Computational solutions of unsteady viscous flows with oscillating boundaries

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Abstract

This paper presents recent computational solutions for two- and three-dimensional unsteady viscous flows with oscillating boundaries based on: (i) an enhanced hybrid-spectral method, and (ii) time-integration methods using artificial compressibility. A comparison is made between the results obtained with a mean-position analysis and with a formulation based on a time-dependent coordinate transformation, better suited for large amplitude oscillations.

1. Introduction

The analysis of unsteady confined flows with oscillating boundaries recently received an increasing research interest for its applications in many engineering problems. Thus, the unsteady annular flows between cylindrical structures executing transverse oscillations, which are of particular interest for the flow induced vibration problems encountered in many engineering systems, have been studied theoretically (based on simplified theoretical models) and experimentally by numerous scientists, such as Chen et al.1, Inada & Hayama2, Mateescu and his coworkers3-6 and others. These simplified theoretical models were proven to be in good agreement with experimental results, at least for simple cylindrical geometries (Mateescu et al.6). However, there is a need for more accurate solutions based on the time-accurate integration of the Navier-Stokes equations and applicable to more realistic and complex geometric configurations. The computational methods developed in this respect should display, in addition to a good accuracy, a very good computational efficiency in order to eventually permit the simultaneous integration of the Navier-Stokes equations and the structural equations of motion required in the study of the dynamics of structures subjected to fluid flows. Several such computational methods have recently been developed based on a pseudo-time-integration with artificial compressibility (Mateescu et al.7-9) and on spectral-collocation or hybrid-spectral formulations (Mateescu et al.10-11), for unsteady viscous flows with oscillating boundaries.

This paper presents several computational solutions recently obtained by this author and his coworkers based on the time-accurate integration of the Navier-Stokes equations for two- and three-dimensional unsteady flows with oscillating boundaries.
2. Computational solutions based on hybrid spectral methods

The enhanced hybrid spectral method is illustrated here for the unsteady viscous flow in the annular space between two eccentric cylinders of radii $a$ and $b$ shown in Fig. 1, where $ea$ represents the eccentricity of the cylinder axes. The flow is referred to the cylindrical coordinates $ax$, $ar$ and $\theta$, where $x$ and $r$ are nondimensional. The central portion of length $al$ of the inner (or outer) cylinder executes transverse oscillations in the longitudinal plane of symmetry, $\theta = 0$, or normal to it ($\theta = 90^\circ$), defined as $a e (x, t) = a e_0 E(x) \cos \omega t = \mathcal{R} [a e_0 E(x) \exp (i \omega t)]$, where $e_0$ is the nondimensional amplitude, $\omega$ the radian frequency and $E(x)$ is the specified axial mode of oscillations. Two fixed cylindrical extensions of the same radii and lengths, $al_0$ and $al_1$, are situated upstream and downstream of the oscillating central portion. At its fixed inlet ($x = -l_0$), the annular passage is subjected to a fully-developed laminar flow defined by the steady axial velocity $U_0 U(r, \theta)$, where $U_0$ represents the mean axial flow velocity, and the function $U(r, \theta)$ defines the known variation of the steady axial velocity in the eccentric annulus $^{10-11}$.

![Fig. 1. Geometry of the annular space between two eccentric cylinders, with the central portion of the inner cylinder executing transverse oscillations in the longitudinal plane $\theta = 0$.](image)

The boundary conditions on the moving portion of the cylinder can be expressed, for oscillations in the plane of symmetry, in the form $u^* = 0$, $v^* = a \left( \frac{\partial e}{\partial t} \right) \cos \theta$, $w^* = -a \left( \frac{\partial e}{\partial t} \right) \sin \theta$, where $u^*$, $v^*$ and $w^*$ are the axial, radial and circumferential velocity components, and $\frac{\partial e}{\partial t} = i \omega e(x,t)$.

The problem is solved in a computational domain $(x, Z, \theta)$ defined by the coordinate transformation

$$Z = 1 - (r - 1) / h(\theta) \quad x = x, \quad \theta = \theta, \quad h(\theta) = \sqrt{b^2 - e^2 \sin^2 \theta} - e \cos \theta - 1 \quad (1)$$

In this computational domain, the unsteady velocity components ($u = u^* - U_0 U(r, \theta)$, $v = v^*$, $w = w^*$) and the unsteady pressure ($p = p^* - p_0^*$) can be expressed in the form

$$u = a \omega e_0 i \exp (i \omega t) \hat{u}(x,Z,\theta) = \sum_{j=0}^{m} \sum_{k=0}^{n} \mathcal{U}_{j,k}(x) T_j(Z) F_k(\theta), \quad (2)$$

$$v = a \omega e_0 i \exp (i \omega t) \hat{v}(x,Z,\theta) = \sum_{j=0}^{m} \sum_{k=0}^{n} \mathcal{V}_{j,k}(x) T_j(Z) F_k(\theta), \quad (3)$$

$$w = a \omega e_0 i \exp (i \omega t) \hat{w}(x,Z,\theta) = \sum_{j=0}^{m} \sum_{k=0}^{n} \mathcal{W}_{j,k}(x) T_j(Z) \varphi_k(\theta) \quad (4)$$
\[ p = \rho a^2 \omega^2 \varepsilon_0 \exp(i \omega t) \hat{p}(x,Z,\theta) = \sum_{j=0}^{m-2} \sum_{k=0}^{n} \mathcal{P}_{jk}(x) T_j(Z) F_k(\theta) \tag{5} \]

where \( T_j(Z) \) represents the Chebyshev polynomials of order \( j \), \( F_k(\theta) \) and \( \mathcal{P}_{jk}(x) \) are orthogonal Fourier functions (e.g. \( \cos k\theta \) and \( \sin k\theta \)), whilst \( \mathcal{U}_{jk}(x), \mathcal{V}_{jk}(x), \mathcal{W}_{jk}(x) \) and \( \mathcal{P}_{jk}(x) \) are a priori unspecified complex functions, and where \( \hat{u}(x,Z,\theta) \), \( \hat{v}(x,Z,\theta) \), \( \hat{w}(x,Z,\theta) \) and \( \hat{p}(x,Z,\theta) \) represent the nondimensional reduced unsteady velocity components and pressure.

The Navier-Stokes equations and the boundary conditions are transformed in the computational domain, via the spectral expansions (2) - (5), into a system of differential equations for the unknown functions \( \mathcal{U}_{jk}(x), \mathcal{V}_{jk}(x), \mathcal{W}_{jk}(x) \) and \( \mathcal{P}_{jk}(x) \). These equations are then discretized using an efficient hybrid approach based on:

(a) An azimuthal Fourier identification with respect to \( \theta \), based on an efficient Fourier expansion of the relative annular clearance \( h(\theta) \).

(b) A collocation approach in the quasi-radial direction \( Z \), imposed at collocation points conveniently distributed within the computational domain (solution accuracy varies exponentially with the number of collocation points\(^1\)).

(c) A mixed central-upwind finite-difference scheme for the \( x \) direction, based on a discretization performed at \( N \) axial stations, \( x_I \) (with \( I = 1,2,\ldots,N \)). The central or upwind schemes are used in function of the local mesh Reynolds number\(^1\).

\[ \beta = 500 \]

\[ \delta = 0 \]

<table>
<thead>
<tr>
<th>( \varepsilon_{rel} )</th>
<th>( \text{Re} )</th>
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<tr>
<td>0</td>
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<tr>
<td>0</td>
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\[ \delta = 1 \]

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<th>( \varepsilon_{rel} )</th>
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<tr>
<td>0.4</td>
<td>1250</td>
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<tr>
<td>0.4</td>
<td>2500</td>
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Fig. 2. Hybrid spectral method. Typical axial variations of the real and imaginary components of the reduced unsteady pressure, \( \hat{p}(x,\theta) \), on the oscillating cylinder at \( \theta = \delta \times 90^\circ \) for \( \beta=500 \), three values of the hydraulic Reynolds number, \( \text{Re}_H \), and two relative eccentricities, \( \varepsilon_{rel} = 0 \) and 0.4.
This efficient hybrid discretization approach leads to the solution of a system of algebraic equations for the \textit{a priori} unknown complex grid values \( u_{j,k,l} = u_{j,k}(x_l) \), \( v_{j,k,l} = v_{j,k}(x_l) \), \( \omega_{j,k,l} = \omega_{j,k}(x_l) \) and \( \phi_{j,k,l} = \phi_{j,k}(x_l) \) which can be expressed in the general matrix form

\[
A \{ u_{j,k,l}, v_{j,k,l}, \omega_{j,k,l}, \phi_{j,k,l} \}^T = B
\]

where \( A \) is a sparse block-tridiagonal matrix (2% non-zero elements or less).

After validation, this hybrid spectral method has been used to obtain for the first time the solution of the three-dimensional unsteady flow between eccentric cylinders, when the inner cylinder executes transverse oscillations following the first flexural beam mode\(^1\). The solutions are illustrated in Figures 2 and 3 for both cases of oscillations in the plane of symmetry (\( \delta = 0 \)) and normal to the symmetry plane (\( \delta = 1 \)).

The typical axial variations of the \textit{real} and \textit{imaginary} components of the reduced unsteady pressure, \( \hat{p}(x,\theta) \), on the oscillating inner cylinder (\( Z=1 \)), are shown in Fig. 2 for an oscillatory Reynolds number \( \beta = \omega a^2 / \nu = 500 \), three values of the hydraulic Reynolds number, \( \text{Re}_H = 2 (b-1) a U_0 / \nu \), and two values of the relative eccentricity, \( e_{\text{rel}} = e / (b-1) = 0 \) and 0.4.

![Fig. 2. Hybrid spectral method. Influence of the hydraulic and oscillatory Reynolds numbers, \( \text{Re}_H \) and \( \beta \), on the typical axial variations of the real and imaginary components of the nondimensional reduced unsteady force, \( \hat{F}(x) \), acting on the oscillating cylinder for a relative eccentricity \( e_{\text{rel}} = 0.4 \).](image)

\begin{table}[h]
\begin{tabular}{|c|c|c|}
\hline
\( \delta = 0 \) & \( \beta \) & \( \text{Re} \) \\
\hline
\( \bullet \) & 500 & 0 \\
\( \triangle \) & 500 & 1250 \\
\( \nabla \) & 5000 & 1250 \\
\( \blacksquare \) & 500 & 2500 \\
\hline
\end{tabular}
\end{table}

\begin{table}[h]
\begin{tabular}{|c|c|c|}
\hline
\( \delta = 1 \) & \( \beta \) & \( \text{Re} \) \\
\hline
\( -\Delta \) & 500 & 1250 \\
\hline
\end{tabular}
\end{table}
Typical axial variations of the real and imaginary components of the reduced nondimensional unsteady force acting on the oscillating cylinder, defined as \( \tilde{F}(x) = \frac{F(x,t)}{\pi \rho \alpha^2 x^2 e_0 \exp(\iota \omega t)} \) and obtained by integrating the unsteady pressure and skin friction, are shown in Fig. 3 for a relative eccentricity \( e_{rel} = 0.4 \).

It was found that the imaginary components are strongly influenced by the increase in the hydraulic Reynolds number \( Re_H = 2(b-1) aU_0 / \nu \), due to the Coriolis effects which become increasingly important for larger values of \( Re_H \).

3. Time-integration method with artificial compressibility

3.1. Two-dimensional solutions for unsteady viscous flows over a backward step with an oscillating floor

The time-integration method with artificial compressibility is illustrated first for the two-dimensional unsteady flows in ducts with a backward-facing step and with an oscillating floor behind the step (configuration shown in Fig. 4). The flow is referred to Cartesian coordinates centered at the step corner, \( Hx \) and \( Hy \), where \( x \) and \( y \) are nondimensional coordinates with respect to the downstream height, \( H \), which is twice the heights of the step and of the upstream channel (\( H/2 \) each). The upstream inlet of the channel is subjected to a fully-developed laminar flow defined by the steady axial velocity \( U(y) = 24 U_0 y (0.5 - y) \), where \( U_0 \) represents the mean axial flow velocity. Just behind the backstep, a portion of the floor of length \( Hl \) executes transversal oscillations defined as \( He(t) \sin(\pi x / l) \), where \( e(t) = e_0 \cos(\omega t) \), in which \( t = U_0 t^* / H \) and \( \omega = \omega^* H / U_0 \) represents the nondimensional time and reduced frequency of the oscillations.

\[
\frac{\partial \mathbf{V}}{\partial t} + \mathbf{G}(\mathbf{V}, p) = 0, \quad \nabla \cdot \mathbf{V} = 0, \quad (7)
\]
where \( \mathbf{V} \), denoting the nondimensional fluid velocity, and \( \mathbf{G} (\mathbf{V}, p) \), which includes the convective derivative, pressure and viscous terms, can be expressed in 2-D Cartesian coordinates in the form

\[
\mathbf{V} = \{ u, v \}^T, \quad \mathbf{G} (\mathbf{V}, p) = \{ G_u (u, v, p), G_v (u, v, p) \}^T
\]

\[
G_u (u, v, p) = \frac{\partial (uu)}{\partial x} + \frac{\partial (vu)}{\partial y} + \frac{\partial p}{\partial x} - \frac{1}{Re} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right),
\]

\[
G_v (u, v, p) = \frac{\partial (uv)}{\partial x} + \frac{\partial (vv)}{\partial y} + \frac{\partial p}{\partial y} - \frac{1}{Re} \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right),
\]

in which \( Re = H U_0 / v \) represents the Reynolds number.

In the present approach, the momentum equation is discretized in real time based on a second order three-point-backward implicit scheme:

\[
\frac{\partial \mathbf{V}}{\partial t} = \frac{1}{\Delta t} \left( 3 \mathbf{V}^{n+1} - 4 \mathbf{V}^n + \mathbf{V}^{n-1} \right), \quad \mathbf{G}^{n+1} = \mathbf{G} \left( \mathbf{V}^{n+1}, p^{n+1} \right). \tag{8}
\]

Thus, equations (7) can be expressed at the time level \( i^{n+1} = (n + 1) \Delta t \) in the form

\[
\mathbf{V}^{n+1} + \alpha \mathbf{G}^{n+1} = \mathbf{F}^n, \quad \nabla \cdot \mathbf{V}^{n+1} = 0, \tag{11}
\]

where \( \alpha = 2 \Delta t / 3 \), \( \mathbf{F}^n = (4 \mathbf{V}^n + \mathbf{V}^{n-1}) / 3 \).

An iterative pseudo-time relaxation procedure using artificial compressibility is then used in order to advance the solution of the semi-discretized equations from the real time level \( t^n \) to \( t^{n+1} \) in the form

\[
\frac{\partial \mathbf{V}}{\partial \tau} + \hat{\mathbf{V}} + \alpha \mathbf{G} = \mathbf{F}^n, \quad \nabla \cdot \mathbf{V} = 0, \tag{12}
\]

where \( \mathbf{V} (\tau) \) and \( \hat{\mathbf{V}} (\tau) \) denote the pseudo-functions corresponding to the variable velocity and pressure at pseudo-time \( \tau \), between the real time levels \( t^n \) and \( t^{n+1} \), and \( \delta \) represents an artificially-added compressibility (the optimum value of \( \delta \) is determined based on the theory of characteristics\(^7\)\(^9\)). An implicit Euler scheme is then used to discretize equations (12) between the pseudo-time levels \( \tau^y \) and \( \tau^{y+1} = \tau^y + \Delta \tau \), and the resulting equations are expressed in function of the pseudo-time variations \( \Delta u = \tilde{u}^{y+1} - \tilde{u}^y \), \( \Delta v = \tilde{v}^{y+1} - \tilde{v}^y \), \( \Delta p = \tilde{p}^{y+1} - \tilde{p}^y \), in the matrix form

\[
[ I + \alpha \Delta \tau \left( \mathbf{D}_x + \mathbf{D}_y \right) ] \mathbf{D}_x = \Delta \mathbf{S}, \tag{13}
\]

where \( \Delta \mathbf{S} = [\Delta u, \Delta v, \Delta p]^T, \alpha = 2 \Delta t / 3, I \) is the identity matrix, and

\[
\mathbf{D}_x = \begin{bmatrix}
M + 1/\alpha & 0 & \partial / \partial x \\
0 & M & 0 \\
\left(1/\alpha \delta \right) (\partial / \partial x) & 0 & 0
\end{bmatrix}, \quad \mathbf{D}_y = \begin{bmatrix}
N + 1/\alpha & \partial / \partial y \\
0 & N + 1/\alpha & \partial / \partial y \\
0 & \left(1/\alpha \delta \right) (\partial / \partial y) & 0
\end{bmatrix},
\]

where

\[
M\phi = \frac{\partial (\tilde{u}^y)}{\partial x} - \frac{1}{Re} \frac{\partial^2 \phi}{\partial x^2}, \quad N\phi = \frac{\partial (\tilde{v}^y)}{\partial y} - \frac{1}{Re} \frac{\partial^2 \phi}{\partial y^2}. \tag{14}
\]

A factored Alternate Direction Implicit (ADI) scheme is used to separate equation (13) into two successive sweeps in \( x \) and \( y \) defined by the equations

\[
[ I + \alpha \Delta \tau \mathbf{D}_y ] \Delta \mathbf{f}^* = \Delta \mathbf{S}, \quad [ I + \alpha \Delta \tau \mathbf{D}_x ] \Delta \mathbf{f} = \Delta \mathbf{f}^*, \tag{16}
\]

where \( \Delta \mathbf{f}^* \) is a convenient intermediate variable vector. These equations are then spatially discretized by central differencing on a stretched staggered grid. A special decoupling procedure, based on the utilization of the continuity
equation, is then used for each sweep to reduce the resulting systems of discretized equations to two sets of decoupled scalar tridiagonal equations\textsuperscript{7-9,12}.

The results obtained with this method for the two-dimensional unsteady flows in ducts with a backward-facing step and with an oscillating floor behind the step are illustrated in Fig. 5. Computations have been performed for a range of values of the oscillatory Reynolds number, $\beta = \omega * H^2 / \nu$, between 5 and 50, and of the Reynolds number, $Re = H U_0 / \nu$, between 100 and 1 000. The nondimensional length of the oscillating floor portion was $l = 0.5 H$, and the upstream and downstream lengths of the computational domain are $l_0 = 1$ and $l_1 = 30$. The real-time step used was $\Delta t = 2 \pi / (20 \omega)$, where $\omega = \beta / Re$.

The unsteady flow over the backward-facing step, generated by the transversal oscillations of the floor portion behind the step, displays an interesting unsteady separated flow pattern\textsuperscript{12}. Two separated flow regions appear on the lower and upper walls, and the position of their separation and reattachment edges substantially vary in time, due to the floor oscillations. This is shown in Fig. 5 a & b for two harmonic cycles defined by: (a) $Re = 1000$ and $\beta = 50$, and (b) $Re = 500$ and $\beta = 25$. Interestingly, it was found\textsuperscript{12} that in the case (b) the separation bubble simply disappears for a certain portion of the oscillatory cycle.

![Fig. 5. Unsteady flow over a backstep with an oscillating floor. Variation in time of the separation and reattachment points positions on the upper and lower walls: (a) Re=1 000, $\beta = 50$, and (b) Re=500, $\beta = 25$.](image-url)
3. 2. Two- and three-dimensional solutions for unsteady annular flows based on time-dependent coordinate-transformation

The time-integration method with artificial compressibility based on time-dependent coordinate transformation is illustrated here for the 2-D unsteady viscous flows between two initially concentric cylinders \( e = 0 \), when the outer cylinder executes transverse translational oscillations defined by \( a e_0 E(x) \exp(i \omega t) \), where \( E(x) = 1 \). The time-dependent coordinate transformation used in this case is

\[
Z = 1 - (r - 1) / h(\theta, x, t) , \quad x = x , \quad \theta = \theta ,
\]

where \( h(\theta, x, t) \) defines the time-dependent azimuthal variation of the relative annular clearance.

In the fixed-grid computational domain obtained by this transformation, the boundary conditions on the oscillating cylinder can be rigorously implemented, which permit to obtain time-accurate solutions even in the case of large amplitude oscillations. In this computational domain, the Navier-Stokes and continuity equations can be expressed in the form

\[
\frac{\partial \mathbf{V}}{\partial t} + Z \mathbf{C} \frac{\partial \mathbf{V}}{\partial Z} + Q(\mathbf{V}, p) = 0 , \quad \mathbf{D} \cdot \mathbf{V} = A \frac{\partial v}{\partial Z} + \frac{1}{r} \left[ A v + Z B \frac{\partial w}{\partial Z} + \frac{\partial w}{\partial \theta} \right] = 0 ,
\]

where \( t \) is the nondimensional time, \( A = \partial Z / \partial r \), \( B = (\partial Z / \partial \theta) / Z \), \( C = (\partial Z / \partial e_x)(\partial e / \partial t) \), \( r = 1 + Z h(\theta, t) \), \( \mathbf{V} = [v, w]^T \) is the nondimensional velocity vector with the radial and circumferential components \( v = v^* / U^* \) and \( w = w^* / U^* \), where \( U^* = a \omega \) in this case, \( p = (p^* - p_0^*) / (\rho U^*_2) \) is the nondimensional unsteady pressure, and where the vector \( Q(\mathbf{V}, p) = [Q_v(v, w, p), Q_w(v, w, p)]^T \) includes the convective derivatives, pressure and viscous terms.

Equations (19), representing the Navier-Stokes and continuity equations transposed in the fixed-grid computational domain, are further discretized in real time based on a second order three-point-backward implicit scheme, and a pseudo-time relaxation procedure with artificial compressibility is used in a similar manner to that presented in the previous section. Similarly, a factored ADI scheme is then used in conjunction with spatial differencing on a staggered grid and with a special decoupling procedure in order to reduce the problem to the solution of several sets of decoupled scalar tridiagonal systems procedure 7-9.

Two-dimensional unsteady annular flow solutions. The solutions obtained for the 2-D unsteady annular flow problem defined above 9 are illustrated in Fig. 5, which shows the typical variations of the real and imaginary components of the reduced unsteady pressure, \( \dot{p} = p / \exp(i \omega t) \), with the relative amplitude of the transverse oscillations, \( e_0 \) (relative to the annular gap, \( ba-a \)). The present solution based on a time-dependent coordinate transformation is compared in Fig. 5 with the solution based on a mean-position analysis (without coordinate transformation). The imaginary components of these solutions (determining the phase lag of the unsteady pressure) were found increasingly different for larger amplitude of oscillations. This shows the importance of the time-dependent coordinate transformation for the analysis of unsteady flows with large amplitude oscillations of the moving boundaries.
Fig. 6. 2-D unsteady annular flows. Typical influence of the relative amplitude of oscillations on the real and imaginary components of the reduced unsteady pressure, $\hat{p}$ on the oscillating cylinder at $\theta=7.5^\circ$. Comparison between:
- ■, solution based on a time-dependent coordinate transformation, and
- □—, solution based on mean-position analysis.

Fig. 7. 3-D unsteady annular flows. Typical axial variation of the reduced unsteady pressure amplitude at $Z=0.127$ and $\theta=7.5^\circ$ for $Re_H=2900$, $\omega=0.416$ and $\varepsilon_0=0.054$. Comparison between:
- ---, solution based on time-dependent coordinate transformation;
- ·· ··, solution based on mean-position analysis;
- •, experimental results.

Three-dimensional annular flow solutions. The solutions obtained with this time-integration method for the 3-D unsteady viscous flow problems with oscillating boundaries are illustrated in Fig. 7 for an annular passage between two concentric cylinders subjected to a fully-developed laminar flow with the mean axial velocity $U_0$, when the central portion (of length $al$) of the outer cylinder executes transversal translational oscillations. Figure 7 shows the typical axial variation of the reduced unsteady pressure amplitude, $\hat{p}(x)$, acting on the oscillating cylinder. In this figure the solution based on a time-dependent coordinate transformation is validated by comparison with experimental results and also compared with a similar solution based on a mean-position analysis; the amplitude of oscillation used in these experiments ($\varepsilon_0 = 0.054$) was just at the limit when the mean position analysis is still reasonably accurate.
4. Conclusions

Recent time-accurate computational solutions are presented for several two- and three-dimensional unsteady viscous flows with oscillating boundaries. These solutions have been obtained with computational methods recently developed by this author and his coworkers, such as an enhanced hybrid-spectral method and time-integration methods using artificial compressibility and based either on a mean-position analysis or on a time-dependent coordinate transformation. These methods have been used to obtain solutions for unsteady viscous flows in 2-D and 3-D concentric and eccentric annular configurations with oscillating inner or outer walls, as well as for unsteady flows over a backward-facing step with an oscillating floor behind the step; some of these unsteady flow solutions were obtained for the first time by using these methods. Very good accuracy and computational efficiency have been displayed by these methods in all problems solved. The annular flow solutions have been validated by comparison with experimental results; a very good agreement was found with the experimental results. The method based on a time-dependent coordinate transformation was found better suited for the case of large amplitude oscillations.

References