Prediction of the Orientations of Main Cracks caused by Hydrofracturing in Strained Rock Samples

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Abstract

We discuss experimental results dealing with crack propagation caused by hydrofracturing in strained rock samples and specify a coupled Boundary Element - Displacement Discontinuity (BE-DD) Method that is used to analyse fracture propagation in laboratory and field experiments. We find that the direction of the main crack propagation in the laboratory experiments may depend on stress concentrations close to edges of initial microcracks. Higher-order coupled BE-DD Method is used to solve two-dimensional problems. A new semi-analytical method to construct three-dimensional edge boundary elements is described.

1 Introduction

This research is a part of the Hot Dry Rock Project dealing with creation of artificial geothermal energy systems and environmentally clean energy extraction from underground hot rocks. A typical system contains two or more wells and a fractured reservoir between the wells at the depth where rocks are hot. Once the
system is created, heat can be extracted from the rock mass surrounding the reservoir by water that flows through fractures. The bigger the reservoir we make the higher the productivity if the system. Hence, the knowledge on how fracture propagation can be controlled using available technical facilities is of significant importance to create the system. To get this knowledge, a number of laboratory and field experiments were conducted by researchers of the National Institute for Resources and Environment (Japan). The ultimate goal of the theoretical work presented herein is to find explanation of the experimental results and develop numerical methods that can be used to simulate the reservoir creation and management.

2 Laboratory experiments and their modelling

In the course of the laboratory experiments, fracture propagation in rock samples is studied. A sample has a cubic shape (Figure 1a). Each side has a length of 20 cm. The sample is made of granite with nearly isotropic elastic properties (Young’s modulus equals $E=50$ GPA, Poisson’s ratio equals $\nu=0.23$).

Let us place the centre (point $O$) of the Cartesian coordinate system at the centre of the sample. The axis $Ox$ is perpendicular to the surface $AA'B'B$. The axis $Oy$ is perpendicular to the surface $BB'C'C$.

A borehole is drilled along the axis $Oz$ through the centre of the sample (Figure 1b). The radius $r$ of the borehole is equal to $r=0.5$ cm. Two packers are used to create a chamber in the borehole (Figure 2).

During laboratory experiments, at first, the sample is loaded by normal external pressure through its surfaces $AA'B'B$, $BB'C'C$, $CC'D'D$, and $DD'A'A$. We call this sides as the external surface. Afterwards, water is pumped up to the chamber and the rock sample is loaded by additional internal pressure $-P_{\text{int}}=22$ MPa. Such a loading results in fracture that cuts the sample. If external pressure is small or equal to zero the fracture propagates along the borehole (Figure 3ab). However, in the experiments with external pressure that is equal to $-P_{\text{ext}}=8$MPa, fracture propagates perpendicular to the borehole axis (Figure 3cd).

![Figure 1: Rock sample and coordinate system (a) and borehole position in the sample (b).](image-url)
Figure 2: Boundary conditions on the borehole surface: normal $P_{int}$ (a) and shear $P_{sh}$ (b) loads, respectively.

Figure 3: Two different crack orientations. Isometric view on radial (a) and transversal (c) cracks and corresponding sample cross-sections (b and d, respectively) by plane $xy$. 
Two sets of numerical simulations using Indirect Boundary Element Method [5] have been made to find out an initial stress field in the sample. In the first set, a model of the sample without a borehole is considered. We study influence of boundary conditions on the external surface on the stress field at the central area of the sample and analyse 4 following cases:

1. Case SSS. Stress boundary conditions on the external surface of the sample: normal constant load $P_{\text{ext}}$ and zero shear loads.

2. Case SSD. Mixed boundary conditions on the external surface: normal constant load $P_{\text{ext}}$ and zero displacements in $z$ direction, together with relevant zero shear loads.

3. Case DSD. Mixed boundary conditions on the external surface: normal constant displacements $u_{\text{ext}}$ and zero displacements in $z$ direction, together with relevant zero shear loads.

4. Case DDD. Displacement boundary conditions on the external surface: normal constant $u_{\text{ext}}$ and zero tangential displacements.

Results of the calculations allow the conclusion that in all above cases the stress state is plane at the central area of the sample. With accuracy of 5% in the 6x6x6 cm central area and 10% in the 10x10x10 cm central area, we have got

$$\sigma_x: \sigma_y: \sigma_z: \tau_{xy}: \tau_{xz}: \tau_{yz} = 1:1:0:0:0:0, \quad (2.1)$$

where the maximum stress values are equal to $-P_{\text{ext}}$.

In the second set, a model of the sample with the borehole is considered (Figure 1b). Stress boundary conditions are set: the external pressure is equal to $-P_{\text{ext}}=8\text{MPa}$, maximum value of the additional internal pressure is equal to $-P_{\text{int}} = 22 \text{MPa}$, the influence of each packer is modelled by applying linear normal and linear shear ($P_{sh} = 0.25P_{int}$) loads along the axis Oz on the part of the borehole surface which contacted with the packers (Figure 2a,b).

In the central region of the sample, the results of the computations are close to those received from the analytical solution of the plane strain problem, but with one exception. Maximum stress concentration is observed on the surface of the borehole:

$$\sigma_r = -P_{\text{int}}, \quad \sigma_\theta = P_{\text{int}} - 2 \cdot P_{\text{ext}}, \quad \tau_{r\theta} = P_{\text{int}} - P_{\text{ext}}, \quad \sigma_z = \tau_{xz} = \tau_{yz} = 0, \quad (2.2)$$

where $\sigma_r$ and $\sigma_\theta$ are radial and circumferential stresses, respectively. Note, that in the framework of the plane strain problem $\sigma_z = \nu(\sigma_x + \sigma_y)$.

### 3 Discussion

Comparing results of the experiments and mathematical modelling, we can observe rather interesting phenomenon. The maximum tensile stress $\sigma_\theta$ is on the surface of the borehole (ref. equations (2.2)). This stress might initiate radial cracks shown in Figure 3,a,b. Therefore, we can expect that only these cracks grow at the first stage of the fracture propagation. However, according to the
experiments, the main crack grows perpendicular to the borehole (Figure 3c,d) just from its surface. Note, the final crack orientation is in accordance with known fact that crack usually grows perpendicular to the minimum principal stress in a compressive stress field. But taking into consideration the stress values close to the borehole (ref. equations (2.2)), we may find rather unusual that the crack begins to grow in this direction.

On the authors’ opinion, following hypotheses could be employed to explain this phenomenon. In both of them, we assume that (1) microcracks should be taken into consideration and (2) the analysis of the first stress intensity factors near to edges of microcracks with different orientations allows us to predict the direction of the main crack growth.

Assuming linear elastic fracture mechanics, we can estimate the first mode stress intensity factor $K_I$ as follows

$$K_I = K_I^* \cdot \sigma_n,$$  \hfill (3.1)

where $\sigma_n$ is driving stress, $K_I^*$ depends on crack geometry [7, 8]. Let us assume that the value $K_I^*$ is constant for a microcrack in the concrete material. Then the value of $K_I$ is defined by driving stress $\sigma_n$ and the main crack propagates in the direction that allows the maximum value of the driving stress. This is the main idea of the first hypothesis. We can predict the direction of the main crack propagation if (1) we consider a number of microcracks that have connections with the borehole and different orientations, (2) estimate driving stresses at the microcrack edges, and (3) choose among of them the microcrack with the maximum driving stress. The position of the last microcrack defines the direction of the main crack propagation.

For example, in a sample under axial tension $\sigma_t^*$ we have $\sigma_n = \sigma_t^*$. The main crack propagates perpendicular to the tension if $\sigma_n > \sigma_t$, where $\sigma_t$ is the tensile strength of the material. For the granite, $7.5 \text{ MPa} \leq \sigma_t \leq 8.8 \text{ MPa}$ (Figure 4). Driving stress for a radial microcrack (ref. Figure 3a,b) equals $\sigma_{nr} = 2P_{ext} - P_{int} = 6 \text{ Mpa}$ (line 1, Figure 4). Here we assume that there is water under driving pressure $-P_{int} = 22 \text{ Mpa}$ in the microcrack while remote stress equals $-2P_{ext} = 16 \text{ Mpa}$. Driving stress for a transversal microcrack (ref. Figure 3c,d) equals $\sigma_{nt} = -P_{int} = 22 \text{ Mpa}$. We have produced the value of $\sigma_{nt} = 22 \text{ Mpa}$ assuming that $K_I^*$ is constant for any of the microcracks. However, we should multiply value $\sigma_{nt}$ by a coefficient $\alpha<1$. Otherwise, we can not explain crack growth along the borehole under $P_{ext} = 0$. The value of the coefficient $\alpha$ should be defined experimentally or by numerical modelling of microcracks. Line 2 (Figure 4) is drawn for $\alpha=0.46$. Since $\sigma_{nt} = -\alpha \cdot P_{int} = 10 \text{ MPa} > \sigma_{nr}$ then the main crack grows perpendicular to the borehole axis (Figure 3c,d) in accordance with the experimental results.
If we employ the second hypothesis, the process of the main crack propagation appears to be as follows. During the experiments, at first, radial microcracks begin to grow along the borehole (Figure 3a,b). Afterwards, close to the microcrack tips, strong tensile stress $\sigma_0$ results in tensile stress $\sigma_2 = 2\nu\sigma_0$ and respective growth of transversal microcracks that initiate the main crack.

Mentioned above linear fracture mechanics approach allows us to get an idea about the directions of the main fracture propagation in the strained rock samples and explain known experimental results. If driving stresses for radial and transversal microcracks smaller than $\sigma_t$ then no fracture is created (Column I, Figure 4). If $P_{ext}$ is small enough then we can have $\sigma_{nr} > \sigma_{nt} > \sigma_t$ and the main crack propagates in radial direction (Column II, Figure 4). We can have $\sigma_{nt} > \sigma_{nr} > \sigma_t$ in the Column III (Figure 4) and predict main crack propagation in transversal direction. This case corresponds to the experiments discussed herein. If $P_{int}$ is big enough then we can have $\sigma_{nr} > \sigma_{nt} > \sigma_t$ and the main crack propagates in radial direction (Column IV, Figure 4).

\[ \begin{array}{cccc}
\sigma_n \\
I & II & III & IV \\
\begin{array}{c}
\text{8.8 MPa} \\
\text{7.5 MPa}
\end{array} & \\
\begin{array}{c}
-\text{P}_{int} = 22 \text{ MPa} \\
-\text{P}_{ext} = 8 \text{ MPa}
\end{array}
\end{array} \]

Figure 4: Interpretation of the experiments. Driving stresses for radial (1) and transversal (2) microcracks, respectively.
4 Coupled BE-DD Method

The above hypotheses of fracture initiation and propagation allow a conclusion that computational models of hydrofracturing must be able to consider explicitly microcracks, analyse accurately stress fields close to their edges, and model an interaction between them and main cracks. Because sizes of the microcracks are much smaller than the sizes of the main ones and, therefore, stress fields near to microcrack edges mainly depend on the stress fields generated by the main cracks, then the modelling must provide us with accurate stress values in very vicinity of edges of the main cracks.

Taking into consideration the size of the problem which might contain many microcracks, the Coupled Boundary Element and Displacement Discontinuity (BE-DD) Method [1] appears to be the least labour and time consuming amongst of the numerical methods that are usually employed in Rock Mechanics to deal with similar problems. This method also allows accurate stress modelling near 2D crack tips if special boundary elements are used in the model.

Analytical influence functions for BE/DD elements were derived [3] and used in the new formulation of the Higher Order Displacement Discontinuity Method (HODDM) to model straight slits in anisotropic media [4, 6]. These functions and the HODDM were implemented into the 2D coupled BE-DD scheme [1] and a number of problems were analysed with a goal to study the influence of the elastic anisotropy on displacements in rock mass and samples. Because of the limited volume of this paper the results of this research will be provided in the report at the Conference and can be distributed by the authors on request.

5 3D edge displacement discontinuity elements

Since we also need accurate stress values in very vicinity of the main crack in 3D analysis, a special technique must be used to model crack edges in such problems. One of the main steps for creating computer codes based on the Displacement Discontinuity Method is the integration of concentrated dipoles on boundary elements. Because the solution for a concentrated dipole has a singularity at the point of application, numerical integration of the concentrated dipoles on boundary elements usually leads to unsatisfactory results in the vicinity of edges of these elements. It is possible to improve the results at points close to the element centres by using analytical integration. Currently, analytical influence functions are known only for constant three-dimensional displacement discontinuity elements in an isotropic medium [2, 7]. Note that such constant elements do not allow an accurate modelling if stress concentration near to the edges. Analytical influence functions are not available for non-constant three-dimensional elements, however the following method can be employed to construct influence functions when receiver nodes or field points are close to the emitter element. The main idea of the method is to construct three-dimensional leaf emitter element as a superposition of infinite band-type one which is loaded by additional system of distributed dipoles.
To explain the idea, let us consider a 3D crack in an anisotropic medium (Figure 5a). Suppose, we need to determine the stresses and displacements assuming linear elastic fracture mechanics close to the edge of the crack by the Displacement Discontinuity Method. As in the 2D problems [4, 6], let us use 3D root edge elements to obtain accurate results. The central part of the crack is approximated by using constant displacement discontinuity elements.

Consider the 3D emitter root edge element E (Figure 5a) as a part of the infinite band-type root element (Figure 5b). The analytical influence functions
for such an infinite element are already known [3]. By applying distributed dipoles with the same intensities, but with opposite signs on the band in the vicinity of the 3D element, we can reach total unloading of the band in these areas (areas for numerical integration). If the last areas are semi-infinite ones we can extract the pure influence of the three-dimensional element. Influence functions for the three-dimensional element are accurate ones because they are constructed from the analytical solution for the infinite element.

Practically, the distributed dipoles are modelled by using displacement discontinuity point integration cells that act on the finite parts of the band. Therefore, we must expect two sources of errors: (1) due to the influence of the semi-infinite parts of the infinite element, and (2) due to numerical integration of point cells in the vicinity of the 3D element. The first of this sources can be successfully eliminated if the area of numerical integration is big enough [4]. We can also eliminate the second source of errors if we use an appropriate flexible scheme of numerical integration [5].

Thus, we are able to extract accurately the pure action of the 3D root edge element. To model the crack edge with finite stresses, we can analogously construct parabolic edge elements. Such elements can be used to model tension-softening or slip-weakening processes at the crack edge [1]. They are also very suitable for the modelling of the stopes in tabular excavations. We are able to employ the above approach to calculate stresses and displacements at the points which are situated in the vicinity of the element's surfaces, or on the surfaces themselves. The same technique can be also employed to find the semi-analytical influence functions for constant inner elements. In this case, we can use three different constant infinite elements (Figure 5c).

6 Conclusions

We have discussed experimental results dealing with crack propagation caused by hydrofracturing in strained rock samples and specified a coupled Boundary Element - Displacement Discontinuity (BE-DD) Method that has been used to analyse fracture propagation in laboratory and field experiments. We have found that the direction of the main crack propagation in the laboratory experiments may depend on stress concentrations close to edges of initial microcracks. Following this conclusion, we emphasise an importance of explicit microcrack consideration in the mathematical models of crack propagation due to hydrofracturing. Higher-order coupled BE-DD Method was used to solve two-dimensional problems. A new semi-analytical method to construct three-dimensional edge boundary elements was described.

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References


