High strength concrete: computational and experimental measurements of stress-strain distributions

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Abstract

The use of high strength concrete (HSC) is increasing faster than the development of a suitable design recommendation, as this new material has become a viable alternative to normal strength concrete (NSC). The authors endeavour to establish a suitable and distinct design criteria in the context of HSC element based on the stress-strain characteristics of HSC beams subjected to pure flexural loads. Test series included concrete strength ranging from 70 MPa to 120 MPa used as a structural elements with various reinforcements. The model analysis results are verified with experimental measurements of the stress-strain distributions. The paper also examines the applicability of the existing design methods in cases where concrete strengths are far beyond present code limits.

1 Introduction

The distribution of the concrete stress in the compression zone is one of the most important characteristics in designing a reinforced concrete flexure member. Significant amount of research development has been conducted in the stress-strain and flexural stress distribution for ultimate design of ordinary concrete structure with compressive strength up to 40 MPa [1]. On the other hand, the development and application of high strength concrete (HSC) with compressive strength up to 120 MPa have been generating increasingly greater interest among civil engineers. This interest stems from the potential commercial use of this "new material" which can be explained by the life cost cycle it offers. However, designers are stretching the ordinary concrete design method beyond its capabilities when used to design structures utilising HSC. Moreover, only a few investigations are available which verified the use
of ACI code rectangular stress block for concretes with strength above 55 MPa, like Laslie et al [2], Kaar et al [3], and Paul Zia [4]. The latter one suggested to revise the design parameters for flexural reinforcement of higher strength concrete. These design parameters are the elastic modulus, modulus of rupture, and the minimum reinforcement value. Moreover, Zia also tried to verify the idealised trapezoidal stress-strain curve which was originally suggested by Jenseon [5]. He concluded that, it might be suitable to use the trapezoidal stress strain curve for application over the full range of concrete strengths.

Furthermore, the major obstacles to the greater use of HSC are lack of knowledge of its basic properties and of its characteristics when used in reinforced concrete. The paper reported herein is intended to measure the characteristic of stress-strain distribution of HSC beams under pure flexural loads until failure, and to evaluate previous, and existing methods, with reference to experimental measurements of stress-strain distribution in HSC elements, and to develop a test method leading to improved and quantitative understanding of the stress distribution in the context of ultimate limit strength of HSC in flexure. Success in this endeavour will permit concrete technologists to make widespread, more precise and cost-effective use of HSC.

2 Experimental consideration

Three concrete mixes (80, 100, and 120 MPa) were selected from series of trial mixes concerning HSC which were conducted at the first phase of this investigation. These mixes were employed for the structural behaviour of HSC beams. Nine beams were cast and tested under flexure loads (see Table:1). The overall length of each beam specimen was 3440 mm. Fig. 3 shows the loading configuration. At each load increment, two different measurements were taken: the compressive concrete strains in the constant moment region through the depth of the beam, the strains in the tensile reinforcing bars. The concrete strains were measured by means of mechanical Demec gauges having a standard length of 200 mm from both face of the beam specimen.

2.1 Stress-Strain relationship

The stress-strain relationships of the concrete mixes were determined in compliance with the BS 1881: part 115, Fig. 1 shows some stress - strain behaviour of typical specimens. It is obviously seen as the concrete compressive strength increases the modulus of elasticity (Ec) increases. Moreover, the shape of the curve is highly linear up to the peak value.

2.2 Strain distribution

For all beams tested the flexural concrete strains along the depth were measured. The strain were measured at each increment load. Furthermore, the average strain of the front and back readings were determined. Fig.2 shows the
Figure 1: Stress-strain relationship of HSC

Figure 2: Strain Distribution along the depth of beam HSC3-1
variation of flexural strains along beam HSC1-3 depth. It could be seen that strains were distributed linearly above the neutral axis throughout all load stages. At each load stage the neutral axis was located.

It is generally noted, as the load is increased, the strain curves tend to flatten out (i.e. the strain increases) moreover, for beams with low tensile reinforcement the neutral axis rose prior to failure, while for heavily steel members the neutral axis remains at the same position as it was prior to failure. On the other hand, it should be noted that the maximum obtained strain value was 0.0031 for beam HSC3-2 at the extreme compression fibre, whereas 0.0021 flexural strain was recorded for beam HSC1-3 at the extreme compression fibre, it was the lowest strain value obtained among all the test members. This indicates the brittleness of higher strength concrete.

The measured steel strains showed that the yield strength of the reinforcement was attained in all beams prior to crushing of the compression concrete. This reveals that all test specimens were under-reinforced sections.

### Table 1: Testing Programme of High Strength Concrete Beams.

<table>
<thead>
<tr>
<th>Beam+</th>
<th>$f_{cu}$, MPa</th>
<th>h, mm</th>
<th>b, mm</th>
<th>d, mm</th>
<th>$E_c$, MPa</th>
<th>$\rho$, %</th>
<th>Neutral axis, mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>HSC1.1</td>
<td>107</td>
<td>250</td>
<td>150</td>
<td>220</td>
<td>40640</td>
<td>1.03</td>
<td>22.93</td>
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<tr>
<td>HSC1.2</td>
<td>97</td>
<td>250</td>
<td>150</td>
<td>220</td>
<td>41000</td>
<td>1.03</td>
<td>22.05</td>
</tr>
<tr>
<td>HSC1.3</td>
<td>85</td>
<td>250</td>
<td>150</td>
<td>220</td>
<td>38550</td>
<td>1.03</td>
<td>26.09</td>
</tr>
<tr>
<td>HSC2.1</td>
<td>105</td>
<td>250</td>
<td>150</td>
<td>212.5</td>
<td>49650</td>
<td>1.42</td>
<td>33.25</td>
</tr>
<tr>
<td>HSC2.2</td>
<td>100</td>
<td>250</td>
<td>150</td>
<td>212.5</td>
<td>43600</td>
<td>1.42</td>
<td>31.85</td>
</tr>
<tr>
<td>HSC2.3</td>
<td>77</td>
<td>250</td>
<td>150</td>
<td>212.5</td>
<td>34000</td>
<td>1.42</td>
<td>27.31</td>
</tr>
<tr>
<td>HSC3.1</td>
<td>107</td>
<td>250</td>
<td>150</td>
<td>215</td>
<td>53000</td>
<td>1.94</td>
<td>34.47</td>
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<td>150</td>
<td>215</td>
<td>39000</td>
<td>1.94</td>
<td>31.30</td>
</tr>
<tr>
<td>HSC4.1</td>
<td>101</td>
<td>250</td>
<td>150</td>
<td>207.5</td>
<td>41200</td>
<td>4.04</td>
<td>64.61</td>
</tr>
</tbody>
</table>

+ Beam nomenclature: for HSC indicates High strength singly section, "1-1" the lowest $\rho$, and the highest $f_{cu}$ grade in each group respectively.

### 3 Computational consideration

#### 3.1 BS 8110 at ultimate state

As mentioned earlier, the ultimate load behaviour of normal reinforced concrete beams is now well understood. Meanwhile, the current design method in the British code relating the flexural strength computation is based on the following essential assumptions:

1. Strains in both concrete and steel reinforcement are directly proportional to the distance from the neutral axis.

2. The ultimate concrete strain at the extreme compression fibre is 0.0035 and the parabolic part of the stress block ends at a strain of $\varepsilon = \sqrt{f_{cu}}/5000$. 


3. Tensile strength of concrete is neglected in all flexural calculations for reinforced concrete. (See Fig. 5)

Accordingly, the ultimate strength of reinforced concrete will be as follows:

\[ M_\tau = A f_t \left( 1 - \rho \frac{k_2 f_t}{k_1 f_{cu}} \right) d \]  

where;

\[ k_1 = 0.45 \left( 1 - \sqrt{\frac{f_{cu}}{52.5}} \right) \]  
\[ k_2 = \frac{\left( 2 - \sqrt{f_{cu}/17.5} \right)^2 + 2}{4 \left( 3 - \sqrt{f_{cu}/17.5} \right)} \]

and \( k_1 \) and \( k_2 \) coefficients concerning to amount and position of internal compressive force in compression block.

### 3.2 Proposed method

Using the strain date along the depth of beams, the neutral axis was located. The stresses corresponding to the strains are calculated using the cylinder stress-strain curve. Therefore, a second degree equation was fitted, using the regression analysis to give the stress at any point at a distance measured from the neutral axis. The equation of the curve was used to get the area of the stress block and the location of the centroid of the stress block by integration. The equation of the stress block is:

\[ f_c = 41155 \varepsilon - 1245809 \varepsilon^2 - 0.67 \]  

Also, from the strain distribution along the beam depth, the strain can be written as a function of neutral axis, and ultimate strain:

\[ \varepsilon = \frac{\varepsilon_u}{c} x \]  

where;

\( \varepsilon_u = \) flexural ultimate strain of each beam.
\( x = \) distance from neutral axis to the given fibre (mm).
\( c = \) neutral axis depth at ultimate (mm).

Substitute eqn. (5) into eqn. (4) yields to:

\[ y_\tau = A \left( \frac{\varepsilon_u}{c} x \right) - B \left( \frac{\varepsilon_u}{c} x \right)^2 - 0.67 \]  

where;

\( y_\tau = f_c = \) stress
\( A = 41155 \) (constant)
\( B = 1245809 \) (constant)

The area (compression force) under the curve is given by integrating between the neutral axis and the top of the beam. This can be illustrate as follows:

\[ C = \int_0^c \left[ A \left( \frac{\varepsilon_u}{c} x \right) - B \left( \frac{\varepsilon_u}{c} x \right)^2 - 0.67 \right] dx \]  

The location of centroid from neutral axis is given by ;
Figure 3: Loading configuration of the beam specimen

Figure 4: The proposed stress block distribution

Figure 5: Ultimate limit state BS8110, [7]
The centre distance from the extreme compression fibre ($x$) can be determined as follows:

$$
\bar{x} = (c - \xi)
$$

(9)

Therefore, the total nominal moment capacity of the beam at ultimate can be determined as follows:

$$
M_n = C.b.(d - \bar{x})
$$

(10)

where;

- $M_n$ = nominal moment at ultimate (N-mm)
- $C$ = area of the compression stress block (N)
- $b$ = width of beam (150 mm)
- $d$ = distance from the extreme compression fibre to the tension steel (mm).
- $\bar{x}$ = location of the centroid from the top compression fibre (mm).

The above procedures are followed in order to determine Neutral axis, the design criteria ($k_2$), and ultimate moment capacity. Table 2 shows the comparison of BS ultimate state method, and the proposed method with the test results obtained from this investigation.

**Table 2: Comparison of theoretical & observed values of ultimate moments capacity**

<table>
<thead>
<tr>
<th>Beam</th>
<th>$f_y$ (MPa)</th>
<th>$Mu_{,test}$ (KN-m)</th>
<th>$Mn_{,theo}$ (KN-m)</th>
<th>$\frac{M_u_{,(test)}}{M_u_{,(theo.)}}$</th>
<th>$Mu_{,(BS)}$ (KN-m)</th>
<th>$\frac{M_u_{,(test)}}{M_u_{,(BS)}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>HSC1-1</td>
<td>470</td>
<td>38.94</td>
<td>33.83</td>
<td>1.15</td>
<td>33.23</td>
<td>1.17</td>
</tr>
<tr>
<td>HSC1-2</td>
<td>470</td>
<td>35.64</td>
<td>33.87</td>
<td>1.05</td>
<td>33.06</td>
<td>1.08</td>
</tr>
<tr>
<td>HSC1-3</td>
<td>470</td>
<td>37.62</td>
<td>33.66</td>
<td>1.12</td>
<td>32.77</td>
<td>1.15</td>
</tr>
<tr>
<td>HSC2-1</td>
<td>470</td>
<td>46.33</td>
<td>42.78</td>
<td>1.08</td>
<td>41.86</td>
<td>1.10</td>
</tr>
<tr>
<td>HSC2-2</td>
<td>470</td>
<td>46.86</td>
<td>42.88</td>
<td>1.09</td>
<td>41.70</td>
<td>1.12</td>
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<tr>
<td>HSC2-3</td>
<td>470</td>
<td>43.56</td>
<td>43.19</td>
<td>1.00</td>
<td>40.71</td>
<td>1.07</td>
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<tr>
<td>HSC3-1</td>
<td>442</td>
<td>67.32</td>
<td>56.46</td>
<td>1.19</td>
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<tr>
<td>HSC3-2</td>
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<td>HSC4-1</td>
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<td>92.42</td>
<td>103.19</td>
<td>0.90</td>
<td>92.00</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Mean 1.08 Mean 1.13
St. Dev. 0.090 St. Dev. 0.080

*The partial safety factor of the reinforcement is not included, whereas of concrete ($\gamma_m$) is included.
4 Conclusions

Based on the work reported here of the stress-strain distribution at ultimate and its application in high strength concrete beams, the following conclusions are made:

(1) For all beam tested the measured maximum flexural strain at extreme compression fibre ($\varepsilon_u$) was 0.0031. This indicates the brittle nature of HSC.

(2) It is obvious that normal design expressions (i.e. BS8110) can be extended well beyond their present range, at least for under-reinforced HSC beams.

(3) Based on the stress-strain data of all specimens tested and the measured flexural strain along beams depth, a stress block is proposed in order to estimate the ultimate moment capacity of under-reinforced HSC beams in flexure. The model would be able to predict the strength in reasonable agreement with experimental results obtained.

5 References


