Vibration models for the stator of electrical machines, measuring techniques and experimental verification

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Abstract

A suitable model for vibrations incorporating the effects of damping is described in this paper. Information on the vibration behaviour of the stator of electrical machines is obtained using an experimental modal-analysis. This paper presents a new measurement technique for the determination of the resonances, and the vibration behaviour at resonance, of stators and rotors of electrical machines. The measuring techniques, instrumentation and experimental arrangements are described. Experimental results on the vibration behaviour of a 120 hp induction motor stator are provided with a view to determine the actual role played by a stator in the production of noise and vibrations.

1 Introduction

In order to reduce the radiated electromagnetic acoustic noise from electrical machines, it is extremely important to understand the vibrational behaviour of stators. Vibrations, and therefore the radiated acoustic noise, can be reduced to a large extent if coincidences between the force-frequencies and the resonant frequencies of the stator of an electrical machine are avoided. Often it is impossible to avoid such coincidences at all operating conditions, particularly under variable frequency operation of electrical machines. By using a suitable vibration-model, a designer can predetermine the resulting vibrations and hence optimize the machine design from the perspective of noise and vibrations. An accurate determination of the vibration characteristics and the resonant frequencies of stators of electrical machines is therefore of paramount importance in the design of quiet electrical machines [1].
From the vibration point of view, the stator of an electrical machine can be modelled as a system consisting of a number of masses interconnected by springs and damping elements to facilitate the analytical solution of the dynamic behaviour of the structure. Although the mass, stiffness and damping are distributed parameters, they are lumped as discrete elements in the vibration-model to assist in the analytical solution. The number of degrees of freedom, which dictates the number of differential equations to be solved, is chosen such that it is sufficient to accurately characterize the system. Generally, the dynamic behaviour of the vibrating structure is described in the classical approach using the differential equations of motion relating the elemental masses, stiffnesses and dampers. Modal-analysis is an alternative form of describing equally well the dynamic behaviour of the structure in terms of the resonant frequencies, normal modes of vibration and the damping associated with the resonance. The modal-parameters can be measured experimentally, and they provide a better insight into the vibration behaviour of electric machine stators [2,3].

The traditional methods of experimental determination of the vibration behaviour involve the use of impulse or hammer excitation, and electromagnetic shakers which can provide a point-excitation. The electromagnetic shaker has a pin that mechanically excites the test object at various frequencies. Due to the use of a point-excitation-system, even the resonances at or near a harmonic of the fundamental excitation frequency could be excited. Further, it is difficult to avoid interference in the measurement from other near-by resonances. This is particularly true for laminated structures such as stators and rotors of electrical machines, where there could be many resonant frequencies that occur close to each other [4,5,6]. The measurement of the modal-parameters are consequently plagued with inaccuracies when determined using a point-excitation-system.

The conventional methods for the measurement of resonant frequencies, vibrations and noise are laborious and very time consuming. In the course of theoretical and experimental investigations, laboratory-techniques have been developed for the measurement of vibrations based on digital processing of signals. The method uses processing of different signals acquired from several transducers with the help of a data acquisition system and a personal-computer. This measurement procedure is quite fast and gives good accuracy.

2 Modal analysis

Modal-analysis is a process of forcing a structure to vibrate predominantly at a selected resonance with its associated mode, thereby eliminating any interference from other resonances of the structure. In order to achieve this, distributed electromagnetic forces are used to induce vibrations in the stator under investigation. The distributed excitation system used is designed to induce vibrations of certain modes. By using a distributed excitation system, sufficient amount of energy can be fed uniformly into the structure. This is
especially required for laminated structures, such as stators and rotors of
electrical machines, where the energy supplied through point excitation is
quickly dissipated within the structure producing widely different vibration
amplitudes at various locations [4,6]. Further, with the use of distributed forces
a particular mode of vibration at a resonant frequency can be studied in detail.
The excitation is so chosen that only the mode of interest is dominantly excited
within a frequency range. The structure would then respond in that principal
mode, and the damping of the structure can be obtained.

2.1 Theory underlying modal analysis

Practical structures, such as stators of electrical machines, are commonly
analyzed using vibration system models having multiple degrees of freedom.
The properties of a continuous structure can be simulated to any desired
accuracy by a system possessing a finite number of degrees of freedom. Each
degree of freedom corresponds to a natural frequency and a mode-shape, and
these can be examined individually. Considering a system with two degrees of
freedom as shown in Fig. 1, the equations of motion of the system are:

\[ m_1 \ddot{x}_1 + (c_1 + c_2)x_1 - c_2 \ddot{x}_2 + (k_1 + k_2)x_1 - k_2x_2 = F_1, \]
\[ m_2 \ddot{x}_2 + (c_2 + c_3)x_2 - c_2 \ddot{x}_1 + (k_2 + k_3)x_2 - k_2x_1 = F_2, \]

which can be written in a matrix form as:

\[
\begin{bmatrix}
  m_1 & 0 \\
  0 & m_2
\end{bmatrix}
\begin{bmatrix}
  \ddot{x}_1 \\
  \ddot{x}_2
\end{bmatrix}
+ \begin{bmatrix}
  c_1 + c_2 & -c_2 \\
  -c_2 & c_2 + c_3
\end{bmatrix}
\begin{bmatrix}
  \dot{x}_1 \\
  \dot{x}_2
\end{bmatrix}
+ \begin{bmatrix}
  k_1 + k_2 & -k_2 \\
  -k_2 & k_2 + k_3
\end{bmatrix}
\begin{bmatrix}
  x_1 \\
  x_2
\end{bmatrix}
= \begin{bmatrix}
  F_1 \\
  F_2
\end{bmatrix},
\]

or

\[
[m] \ddot{x} + [c] \dot{x} + [k] x = F.
\]

In solving the above equations for the response of \( \{x\} \) for a particular
set of exciting forces, the major obstacle encountered is the coupling between
the equations. In Eqn. 1, it can be seen that while the stiffness matrix is

![Figure 1 : Two degrees of freedom system.](image)
symmetric it is not diagonal resulting in an elastic coupling.

If the system of equations, could be uncoupled so that we obtain diagonal matrices for mass, damping and stiffness, then each equation will be similar to that of a single degree of freedom system, which can be solved independently of each other. As known, this process of deriving the system response by transforming the equations of motion into an independent set of equations is called modal analysis.

There are several types of damping that are present in real systems, and they are commonly classified as viscous, hysteretic, frictional, aerodynamic, etc. Although it is very difficult to ascertain the exact nature of damping present in a system, certain simplifying assumptions can be made to account for them. Firstly, it is assumed that the distribution of the damping is proportional. By proportional damping it is implied that the damping matrix, \([c]\) is proportional to the stiffness matrix \([k]\) or the mass matrix \([m]\), or to some linear combination of the two. The previous studies [1,4-6] of machine-stators indicate the presence of well defined mode shapes at all the resonant frequencies, thereby indicating the presence of proportional damping. Due to the assumption of proportional damping, the co-ordinate transformation which diagonalises the mass and stiffness matrices will also diagonalise the damping matrix.

The required co-ordinate transformation is one that decouples the system with regard to inertia and elasticity, and therefore yields diagonal mass and stiffness matrices. The orthogonal properties of the mode-shapes are used for this purpose. The \(\{x\}\) co-ordinates are transformed to \(\{\eta\}\) co-ordinates using the following relationship:

\[
\{x\} = \{\phi\} \{\eta\},
\]

where \(\{\phi\} = [\phi_1, \phi_2, ..., \phi_n]\) is referred to as the modal matrix, \(\phi_n\) is a mode-shape vector, and \(\{\eta\}\) is called modal co-ordinates. Therefore,

\[
[m] \{\phi\} \{\dot{\eta}\} + [c] \{\phi\} \{\ddot{\eta}\} + [k] \{\phi\} \{\eta\} = \{F\}.
\]

Pre-multiplying both sides by the transpose of the modal matrix \(\{\phi\}^T\), and using the orthogonal properties of the modal matrix which are given in following relationships:

\[
\]

Since \(M_i, C_i\) and \(K_i\) are diagonal matrices, we obtain a set of independent equations of the form:

\[
M_i \ddot{\eta}_i + C_i \dot{\eta}_i + K_i \eta_i = \{\phi_i\}^T \{F\} = F_i,
\]

where each equation represents a single degree of freedom system, \(M_i, C_i\) and \(K_i\) are the \(i^{th}\) modal mass, \(i^{th}\) modal damping and \(i^{th}\) modal stiffness, respectively. Eqn. 5 forms the basis for the use of distributed force excitation of structures. Thus, if the excitation force is chosen such that it resembles a particular mode of vibration, the system response will be similar to that of a
system with a single degree of freedom. This would permit each of the resonant frequencies to be individually examined.

The vibration response of a continuous structure (i.e. multiple degree of freedom system) to any force can be represented by the superposition of the various responses in their individual modes, considering each mode to respond as a single degree of freedom system. The forced vibration response of a continuous structure can therefore be obtained in terms of its modal parameters provided the mass, damping and stiffness of the structure are accurately known. It is obvious that the mass and the stiffness of the structure are needed to determine the resonant frequencies of the structure. For the determination of forced response, in addition it is required to know the damping present in the structure.

3 Experimental arrangements

In the following sections, the design details of the stator models, measurement set-up and the various excitation systems used are described.

3.1 Stator models

Model I is a smooth thick cylindrical shell made of mild steel. Fig. 2 shows the stator-shell model, which has the same dimensions as that of a 120 hp induction motor stator. Model II is made by stacking stator laminations, which were punched for the 120 hp induction motor mentioned above. Fig. 3 shows the cross-section of the laminated stator of the 120 hp induction motor. 14 gauge copper wires are installed in the slots of Model II so that they resemble the actual windings installed in an electrical machine stator.

**Figure 2**: Dimensions of the stator models.  
**Figure 3**: Cross-section of the laminated stator.
3.2 Measurement set-up

Although a variety of instruments for the measurement of sound and vibrations are commercially available, there exists a need to develop an experimental set-up that is particularly suitable for the study of noise and vibration problems of electric machines. The experimental set-up developed for such purposes is described in the following.

Fig. 4 shows the schematic block diagram of the measurement system. The measurement system acquires signals from accelerometers, microphones and search-coils. Vibration measurements usually require a modal analysis which involves the study of the vibration distribution along the machine/stator surface. To conduct a modal analysis, it is therefore necessary to measure the vibration signals from different locations at the same instant of time. In order to achieve this requirement a high speed data acquisition board with a capability of simultaneously acquiring data from the channels is used. In the case of microphones, the experimental arrangement multiplexes sequentially the microphone inputs and processes the information according to the test-code of a given standard.

The heart of the data acquisition and measurement system is a high speed data acquisition board. In the present system, the data acquisition board has the provision for 16 channels of single-ended analog inputs. Further, this board has the data acquisition speed of 250 kHz for A/D conversion with the resolution of 12 bits. For simultaneous acquisition, four sample-and-hold amplifiers are used to track and hold the various input channels while the A/D converter digitizes the amplifier outputs sequentially. Thus, a maximum sampling frequency of 50 kHz per channel is possible for simultaneous acquisition from four channels. Depending on the span of the search-coil used and the size of the machine, the voltage at the terminals of the coils may range from 0.2 V to tens of volts. Isolated wide-band signal conditioning modules are used to condition these signals before they are fed to the A/D converter. Piezoelectric accelerometers are used to measure vibrations. The vibration signals are pre-conditioned using charge-amplifiers. Condenser microphones are used in conjunction with microphone preamplifiers and interfaced to the data acquisition board.

The final step in the measurement process involves the processing of the acquired signals and displaying the results. The data acquisition board has an on-board memory which allows storage of $2^{18}$ samples without the intervention of the PC's processor. A continuous data transfer mode is used that allows half of the on-board memory to be filled while the other half of the on-board data buffer is emptied into the PC-system memory. This permits the processing of the signals in almost real-time. The software developed for processing of the signals allows monitoring of the signals in the time-domain as an oscillogram with a continuous display of its peak and rms values. In addition, an FFT analysis is used to determine the frequency spectra of the vibration, sound and the force signals.
**Figure 4**: Schematic diagram of the measurement set-up.
3.3 Excitation forces

In the present experimental set-up, it is possible to induce vibrations in the stator models using different excitation systems. The hammer excitation, and the excitation achieved with distributed electromagnetic forces are described in the following. In the previous investigations [5,6], the stator models were tested using a magnetic shaker.

3.3.1 Impulse response

In this method for investigating the vibration behaviour of the stator models, hammer excitation technique is used to obtain the impulse response. In the theoretical sense, an impulse is a function of infinite amplitude with zero width having unity area. Thus, its frequency domain has unity amplitude at every frequency. In practice, the model is struck sharply with a force small enough to elicit a vibration response. Although this excitation deviates from that of an impulse, it does excite all the significant resonant frequencies of the model. This method provides a quick procedure for finding the significant resonant frequencies. Since the energy distribution of the forcing spectra is actually a function of the pulse duration, the resonance amplitudes diminish with increasing frequencies. The vibration signal is measured with the help of 4 to 6 accelerometers, and the analog signal from one of the accelerometers is used to trigger the acquisition of the data. The ensuing vibrations are recorded, and the Fourier-analysis will reveal the significant resonant peaks of the model being tested.

3.3.2 Distributed electromagnetic forces

In the present experimental set-up, it is desired to prominently induce circumferential vibration modes of 0, 1, 2, 3 and 4. The mode of vibration is defined as the number of cycles of deformation around the circumference of the stator. In order to achieve this, electromagnetic forces having the distribution similar to the mode of interest are used. The electromagnetic forces are produced using a 2-pole or a 4-pole winding, which are installed on a stationary rotor structure. These windings are energised using a single-phase variable-frequency power supply to produce pulsating air-gap fields. The action of the pulsating magnetic fields on the stator surfaces produces a pulsating force that acts on the stator. These forces are called Maxwell's forces, and it is proportional to the square of the magnetic flux-density [7]. The 2-pole and the 4-pole windings are designed to have no mutual flux-linkages, which would also permit their combined operation. Further, provisions are made to connect the coils of the 2-pole or 4-pole windings in a 3-phase configuration, which results in rotating force distributions.

Mode 0, or uniform force distribution around the circumference can be obtained using either the 2-pole or 4-pole winding. Mode 2, or elliptical force
distribution around the circumference is obtained with the help of the 2-pole winding. The force distribution for mode 4 is obtained using the 4-pole winding. The combined excitation of the 2-pole and the 4-pole windings yields the force distributions necessary for mode 1 and mode 3 vibrations.

4 Experimental results

The impulse responses of the stator-shell model and the laminated stator are shown in Fig. 5 and 6, respectively. The amplitudes of acceleration levels are plotted on a logarithmic scale versus frequency. The acceleration amplitudes are shown in mm/sec², and also in dB relative to $10^{-2}$ mm/sec².

The response of any vibration system to an impulse excitation will be to excite all its significant resonances. The various peaks in the impulse response, therefore, correspond to the resonances of the model. These peaks correspond very well to the resonances reported by the authors in references [5,6]. Table 1 lists the resonances of the stator-shell model investigated in this study. These resonances are selected such that they represent resonances of circumferential modes $n=0$ to 5. Further, for each of the circumferential modes of vibration an axi-symmetric and an anti-symmetric resonance are chosen. The various peaks of interest in the impulse response are identified according to Table 1. The selected resonances of the laminated stator are provided in Table 2. In the case of the laminated stator, no resonances associated with anti-symmetric longitudinal modes were present.

The present experimental set-up and the measurement techniques permit the calculation of the damping-ratio associated with each mode of vibration with more accuracy. The amount of damping present at a particular mode is derived from the amplitude response curve at the resonance. Damping is determined from the sharpness of the peak, and is normally measured in terms of the Loss-Factor given by the ratio:

$$\zeta = \frac{\omega_2 - \omega_1}{2 \omega_0},$$

where $\omega_0$ is the natural frequency and $\omega_1$ and $\omega_2$ are frequencies on either side of the natural frequency where the peak amplitude is reduced by 3 dB, and $\zeta$ is called the damping-ratio. To eliminate any interference from other near-by resonances, a least-error square curve-fit is used through the experimental data points to obtain the damping-ratio. The damping model, which is general enough to cover viscous, hysteretic and frictional damping, is used for the curve fitting procedure.

Figs. 7 and 8 show the vibration response at 3197 Hz, $n=3$, $m=0$ resonance of the stator-shell model. There is a resonance of the stator-shell model at 3139 Hz associated with $n=2$, and $m=2$. The benefit of distributed forces is its ability to selectively excite resonances. Thus, the damping is determined from the vibration response with little interference from the neighbouring resonances. Fig. 9 shows the vibration response at $n=2$, $m=0$. 

**Table 1**: Measured damping-ratios for resonances of the stator-shell model with different excitation force distributions.

<table>
<thead>
<tr>
<th>S.No</th>
<th>Freq</th>
<th>n</th>
<th>m</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
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* : Could not be measured, n = circum. mode, m = long.mode.

A = Impulse response, B = Pulsating force-distribution with 2-pairs of force-poles,
C = Pulsating force-distribution with 4-pairs of force-poles, D = Pulsating force-distribution with 1&3-pairs of force-poles
E = Rotating force-distribution with 2-pairs of force-poles

**Table 2**: Damping-ratios measured at selected resonances of the 120 hp laminated stator with windings using different excitation forces.

<table>
<thead>
<tr>
<th>S.No</th>
<th>n</th>
<th>m</th>
<th>Freq, Hz</th>
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<th>B</th>
<th>C</th>
<th>D</th>
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resonance at 470 Hz for the laminated stator model. The damping-ratios measured at the selected resonances of the stator-shell and laminated stator are provided in Tables 1 and 2, respectively. It is observed that the damping present in the laminated stator is about 10 to 100 times of that present in the solid stator-shell model.

**Figure 9** Impulse response at 470 Hz for the laminated stator.

### 5 Conclusions

A versatile system for the measurement of acoustic noise, vibrations, resonant frequencies and electromagnetic forces is described in this paper. The measurement system is relatively economical. It has the advantage of acquiring and processing various signals simultaneously, which results in significant savings of labour and time.

The advantage of the distributed excitation forces over the magnetic shaker which provides a point excitation is in its ability to excite resonances selectively. Further, the distributed forces facilitate injection of considerably more vibrational energy into the test model.

### 6 References