Comparison of a truss model for confinement with UBC and NZS requirements
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Abstract

A truss model with structural instability, recently proposed for the confinement of concrete columns, is compared with the requirements of Uniform Building Code and New Zealand Standards, as regards the amount and spacing of transverse reinforcement. The truss model is found in a satisfactory approximation with the UBC. Whereas the NZS require a larger amount of transverse reinforcement, by assuming that this compensates the ductility loss due to a possible heavy axial compression.

1. Introduction

The last twenty-five years, a lot of test results and empirical formulae, based on them, have been published, concerning the confinement of concrete columns [1, 2, 5-7]. The above have been taken into account in the formulation of requirements of various seismic codes [3, 8].

Recently, a theoretical truss model has been proposed [4], which interprets the premature global instability of concrete columns, with poor transverse reinforcement, as due to buckling of interior vertical concrete struts and not to hoop fracture, as was previously interpreted [2,5].

Here, by an application on a typical reinforced concrete column, the results of the proposed truss model will be compared with the relevant requirements of the Uniform Building Code [8] and the New Zealand Standards [3], as regards the amount and spacing of transverse reinforcement.

2. UBC and NZS requirements

According to the Uniform Building Code [8], the mechanical ratio $\rho_t$ of transverse reinforcement, along a principal direction of a rectangular column
section, must be
\[
\rho_t = \frac{A_t}{b s} \cdot \frac{\sigma_{ty}}{\sigma_{cy}} > 0.09,
\]
where \(A_t\) the total section of transverse bars and \(b\) the width of column section, perpendicular to the principal direction under consideration, \(s\) is the spacing of confined sections over the height of column, \(\sigma_{ty}\) the tensile yield strength of transverse reinforcement and \(\sigma_{cy}\) the compressive strength of unconfined concrete.

The spacing \(s\), of confined sections over the height of column, must be, according to UBC,
\[
s < \min\left(\frac{b_o}{4}, 10\text{cm}\right),
\]
were \(b_o\) is the minimum dimension of column section.

The New Zealand Standards [3] require a larger value for the mechanical ratio \(\rho_t\) of transverse reinforcement, by assuming that this will compensate the ductility loss due to a possible heavy axial compression. So, according to NZS, must hold
\[
\rho_t > 0.12 \left(0.33 + 1.67 \frac{N}{\sigma_{cy} bd}\right),
\]
where \(d\) is the column section depth and \(N\) is the compressive axial load of column, which must be kept less than an upper bound,
\[
N < 0.6\sigma_{cy}bd.
\]
For the spacing \(s\) of transverse reinforcement, the NZS prescribe
\[
s < \min\left(\frac{b_o}{5}, 15\text{cm}, 6D_l\right),
\]
where \(D_l\) is the minimum diameter of longitudinal reinforcement.

3. The proposed truss model

3.1. Description of model [4]

A concrete column with rectangular section is considered, with neither concrete cover nor longitudinal reinforcement, subjected to concentric axial compression (fig.1). A part of this column, between two consequent confined sections, is simulated by a plane truss. By exploit of the symmetry with respect to horizontal axis, the half of this structure is studied (fig.2).

The model consists of a single series of equal elementary square trusses. The sections of concrete bars of model, as noted in fig.2, have been obtained from the simulation of a concrete prism, under biaxial stress state, by an elementary square truss.

The bars of truss model obey nonlinear uniaxial stress-strain laws \(\sigma-\epsilon\) of concrete or steel (fig.3). The compressive strength \(\sigma_{cu}\) of concrete bars of model is assumed 1.5 times that of unconfined concrete \(\sigma_{co}\); this strength is maintained constant up to a strain \(\epsilon_{cu}=0.030\), after which a gradual crushing occurs (fig.3a). These assumed large strength and particularly ductility potentials, of the individual concrete bars of the model, are not exploited when
the column is poorly confined by transverse reinforcement.

### 3.2. Trends of truss model

When the proposed truss model is applied on a concrete column, for a constant spacing \( s \) and gradually increasing mechanical ratio \( \rho_t \) of transverse reinforcement [4], a series of monotonic axial stress-strain curves \( \sigma-\varepsilon \) of column are obtained (fig.4), on which the following trends are observed.

For a rather small \( \rho_t \), the complete strength potential of concrete is reached. Then, for increasing \( \rho_t \), the strength is maintained constant, whereas the ductility continues to increase.

For \( \rho_t \) lower than a value \( \rho_d \), a premature global instability is observed (the strength suddenly becomes zero for a specific strain \( \varepsilon \) of column), which is delayed as the \( \rho_t \) increases (fig.4). The same phenomenon has also been previously observed in various tests and was interpreted as due to hoop fracture[2, 5]. However, the proposed here truss model interprets this premature global instability of column as due to buckling of interior vertical concrete struts.

For quite large values of \( \rho_t \geq \rho_d \), the buckling of interior vertical concrete struts is absolutely prevented and the complete ductility potential of compressed concrete is exploited. The value \( \rho_d \) can be called minimum \( \rho_t \) for ductility.

However, even for large values \( \rho_t \geq \rho_d \), because of the assumption of crushing of concrete bars of truss model, for large strains \( \varepsilon > 0.030 \), a gradual crushing of column occurs (fig.4). After the completing of this crushing, a small compressive strength of column remains, due to the compressed now diagonal concrete bars, confined by the transverse reinforcement. Finally, for a very large \( \varepsilon \), the transverse reinforcement is fractured in tension. However, this happens under the small remaining strength, so it has not a practical significance.

### 3.3. Preliminary design by hand

In a series of numerical examples, the failure mode of truss model is observed (fig. 5), in which the transverse reinforcement is in tensile yield, an extensive cracking of concrete occurs and an interior vertical concrete strut, bearing a compressive axial load \( V \), tends to buckle by pushing the whole part of structure beyond it to slip, to the direction it is going to buckle, that is, in fig.5, the shadowed right part of the truss tends to slip to the right.

We write the equilibrium condition of the right shadowed part of the truss along the horizontal direction,

\[
\Sigma F_x = V \frac{u_a}{a} - T_y = 0 ,
\]  

where \( T_y \) is the tensile yield force of the transverse steel bar of model and \( u_a / a \) is the maximum inclination, in which the concrete strut under consideration can
equilibrate. Thus
\[ u_n = \frac{T_s}{V} = \frac{\sigma_y A_s / 2}{\sigma_c 2 A_i} = 1.2 \frac{\sigma_{0e}}{\sigma_c}. \] (7)

Now, we write the stiffness equation, along the horizontal direction, for the right shadowed part of structure of fig. 5.
\[ (K_{VE} - K_{VG} + K_{SH})u + p = 0, \] (8)
where \( u \) is the horizontal translation of the right part of the structure. We observe that the total stiffness consists of the following three terms:

1. The elastic stiffness of the vertical concrete strut
\[ K_{VE} = \frac{E_{un} 2 A_i}{a} \left( \frac{u_n}{a} \right)^2, \] (9)
where \( E_{un} \) is the unloading elasticity modulus of concrete.

2. The geometric stiffness of the concrete strut
\[ -K_{VG} = -\frac{V}{a} = -\frac{\sigma_c 2 A_i}{a}, \] (10)
which is negative because of the compressive loading.

3. The stiffness of transverse steel bar of model
\[ K_{SH} = \frac{E_{sh} A_i / 2}{a}, \] (11)
where \( E_{sh} \) is the elasticity modulus of transverse steel in the strain-hardening region.

Numerical examples show that the elastic stiffness \( K_{VE} \) plays usually the main role in counteracting the negative geometric stiffness \( -K_{VG} \) of the concrete strut. So, we ignore the strain-hardening of transverse reinforcement, for safety reasons, \( K_{SH}=0 \). In order to avoid a global instability of the column, the total stiffness, consisting now of only two terms, must be positive,
\[ K_{VE} - K_{VG} > 0. \] (12)
By substituting eq. 9,10 into eq. 12, we obtain
\[ \frac{u_n}{a} > \sqrt{\frac{\sigma_c}{E_{un}}}. \] (13)
And, by substituting the eq. 7,
\[ \rho_t > \frac{5 \sigma_c}{6 \sigma_{0e}} \sqrt{\frac{\sigma_c}{E_{un}}}. \] (14)
In the second part of this inequality, the expression \( \sigma_c \sqrt{\sigma_c / E_{un}} \) is variable and includes two parameters of confined concrete, its stress \( \sigma_c \) and unloading elasticity modulus \( E_{un} \). We can be based on the model of J.B. Mander [2] for confined concrete, which includes unloading, in order to find the maximum value of the expression \( \sigma_c \sqrt{\sigma_c / E_{un}} = \sigma_c \sqrt{\Delta \varepsilon} \), where \( \Delta \varepsilon \) is here the strain increment of confined concrete, recovered after a complete unloading. Then,
according to eq. 14, the mechanical ratio of transverse reinforcement must be

$$\rho_t = \frac{5}{6\sigma_{\text{co}}} \max(\sigma_c \sqrt{\Delta e}).$$  \hspace{1cm} (15)

As regards the axial compressive strength of column, numerical examples, by the proposed truss model, show that, when this strength is reached, the diagonal concrete bars have very small forces. So, by assuming that only the vertical concrete struts contribute to column strength, we obtain the strength of column before spalling, according to fig. 1, 2, 3,

$$\sigma_u = \frac{\sum V_u}{bd} = \frac{n \sigma_{\text{cu}} A_i}{bd} = \frac{5}{6\sigma_{\text{cu}}},$$

where $V_u$ is the ultimate strength of a vertical concrete strut and $n=2d/s$ is the number of elementary square trusses of the model.

And the strength loss of column, due to spalling of the exterior concrete struts, will be

$$\Delta \sigma = \frac{2\sigma_{\text{cu}} A_i}{bd} = \frac{5}{6\sigma_{\text{cu}}} \frac{s}{2d}. \hspace{1cm} (17)$$

4. Applications

By an application on a typical reinforced concrete column, the results of the proposed truss model will be compared with the requirements of the UBC and NZS codes. A shown in fig. 6, the column has a square section with side 40cm, there is no concrete cover, there are 12 longitudinal bars with diameter $D_l=20\text{mm}$, distributed around the perimeter of the section, and four transverse bars along each principal direction of the section. The strength of unconfined concrete is $\sigma_{\text{cu}}=30\text{MPa}$, the yield stress of longitudinal steel $\sigma_y=400\text{MPa}$ and that of transverse steel $\sigma_y=300\text{MPa}$. The spacing $s$ and the diameter $D_t$ of transverse reinforcement are sought.

According to eq. 2 of UBC,

$$s < \min(40\text{cm}/4, 10\text{cm}) = 10\text{cm},$$

whereas the eq. 5 of NZS requires

$$s < \min(40\text{cm}/5, 15\text{cm}, 6\times2.0\text{cm}) = 8.0\text{cm}.$$  

So, a spacing $s=8.0\text{cm}$ is chosen, for which the simplified formulae 16, 17, based on the truss model, and fig. 3a where $\sigma_{\text{cu}}=1.5 \sigma_{\text{co}}=1.5\times30\text{MPa}=45\text{MPa}$, give a column strength before spalling $\sigma_u = \frac{5}{6}45\text{MPa}=37.5\text{MPa}$ and a strength loss due to spalling $\Delta \sigma = 37.5\text{MPa} \frac{8.0\text{cm}}{240\text{cm}} = 3.75\text{MPa}$.

According to eq. 1 of Uniform Building Code, the mechanical ratio of transverse reinforcement must be

$$\rho_t = \frac{4\pi D_t^2/4}{8.0\text{cm} \times 40\text{cm}} \times \frac{300\text{MPa}}{30\text{MPa}} > 0.09.$$
From this inequality results \( D_t > 0.9575 \text{cm}, \) that is the UBC requires a diameter of transverse reinforcement \( D_t = 10 \text{mm}. \)

Now, we are going to apply the proposed truss model on the present example. First, we try \( D_t = 8 \text{mm}. \)

In order to find the maximum value of the expression \( \sigma_c \sqrt{\Delta \varepsilon} \) of eq. 15, we are based on the model of J.B. Mander for confined concrete \([2]\), which, for the present application with \( D_t = 8 \text{mm} \), gives the diagrams of fig. 7, representing, on one hand, the variation of the stress \( \sigma_c \) and the unloading elasticity modulus \( E_{un} \) of confined concrete, as well as of the \( \Delta \varepsilon = \sigma_c / E_{un} \), with respect to the strain \( \varepsilon_c \) (fig. 7a) and, on the other hand, the variation of the quantity \( \sigma_c \sqrt{\Delta \varepsilon} \) with respect to \( \varepsilon_c \) (fig. 7b). From the latter, the sought maximum value is obtained, \( \max(\sigma_c \sqrt{\Delta \varepsilon}) = 2.315 \text{MPa}. \) Now, by applying the eq. 15, we find the amount of transverse reinforcement, required by the truss model, in this specific case,

\[
\rho_t > \frac{\sigma}{6} \frac{2.315 \text{MPa}}{30 \text{MPa}} = 0.06431.
\]

However, the diameter \( D = 8 \text{mm}, \) which is now tried, gives a

\[
\rho_t = \frac{4\pi 0.8^2/4 \times 300}{840 \times 30} = 0.06283 < 0.06431,
\]

so, it is not sufficient.

Then, we try a larger diameter of transverse reinforcement, \( D_t = 9 \text{mm}. \) By following the same as above procedure, we find \( \max(\sigma_c \sqrt{\Delta \varepsilon}) = 2.590. \) Thus

\[
\rho_t = \frac{4\pi 0.9^2/4 \times 300}{840 \times 30} = 0.07952 > 0.07194 = \frac{5 \times 2.590}{6 \times 30},
\]

that is the diameter \( D_t = 9 \text{mm} \) is sufficient, according to the proposed truss model.

We observe that the Uniform Building Code requires \( D_t = 10 \text{mm}, \) in the present application, whereas the proposed truss model recommends \( D_t = 9 \text{mm}. \) That is, there is a satisfactory approximation between the UBC and the proposed truss model.

Now, we come to the eq.3.4 of New Zealand Standards, which require a larger amount of transverse reinforcement

\[
\rho_t = \frac{4\pi D_t^2/4 \times 300}{840 \times 30} > 0.12(0.33 + 1.67 - 0.6) = 0.16.
\]

From this inequality is obtained \( D_t > 1.277 \text{cm}, \) that is a diameter \( D_t = 13 \text{mm} \) is required by NZS, which is quite larger than the values \( D_t = 9 \text{mm} \) and \( D_t = 10 \text{mm} \), recommended by the truss model and the UBC, respectively.

In fig. 8, the stress-strain curves \( \sigma-\varepsilon \) of confined concrete, for the present application, are represented, on one hand the \( \sigma-\varepsilon \) curves for \( D_t = 9, 10 \) and \( 13 \text{mm} \) determined by J.B. Mander’s model \([2]\), and, on the other hand, the \( \sigma-\varepsilon \) curve for \( D_t = 9 \text{mm} \) determined by the proposed here truss model.

The larger \( \rho_t \) required by NZS is based on the assumption that this can compensate the ductility loss due to a possible heavy axial compression. In fig. 9, the moment-curvature curves \( M-\varphi \) of a specific column section are
represented, for various values of the compressive axial load \( N \). And it is clearly demonstrated that, as the \( N \) increases, the flexural strength of column section is increased, whereas the curvature ductility is significantly reduced. It is also observed, in fig.9, that the ductility mainly depends on the ultimate curvature, which can be estimated, in a simple way, with satisfactory approximation, on the basis of fig.10 which represents the ultimate state of a column section, in which the concrete begins being fractured in compression. Indeed, from the equilibrium of forces perpendicular to the section, the neutral axis depth \( x \) is obtained,

\[
x = \frac{N}{\sigma_{cu} b},
\]

whereas the ultimate curvature ductility is

\[
\varphi_u = \frac{\varepsilon_{cu} x}{x}.
\]

By substituting the eq.18 into eq. 19, we obtain

\[
\varphi_u = \frac{\varepsilon_{cu} \sigma_{cu} b}{N}.
\]

This equation shows that the curvature ductility is increased with the strength \( \sigma_{cu} \) and ultimate strain \( \varepsilon_{cu} \) of confined concrete. So, the NZS code, based on J.B. Mander’s model [2], recommends a larger amount \( p_t \) of transverse reinforcement, which will increase the \( \varepsilon_{cu}, \sigma_{cu} \) of confined concrete, thus increasing the curvature ductility of the section, too, according to eq.20. However, the J.B. Mander’s model, on which the NZS recommendations are based, is rather optimistic and more suitable for spiral members and does not closely agree with other confinement models [1, 6, 7], which are more conservative. So, it is more proper not to rely on the increased \( p_t \) recommended by NZS, but to keep from NZS only the requirement of eq.4 for a limit compressive axial load, in agreement with eq.20, which shows that a reduced \( N \) increases the curvature ductility.

According to fig.11, the compressive axial force of a column consists of two parts, \( N = N_o + \Delta N \), where the \( N_o \) is due to the static vertical loads, whereas the \( \Delta N \) is due to the seismic overturning moments of the frame and is significant particularly in the exterior columns of groundfloor of a tall building. The \( \Delta N \)'s can be reduced by reduction of the overturning moment \( \Sigma Q_u H \) (fig.11), which cannot be achieved by the reduction of the \( \Sigma Q_u \) without reducing the shear strengths \( Q_u \), that is the sections of columns, which is not prudent; but the overturning moments can be drastically reduced by reducing the leverarm \( H \), that is the height of the building. Alternatively, in order to increase the ductility, we can reduce the static part \( N_o \) of the axial forces of columns, either by reducing the vertical static loads or by increasing the number of columns. Finally, in order to increase the curvature ductility of column sections, we can also, according to eq.20, to increase the width \( b \) of columns or the quality of concrete which means increase of \( \sigma_{cu} \).

5. Conclusions

5.1. The proposed truss model, for confinement of concrete columns, interprets the premature global instability of a column with poor transverse
reinforcement, as due to buckling of interior vertical concrete struts, whereas previous models used to interpret this global instability, observed in test results, as due to fracture of transverse reinforcement.

5.2. A satisfactory approximation is observed between the results of the proposed truss model and the requirements of the Uniform Building Code. So, the truss model is reliable and can be proved useful for an insight in the phenomenon of confinement, as well as for a preliminary design, by a hand calculator, of the confinement of concrete columns.

5.3. The New Zealand Standards recommend a larger amount of transverse reinforcement, by assuming that a further increase of strength and ductility of confined concrete core is achieved, so that to compensate the loss of curvature ductility of column sections due to heavy additional compression, caused by the seismic overturning moments of the building. However, this assumed further increase of strength and ductility of confined concrete, suggested by J.B. Mander's model, is rather optimistic and does not closely agree with other confinement models, which are more conservative. So, instead of relying on increased confinement, for the improvement of curvature ductility of column sections, we can keep the factored compressive axial forces of columns below a limiting value, as also recommended by NZS. This can be achieved by reduction of height of building, by reduction of vertical static loads, or by increase of number of columns, of concrete quality or increase of dimensions of column sections, particularly of the exterior columns of groundfloor in a tall building.

5.4. The strength loss of a column due to spalling and the ultimate strength of confined concrete core mainly depend and can be controlled by the spacing of transverse reinforcement over the height of column. Estimations of their magnitudes can be obtained by simple formulae based on the proposed truss model.

6. References

6. Sheikh S.A., Uzumeri S.M. Analytical model of concrete confinement in


Figure 1. Confined concrete column under concentric axial compression.

Figure 2. Truss model of half part of column between two consequent confined sections.

Figure 3. Primary stress-strain curves $\sigma$-$\varepsilon$ of concrete and steel bars of model.

Figure 4. Monotonic axial stress-strain curves $\sigma$-$\varepsilon$ of a column, for constant spacing $s$ and gradually increasing amount $\rho$ of transverse reinforcement, obtained by the truss model.
Figure 5. Failure mode of truss model.

Figure 6. Given and sought data of the application.

Figure 7.a. Stress-strain curve $\sigma_c$-$\varepsilon_c$ of confined concrete with variation of the unloading elasticity modulus $E_{un}$ for the present application with $D_l=8$mm, obtained by J.B. Mander's model [2]. b. Variation of the expression $\sigma_c \sqrt{\Delta \varepsilon}$ with respect to the strain $\varepsilon_c$. 
Figure 8. Monotonic axial stress-strain curves $\sigma_c-\varepsilon_c$ of confined concrete, for $D_t=9,10,13\text{mm}$, obtained by J.B. Mander’s model [2], as well as the $\sigma_c-\varepsilon_c$ curve for $D_t\geq 9\text{mm}$, obtained by the truss model.

Figure 9. Moment-curvature curves $M-\varphi$ of a specific reinforced concrete section, for various values of the compressive axial force $N$.

Figure 10. Ultimate state $U$ of a reinforced concrete section, in which the concrete begins being fractured in compression.

Figure 11. Deformed shape and reactions of groundfloor columns, due to static and seismic loads of the building.