combined transverse and angular motion, at the same frequency, and in a uniform inflow. A 3-D unsteady vortex-lattice technique is applied to model the flow around the system, including leading-edge separation and dynamic-stall effects. A free-wake analysis is incorporated in order to account for the effects of non-linear wake dynamics, at high translation velocities and at increased amplitudes of oscillatory motion. Numerical results are presented vs. experimental measurements for the thrust coefficient and the efficiency of the system over a range of motion parameters, including reduced frequency, Strouhal number, feathering parameter and phase lag between oscillatory motions. These results indicate that high efficiency can be obtained under conditions of optimal wake formation. Thus, the present method can serve as a useful tool for assessment and the preliminary design of this kind of non-conventional propulsive systems with optimised performance.

1 Introduction

The optimum propulsive performance of thrust-producing harmonically oscillating foils depends on a large number of parameters. Recent theoretical and experimental investigations, Anderson et al [1], report maximum measured efficiency of 87%, under conditions of optimal wake formation. These conditions correspond to large amplitude of transverse motion-to-chord ratio (of order 1), large maximum values of the instantaneous angle of attack (between 15° and 25°), and phase lag between oscillatory motions 75° - 90°, when the
pivot point for the angular motion is at the 1/3 chord length from the leading edge. Under these conditions, very high propulsive efficiency (of order 1) is obtained in the range of Strouhal numbers in the interval from 0.1 to 0.4, accompanied by a thrust production ranging from small to large values. This observation permits the exploitation of flapping-foil devices for the optimum production of thrust with stability, Czarnowski [2]. Visualisation experiments in 2D flapping foils, Anderson et al [1], Freymouth [3], have shown that the high values of the thrust and of the efficiency are associated with the formation of a leading-edge vortex, also referred to as dynamic-stall vortex, which, for specific parametric combinations, subsequently is amalgamated with the trailing edge vorticity.

In order to account for the 3D effects, in the present work a vortex-lattice technique coupled with a non-linear wake model, previously developed for the attached-flow problem around thrust-producing oscillating wings of moderate aspect ratio, Belibassakis et al [4], is extended to model the unsteady leading-edge separated flow and dynamic-stall effects, which are most significant in the above range of parameters. Following similar applications concerning the modelling of 3D dynamic-stall effects on the flow over the blades of wind turbines, Voutsinas & Riziotis [5], Riziotis & Voutsinas [6], the extra unknown, namely the rate at which vorticity is shed from the leading-edge separation point to the associated vortex sheet, is specified by means of the 2D semi-empirical ONERA (Office National d'Etudes et de Recherche Aerospatiales) model, Petot [7], Bierbooms [8], applied in a section-by-section procedure over the whole span of the wing.

2 Modelling of the unsteady flow over oscillating wing tails

For the description of the kinematical characteristics of the oscillating wing tails and of the induced flow dynamics two reference systems are used; see Belibassakis et al [4]. The inertial coordinate system, \((X,Y,Z)\), and the body-fixed, moving coordinate system, \((x,y,z)\), that is translated with constant velocity \((U)\) with respect to the former, executing simultaneously a combined transverse \((Z = Z_o + h(t), h(t) = h_o \sin(\omega t))\) and angular \((\theta(t) = \theta_o \sin(\omega t + \psi))\) oscillatory motion at the same frequency \((\omega = 2\pi f)\). The most important non-dimensional parameters describing the kinematics and dynamics of the system are the reduced frequency \(k = \omega c / 2U\) and the feathering parameter \(\chi = \theta_o U / h_o \omega\), where \(c\) stands for the chord of the wing and \(h_o, \theta_o\) denote the amplitudes of the transverse and angular oscillation, respectively. In the present work the Strouhal number \(St = 2 f h_o / U\) and the maximum instantaneous angle of attack \(\alpha_o\), defined by

\[
\alpha_o = \max[\alpha(t)], \quad \alpha(t) = \tan^{-1}\left(\frac{dh / dt}{U}\right) - \theta(t),
\]

are used instead of the former non-dimensional parameters, because of their direct relevance to the dynamics of the wake, Anderson et al [1].
2.1 Formulation of the problem and representation of the flow field

The flow around the oscillating wing(s) is assumed to be inviscid and incompressible. As each lifting component is steadily translated with a mean velocity $U$, the vorticity created in the boundary layers of its upper and lower surfaces is continuously shed into the wake. Furthermore, this vorticity, which is subsequently convected away from the wing with the local velocity, is assumed to be concentrated into sheets of infinitesimal thickness, constituting the trailing vortex wakes. It is also assumed that the flow outside these vortical regions is irrotational.

Under the previous assumptions, the mathematical problem concerning the unstalled flow around the oscillating wings, consists of the Laplace equation, the solid (no-entrance) boundary condition, the kinematical and the dynamical conditions on the free vortex sheets, and the Kutta condition along the trailing edge and the tip(s) of the wing, see Ref. [4]. For moderate values of the maximum angle of attack, this problem is effectively treated in the framework of viscous-inviscid interaction techniques. In particular, an unsteady vortex-lattice technique, coupled with a non-linear wake model, has been developed in Ref. [4], and successfully applied to the problem of thrust production by oscillating wings. By using Green's theorem, the disturbance velocity field (due to the presence and the motion of each lifting body) is represented by means of singularity distributions, as follows

$$\ddot{u} = \frac{1}{4\pi} \left\{ \int_{S_b} \left( \frac{\sigma}{r^3} + \frac{\hat{\gamma} \times \hat{r}}{r^3} \right) dS + \int_{S_w} \frac{\hat{\gamma} \times \hat{r}}{r^3} dS \right\},$$

where $\hat{\gamma}$ is the surface vorticity, associated with the tangential discontinuity of the velocity field on the solid boundary ($S_b$) and on the vortex sheets ($S_w$), and $\sigma$ stands for the source-sink distribution on the body. These singularities are represented by a discrete source- and vortex-lattice extended over their support.

For attached flows the wake vorticity is shed from the separation lines, which in this case consist of the trailing edge and the tip(s) of the wing, at a rate determined by Kutta condition. The problem is solved by a time-marching technique permitting the modelling of continuous shedding and transport of vorticity in the wake, and the simultaneous deformation of the vortex sheets.

2.2 Static-stall vs. dynamic-stall characteristics

In all lifting bodies operating in steady-flow conditions, the first sign of approaching stall is, usually, the appearance of a region of limited flow separation in the suction side near the trailing edge. This effect is associated with the progressive fall of the rate of increase of the lift coefficient with incidence, which prior to this point was increasing linearly. We define this point as $\alpha_s$, see Fig. 1. As the angle of attack is further increasing, the region of flow
separation gradually extends all over the suction side, and finally, at relatively large angles of attack, the flow separates immediately after the leading edge. In this interval of angles of attack the lift coefficient attains its maximum value, after which it drops, defining the static-stall region. One parameter which is closely related to the non-linearity of the system in the static-stall region is the loss of lift due to stall, defined by

$$\delta C_l(\alpha) = C_l(\alpha)|_{\text{linear}} - C_l(\alpha),$$

where $C_l(\alpha)|_{\text{linear}}$ is the lift coefficient at small angles of attack extrapolated to the stall region, see Fig. 1, and $C_l(\alpha)$ is the measured value of the lift coefficient. Thus, $\delta C_l(\alpha)$ is zero at small angles and becomes significant for $\alpha > \alpha_s$. This function together with the experimental curve for $C_l(\alpha)$ are shown in Fig. 1 for the NACA0012 section (for which $\alpha_s \approx 12^\circ$), from Abbot & Doenhoff [9].

In the case of unsteady lifting flows, where the angle of attack oscillates within an interval, if the instantaneous angle of attack takes values that are the same order with the static-stall angle $\alpha_s$, or greater, then large hystereses develop in the fluid-dynamic forces and moments with respect to their steady

![Figure 1: Experimental lift coefficient and loss of lift due to stall for NACA0012 wing section, from Abbot & Doenhoff [9].](image-url)
counterparts at the same angle of attack, Mc Croskey [10]. In this case, the maximum values of the unsteady lift, drag and pitching moment coefficients can greatly exceed their static counterparts. The term dynamic stall is used to refer to unsteady separation and stall phenomena associated with the flow around lifting bodies that are forced to execute oscillatory motion corresponding to time variations of angle of attack in this region of parameters.

2.3 Modelling of leading-edge flow separation

In the present work the vortex-lattice method developed in Ref. [4], for treating non-separated flows around oscillating wing-tails, will be extended to the case of leading-edge separated flows, corresponding to operating conditions which are characterised by higher values of the maximum angle of attack.

Remaining in the framework of viscous-inviscid interaction techniques, an additional vortex sheet is introduced to the model, associated with the vortical flow induced by separation. In order to solve the potential-flow problem outside the vortical surfaces, additional information characterising the simplified effects of viscosity must be supplied to the potential-flow solver; see, e.g., Katz & Plotkin [11]. Thus, except of the location of the additional separation point, the strength of the vortex sheet must also be provided to the model.

Figure 2: 3D plot of the wake shape at the root sections of stationary flat plate wing with $AR=2$, at $a=15^\circ$. 
Following similar applications associated with the modelling of 3D dynamic-stall flow, Voutsinas & Riziotis [5], Riziotis & Voutsinas [6], in the present work this location is fixed at the leading edge of the wing. The other free parameter, namely the rate at which vorticity is shed from the leading-edge separation point to the associated vortex sheet, will be specified by means of the ONERA model, Petot [7], applied in a section-by-section procedure over the whole span of the wing. The discrete scheme materialising the shedding of vorticity in the case of leading edge separation is depicted in Fig. 2 for a stationary flat-plate wing starting from the rest. In this figure, the trailing and the leading-edge vortex sheets at the earliest stages of development are shown, and the orientation of vorticity in the formed closed vortex rings is as implied by the application of Kelvin's theorem, Katz & Plotkin [11], in a closed circuit surrounding the lifting body.

The ONERA model, Petot [7], Bierbooms [8], belongs to the class of engineering models developed for the prediction of 2D dynamic-stall flow characteristics in the context of blade element theory. According to this model, stall is accounted for by solving a forced system of differential equations providing, along with other quantities, the strength of the separated vorticity. The model coefficients, including also the forcing terms, are usually determined from wind-tunnel measurements. The differential equation for the non-linear circulation \( \Gamma_{\text{sep}} \) associated with dynamic-stall effects is

\[
\frac{d^2 \Gamma_{\text{sep}}}{dt^2} + \frac{A}{\tau} \frac{d \Gamma_{\text{sep}}}{dt} + \frac{R}{\tau^2} \Gamma_{\text{sep}} = -\frac{c}{2} \left( \frac{R}{\tau^2} \omega_r \delta C_{L} + \frac{E}{\tau} \frac{dw_n}{dt} \right). \tag{4}
\]

In the above equation \( c \) is the chord length of each section of the oscillating wing, \( \tau = c/(2 \omega_r) \) is a time constant, and \( \delta C_{L} \) is the static loss of lift due to stall, Eq. (3), corresponding to the instantaneous angle of attack \( \alpha(t) \) at each section. In Eq. (4), \( \omega_r \) and \( w_n \) are the average parallel and normal velocity components, respectively, at the forward part of the airfoil chord. Also, in the same equation \( A, R, E \) are experimentally determined coefficients, which are dependent on the geometry of the section. In the case of a NACA0012 wing section these coefficients are given by the following formulae, Bierbooms [8],

\[
A = 0.25 + 0.1 \left( \delta C_{L} \right)^2, \quad \sqrt{R} = 0.2 + 0.1 \left( \delta C_{L} \right)^2, \quad E = -4.01 \left( \delta C_{L} \right)^2, \tag{5}
\]

Since \( \delta C_{L} = 0 \) for \( \alpha(t) \leq \alpha_s \), from Eqs. (4) and (5) it is obvious that the ONERA model results in \( \Gamma_{\text{sep}} = 0 \) in all cases characterised by \( \alpha_s < \alpha_s \). This fact renders the present model fully compatible with the non-separated flow vortex-lattice model, developed in a previous work (see Belibassakis & Politis [4]), in the range of operating conditions corresponding to \( \alpha_s < \alpha_s \).
Due to its 2D origin, the ONERA dynamic-stall model requires some modifications to apply to 3D problems of the same character. Thus, at a first stage, the fully attached flow problem is solved, in order to determine the distribution and the history of the angles of attack at all sections of the lifting body. At a second stage, this information is exploited for determining at each discrete time step the coefficients of the differential equation (4), and satisfying all requirements (equation/boundary conditions) of the problem along with the production, shedding and transportation of vorticity from the leading edge.

3 Numerical results and discussion

3.1 Impulsively started flat plate at high angles of attack

In Fig. 3 the normal force coefficient \( C_n \) as calculated by the present method for a stationary flat plate of \( AR=2 \) is compared with experimental data from Parkinson et al [12]. It can be observed from Fig. 3 that present method provides good results at low and moderate angles of attack, where separation effects are negligible. Furthermore, at higher angles of attack, in the neighbourhood of the static stall angle \( (\alpha \approx 17^\circ) \) and beyond that point, up to angles of order \( (\alpha \approx 22^\circ) \), the present method produces satisfactory predictions.

![Figure 3: Normal force coefficient of stationary rectangular wing (AR=2).](image-url)
3.2 Flapping wings at various operating conditions

In a recent theoretical and experimental study Anderson et al [1] report integrated thrust, power and efficiency measurements in the case of harmonically flapping foils with NACA0012 sections, producing thrust at various operating conditions. Tests were conducted in a tank, at an average \( Re \approx 40000 \). To demonstrate the beneficial influence of dynamic-stall effects on the efficiency of the system, under operating conditions corresponding to optimal wake formation and at increased maximum angles of attack, in Fig. 4 the history of the lift and the thrust coefficient of a flapping wing of \( AR=7 \) is shown, as calculated by the present method (solid lines), and as calculated by the fully attached-flow model (dashed lines), Belibassakis et al [4]. In this case, the operating conditions are: \( \theta_0 / c = 0.75 \), \( \psi = 75^\circ \), \( St = 0.35 \), leading to a maximum (instantaneous) angle of attack \( \alpha_o = 22^\circ \). We can observe in Fig. 4 the modified sinusoidal waveform of the angle of attack, which, in general, is experimentally found to affect dynamic-stall inception and evolution; see, e.g., Anderson et al [1] and the references cited therein. We can also see in this figure that, in the above operating conditions, the inclusion of unsteady leading-edge separation in our model has a tendency to increase slightly the lift and the thrust coefficients of the flapping wing (solid lines), as compared to the results obtained by the attached-flow model (dashed lines). This fact produces also a secondary beneficial influence on the calculated efficiency.

In Figure 5, the thrust coefficient and the efficiency of the flapping wing with the same, as before, geometrical characteristics (trapezoidal planform, \( AR=7 \), NACA0012 sections) is compared with experimental data obtained for a 2D flapping foil of the same section, Anderson et al [1]. The relatively large aspect ratio of the wing makes feasible such a comparison. Numerical predictions are obtained by the present model in the range of operating conditions corresponding to \( St = 0.2 - 0.4 \), and for the following characteristic cases,

**Case 1** : \( h_o / c = 0.75 \), \( \psi = 90^\circ \), \( \alpha_o = 14^\circ - 16^\circ \).

**Case 2** : \( h_o / c = 0.75 \), \( \psi = 75^\circ \), \( \alpha_o = 19^\circ - 22^\circ \).

These values of the motion parameters correspond, in general, to optimum operating conditions for the unsteady propulsion system. In all the above cases the maximum angle of attack during the cycle of operation exceeds by a value of \( 2^\circ \) to \( 10^\circ \) the static-stall angle, rendering the dynamic-stall effects more or less important. Numerical predictions obtained by the present model are in a relative agreement with experimental measurements, indicating further that very high levels of efficiency can be achieved by 3D thrust-producing systems of flapping wings. Moreover, it can be observed in Fig. 5 that the efficiency of the system remains practically at a very high level over an extended region of \( St \), permitting, thus, small or significant thrust production with stability. This result could be found very useful in the design of non-conventional unsteady propulsion systems with flapping wing-tails.
Figure 4: Variation of angle of attack, lift and thrust coefficients in the case of a flapping wing of $AR=7$, during the first cycles of operation starting from rest.

Figure 5: Thrust coefficient and efficiency of a flapping wing of $AR=7$ vs. experimental data for a 2D flapping foil with the same section (NACA0012).
4 Conclusion

A 3D unsteady vortex-lattice technique coupled with a non-linear wake model is applied to the analysis of thrust-producing flapping wings, considered as an unsteady propulsion system. Numerical results verified by experimental data indicate that high levels of efficiency (of order 85%) can be obtained, in the range of operating conditions corresponding to large amplitudes of motion and optimal wake formation. As the maximum angles of attack are increased above the static-stall angle, the modelling of leading-edge separation is important, in order to account for dynamic-stall effects, that may further increase the efficiency. Another parameter with a beneficial influence on the propulsive characteristics of the system is wing’s flexibility. Future work is planned towards this direction.

References