Non-linear Wave Forces on Large Structures
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Abstract

In the present work a non-linear dispersive wave model is developed for the estimation of the horizontal forces on large submerged structures where the effects of the wave diffraction are significant and the inertia force becomes dominant. The model is based on the Boussinesq type of equations with appropriate reflective boundary conditions on the body surface. The Smagorinsky approximation is used for the representation of the damping by eddies smaller than the computational grid size. Model results for large cylinders as well as for square and rectangular section caissons are presented and compared with experimental data.

1. Introduction

Morison equation is an empirical formula for the estimation of the forces on a small submerged pile structure in the presence of surface waves. This relationship, which involves both a drag and an inertia component of force, assumes that the object is so small as not to disturb the incident wave field. As the size of the object in relation to the incident wave length increases, the Morison equation is no longer valid. The incident wave gets scattered when encounters a large object, necessitating a diffraction theory approach to the calculation of wave forces.

Several second order solutions have been proposed based on a Stokes perturbation method [1], [7], or on an eigenfunction expansions [2]. However the above solutions are valid only for axisymmetric bodies and not in a complicated case where the presence of other structures and the shore boundary affects the hydrodynamic field. In the present work the horizontal forces on a large structure are estimated from the
hydrodynamic field resulting from a nonlinear wave model based on the Boussinesq equations with improved linear dispersion characteristics.

2. Numerical Model

Boussinesq equations are widely used for the simulation of the propagation of the non linear dispersive regular and irregular waves in the nearshore region. The present model is based on a new high-order numerical scheme proposed by Wei and Kirby [9].

Special attention has to be given for the boundary condition related to the presence of the structure since the wave force is derived form the hydrodynamic field near the body. The (reflection) boundary condition on the body surface is written [9]:

\[
\mathbf{u} \cdot \mathbf{n} = 0
\]

\[
\nabla \zeta \cdot \mathbf{n} = 0
\]

\[
\partial u_T / \partial n = 0 \quad x \in \partial \Omega
\]

where \( \mathbf{n} \) is the outward normal vector, \( \mathbf{u}=(U,V) \) the horizontal velocity, \( \Omega \) the fluid domain, \( \partial \Omega \) the boundary, \( u_T \) is the velocity component tangent to the boundary, \( \zeta \) is the surface elevation and \( x \) a position in the domain.

Usually only the first of the above conditions is applied. However setting \( \mathbf{u} \cdot \mathbf{n} = 0 \) covers only a portion of the condition. The second condition is consistent with the usual physical motion of total reflection at a vertical barrier and the last one imposes a no-shear condition for the flow along the wall.

In the numerical integration of a 3-D open channel turbulent flow it is necessary to introduce an eddy viscosity, which represents physically the damping by eddies smaller than the computational grid size. After the depth averaging, to filter out the vertical velocity profile, the residual stress, have the following form in a 2-D horizontal model [4], [5]:

\[
\frac{\partial}{\partial x} \left( E \frac{\partial u}{\partial x} \right) + \frac{1}{2} \frac{\partial}{\partial y} \left( E \left( \frac{\partial u}{\partial y} + \frac{\partial \nu}{\partial x} \right) \right)
\]

(1)

where \( E \) is the sub-grid eddy viscosity having the isotropic shear-dependent form proposed by Smagorinsky:

\[
E = \rho \left[ \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial \nu}{\partial y} \right)^2 + \frac{1}{2} \left( \frac{\partial u}{\partial y} + \frac{\partial \nu}{\partial x} \right)^2 \right]^{1/2}
\]

(2)
in which the mixing length $l$ is determined by:

$$l = c \Delta x$$

(3)

where $\Delta x$ is the grid size and $c$ an empirical constant.

In 3-D modelling the value of $c=0.1825$ is used. However, in 2-D horizontal flow it must be expected a different value for $c$. Love and Leslie [4], in their study on Burgers' equation (depth averaged non linear shock wave propagation), suggested for $c$ the value of 0.4. A similar value was also proposed by Madsen et al. [5] for depth integrated flows. This value of $c$, $c=0.4$, is also adopted here.

In flow problems dominated by wave motion an open boundary condition must allow waves generated in the domain of interest to pass through the boundary without undergoing significant distortion and without influencing the interior solution. In the present problem, the outgoing waves at the offshore boundary are described using the Orlanski [6] condition.

In Figure 1 the surface elevation and the velocity filed around a vertical circular cylinder in water of finite depth is shown. The radius $r$ of the structure is $r=50$ m, the wave height $H=1$ m, the length $L=100$ m and the water depth $d=10$ m. The dimensions of the body are in the same order with the wave length and hence the presence of the structure disturbs the incident wave field. The figure also reveals the importance of the effects of the diffraction behind the pile and the reflection on it.

The total horizontal force on a structure is estimated by integrating the force due to pressure over the whole body:

$$F = \int_P n \cdot dS$$

(4)

where $n$ is a unit normal vector into the fluid, $P$ the pressure and $S$ the surface of the structure.

In a Boussinesq model the pressure $P$ is given by:

$$P = \rho g (\zeta - z) + \rho \left( \frac{(d+z)^2 - h^2}{2} \right) \left( \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} \right)$$

(5)

$\rho$ is the fluid density, $d$ is the water depth and $h=d+\zeta$. 

Figure 1. Surface elevation and the velocity field around a vertical circular cylinder.
3. Model results

In Table 2 a comparison between the present numerical solution and experimental data (given by Chakrabatri [1]) is presented. The dispersion parameter \( \sigma = 2\pi d/gT^2 \) takes values from \( \sigma = 0.12 \) to \( \sigma = 0.3 \) (\( d \) = water depth and \( T \) = wave period). The other design parameters are also chosen to be representative of ocean engineering applications: \( a/d = 0.862 \) and \( H/a = 0.261 \) (\( a \) = radius of the cylinder, \( H \) = wave height). The maximum forces \( F_{\text{max}} \) are normalised with respect to the weight of the water displaced by the cylinder and the dimensionless wave height \([= \rho g a^2 d(H/2a)]\). The coefficient \( C_M \) is defined as: \( C_M = F_{\text{max}}/[\rho g a^2 d(H/2a)] \). The numerical model predicts well the applied wave forces on the large pile.

Table 2. Calculated horizontal force on a vertical circular cylinder and comparison with experimental data; \( a/d = 0.862 \) and \( H/a = 0.261 \).

<table>
<thead>
<tr>
<th>( \sigma )</th>
<th>0.12</th>
<th>0.19</th>
<th>0.22</th>
<th>0.25</th>
<th>0.30</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_M H/2a ) (exp. data)</td>
<td>0.15148</td>
<td>0.12677</td>
<td>0.11781</td>
<td>0.10027</td>
<td>0.08057</td>
</tr>
<tr>
<td>( C_M H/2a ) (model)</td>
<td>0.145</td>
<td>0.120</td>
<td>0.111</td>
<td>0.095</td>
<td>0.078</td>
</tr>
</tbody>
</table>

The model is also used for the calculation of the wave forces on square and rectangular section caissons under typical conditions. Experiments by Shankar and Subbiah [8] are reproduced numerically. The experiments were conducted in a wave flume 90 cm x 90 cm in cross-section and 31.4 m in length. Waves of 1 to 2 seconds period and 3.5 cm to 18 cm height were generated at three water depths of 30 cm, 40 cm and 50 cm. The width \( b \) of the square structure was \( b = 18 \) cm. In the case of the rectangular caisson the dimension were \( b \times l = 24 \) cm x 18 cm (where \( b \) is the width normal to wave direction). The comparison between experimental data and model results is shown in Figures 2 and 3.

4. Conclusions

A non linear dispersive wave numerical model with appropriate reflective boundary conditions and the introduction of an eddy viscosity, to represent the damping by eddies smaller than the computational grid size,
Figure 2. Normalised maximum wave forces on square caisson for $b/L=0.0963$, $d/L=0.1925$.

Figure 3. Normalised maximum wave forces on rectangular caisson for $b/L=0.0924$, $d/L=0.1925$, $b/l=4/3$. 
can predict with good accuracy the horizontal forces on large submerged structures where the effects of the wave diffraction are significant and the inertia force becomes dominant.

References


