Indirect determination of the fracture properties through three point bending tests

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Abstract

The direct determination of the mode I softening function requires sophisticated laboratory equipment, therefore an indirect estimate of the material properties, based on tests achievable in standard laboratories, is often preferred. In this latter case, however, identification procedures, responsible for nonneglectable error sources, must be resorted to. The paper aims to analyse the performances of two alternative procedures usually followed to estimate the material parameters of the cohesive crack model. Experimental results from three point bending tests on notched high strength concrete specimens are used on purpose.

1 Introduction

The mode I softening function, the basic ingredient in the cohesive crack model to predict the structural response of brittle disordered materials, can be determined from experimental results in a direct or indirect way.

The direct determination of the fracture properties from tensile tests presents some practical problems due to the global unstable behaviour of the specimen and to the difficulty in achieving a uniform crack opening, so that it cannot be accomplished in standard laboratories. Therefore, the indirect determination of the softening function is often preferred and the fracture properties are evaluated from conventional three or four point bending tests. In order to determine the softening function an inverse problem has to be solved and two main approaches can be followed: (i) the structural response is separated in a number of different regions primarily influenced by a restricted number of parameters; the "regional" behaviour is then approximated by explicit expressions to obtain a system of equations in the unknown parameters; (ii) the structural response is analysed as a whole and nonlinear optimization techniques, involving iterative, finite element modelling of the structural behaviour, are used.
The paper aims to discuss potential problems relevant to the above approaches; to this end the identification of the cohesive crack model parameters from three point bending (3PB) tests on high strength concrete (HSC) is analysed, giving emphasis on the computational aspects and the results accuracy.

2 Numerical models

The key step for the indirect determination of the fracture properties relies on appropriate numerical simulations of the material and the structural behaviour of the tested specimens, it is hence useful to discuss briefly the models adopted.

2.1 Material model

When dealing with 3PB tests on notched specimen it is customary to consider a linear-elastic behaviour in the whole specimen except that in the fracture process zone where the material behaviour is assumed orthotropic and degrading at the onset of the peak tensile stress $f_t$. As long as the monotonic mode I is concerned, only tractions $f_{cr}$ orthogonal to the crack surface can develop, whose intensity decreases with the opening $w$ between the opposite faces of the cohesive crack. A commonly accepted model is given by the bilinear softening function defined according to 4 parameters, Petersson, see fig. 1:

$$f_{cr} = f_t \left[1 - (1 - \alpha) \frac{\xi}{\beta}\right] \quad \text{for} \quad \xi = w / w_o \leq \beta$$
$$f_{cr} = f_t \left[\alpha(1 - \xi)/(1 - \beta)\right] \quad \text{for} \quad \xi = w / w_o > \beta$$

that reduces to linear softening when the normalised kink point coordinates $\alpha = f_k/f_t$ and $\beta = w_k/w_o$ satisfy the relation $\alpha + \beta = 1$.

2.2 Structural model

A self-developed finite element (f.e.) program has been used, aiming to be able to couple the desired constitutive relation with the desired f.e. type, Goretti. The modified Ricks method, proposed by DeBorst, has been implemented in the solution strategy to capture possible softening and snap-back branches of the response together with element underintegration to account for shear locking problems. A plane stress state has been assumed and a total of 182 linear elastic bidimensional f.e. plus 41 nonlinear interface elements has been considered. The interface elements, located at midspan in correspondence to the crack surface, behave according to eqn. (1) and become active only if $f_t$ is attained. Finally, in view of the optimization procedure, the stiffness matrix has been condensed to limit the structural model to only 42 degrees of freedom.

3 Experimental tests and facilities

Conventional 3PB tests on notched specimens were carried out. The HSC specimens were prepared according to the quantities listed table 1. The castings were steam cured for 1 day and afterwards stored indoors for 24 months. A total amount of 20 specimens were prepared: 6 rectangular prisms (150x150x600mm)
to be tested through 3PB tests plus 4 cylindrical specimens (Φ160mm, height 320mm) and 10 cubic specimens (150x150x150mm) to have separate evaluation of the splitting and compressive strengths, see table 2.

The rectangular prisms were initially used to measure the Young modulus E and Poisson ratio ν, then one day before testing were sawn to provide an initial relative notch depth $\alpha_0 = 0.6$, 2mm wide.

**Table 1. Concrete type and composition**

<table>
<thead>
<tr>
<th>cement type</th>
<th>cement content (Kg/m$^3$)</th>
<th>crushed aggregates max dia. (mm)</th>
<th>w/c ratio</th>
<th>concrete density (Kg/m$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Portland</td>
<td>525</td>
<td>400</td>
<td>30</td>
<td>0.40</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>2335</td>
</tr>
</tbody>
</table>

**Table 2. Mechanical properties of tested concrete (avg. values)**

<table>
<thead>
<tr>
<th>compressive strength (MPa)</th>
<th>splitting strength (MPa)</th>
<th>Young modulus (MPa)</th>
<th>Poisson ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>98.52</td>
<td>4.14</td>
<td>44.5</td>
<td>0.15</td>
</tr>
</tbody>
</table>

The test rig used is shown in fig. 2 and it was selected so as to be able to carry out stable 3PB tests. The specimen, prepared to comply with the RILEM recommendations for the supports and loads arrangements, was mounted into a stiff steel frame (500 KN/mm) deputed to transfer the external load into applied displacement to the specimen. The load $P$ was measured using a load cell (250±0.05 KN base), whereas the vertical displacement at midspan $u$ and the crack mouth opening displacement $c_{mod}$ were measured using HBMW10 displacement transducers (10±0.007mm base) installed on both the lateral faces of the specimen to check the simmetry of the structural response. All the data were recorded using a HBM-UPM60 A/D converter connected to a PC.
4 Curve fitting procedures

Broadly speaking, two alternative procedures can be followed to determine the softening function parameters. The first is based on a selective hierarchical estimate and proceeds parameter by parameter, whereas the second involves a minimization technique aiming at a simultaneous identification of all the parameters. It is the purpose of the paper to compare the effectiveness of the two procedures when used in conjunction with the test case studied. In this case the total number of parameters to be determined is equal to 6 since the response is affected by both the elastic region contribution \((E, v)\) and the fracture process zone contribution \((f_l, W_o, \alpha, \beta)\). However, the number of free parameters has been reduced to 4 since the values for \(E\) and \(v\) have been fixed in advance according to the tests results, see table 2. The above number can be further reduced provided that some confidence level is placed on the validity of those empirical expressions relating the sought parameters, such as compressive and tensile strengths or compressive strength and fracture energy, e.g. CEB.5

4.1 Curve fitting via selective identification

In this case an attempt is made to separate the structural response, i.e. the load-displacement curve, in different regions primarily influenced by a restricted numbers of parameters, to find out simplified analytical expressions relating the material properties region by region. The problem is then solved if it is possible to write down a number of expressions at least equal to the number of the sought parameters. A clear example is represented by a method proposed in Planas et al.6 that is hereafter reconsidered. Since the elastic parameters and the tensile strength can be estimated by separate tests, see table 2, only 3 further measurable quantities, implying some relationship among the remaining material parameters, should be determined.

They are choosen as follows: the fracture energy \(G_f\), i.e. the area under the softening function; the first order moment of the softening function with respect to the stress axis \(w G_f\) and the initial slope \(H_1\) of the softening function, fig. 1:

\[
G_f = 0.5 f_l W_o (\alpha + \beta) \\
\bar{w} G_f = f_l W_o^2 (\alpha + \alpha \beta + \beta^2) / 6 \\
H_1 = f_l (\alpha - 1) / W_o \beta
\]

The above three left hand side quantities can be simply computed using the load-displacement curve \(P-u\) measured in a stable 3PB test. In particular, \(G_f\) is computed according to the RILEM recommendations4 from the area under the \(P-u\) curve once compensated of any dead load effect; \(\bar{w} G_f\) is estimated using the far end portion of the \(P-u\) curve together with the assumption of rigid body motion at collapse and \(H_1\) is estimated using the peak load \(P_{\text{max}}\) together with the assumption of small specimen size. The following equations holds:
\[ G_f = [\alpha(1 - \alpha_o)]^{-1} \int_u P(u) du + \text{dead load effects} \quad (5) \]

\[ \bar{w} G_f = 4K/tL \quad (6) \]

\[ H_1 = \left( E/2b \right) \left[ \left( P_{\text{max}}/bt\lambda_1 \right)^{-1/\alpha} - 1 \right] \quad (7) \]

where the coefficients \( B \) and \( m \) can be derived from the Bazant\(^7\) size law on the nominal stress: \( \sigma_N = P_{\text{max}}/bt = Bf_t(1+b/b_o)^m \) being \( b_o = EG_f/f_t^2 \) the characteristic length of the material, whereas the coefficient \( K \) is derived from the fitting \( Pu^2 = K \) of the far end portion of the compensated \( P-u \) curve. The approximations inherent in the above formulae, the use of material parameters evaluated from different tests and the propagation of errors when the formulae are used hierarchically are estimated in Valente et al.\(^8\). It is found that for \( G_f \) the error varies from 2\% to 8\% ranging from small to large size specimens; on the contrary the rigid body motion assumption for \( \bar{w} G_f \) can be directly checked by the comparison between the far end portion of the \( P-u \) and \( P-cmod \), but to avoid unacceptable errors it is vital to measure the tail of the \( P-u \) curve as far as possible annihilating the dead load effects; finally, the error in \( H_1 \) is found to be strongly dependent on the \( \alpha \) and \( \beta \) parameters and increases as the brittleness number \( \eta = b/b_o \) increases.

### 4.2 Curve fitting via minimization techniques

In this case the whole response is used and the material parameters are simultaneously and iteratively updated until an error function, measure of the distance between the experimental curve and the numerical one, is minimized, Ulfkjær & Brinker.\(^9\) This procedure has the advantage that no prior information on the material properties is required, but on the other hand shows highly time consuming since it requires a full finite element computation of the structural response at each iteration step. A further complication concerns uniqueness problems typical of inverse problems as the one considered, e.g. Dahlquist & Bjork\(^10\), therefore care must be taken in choosing the error function \( \Delta \), the weights \( \psi \) and possible constraints to the variables. Since the fracture energy plays the most significant role in the fracture process, \( \Delta \) has been selected in the form of an energy measure:

\[ \Delta = \psi_1 \int |P^m - P^c| d(u) + \psi_2 \int |P^m - P^c| d(c\text{mod}) \quad (8) \]

where the suffixes "\( m \)" and "\( c \)" stand respectively for measured and computed and the absolute values for the increments \( d(u) \) and \( d(c\text{mod}) \) have been introduced in the numerical integration in order to overcome uniqueness problems caused by snap-back branches in the load-displacement curves. Finally it is important to note that, being \( \Delta \) a global measure of the error, some spread in the estimate of the individual values of the material parameters can be observed, as shown in Valente et al.\(^8\), even if the whole curve is fitted within the required accuracy.
5 Results

The two procedures above have been used to estimate the softening function parameters from the experimental results. The outcomes are listed in table 3 and 4 respectively for the so called "selective" and "simultaneous" identification procedures, where the percent average deviation between the measured and computed response load $P$, evaluated from the $P-u$ curve, is also reported. In the "selective" procedure $G_f$, $\bar{w} G_f$ and $H_f$ have been computed using eqns. (2-4) and the material parameters have been identified from eqns. (5-7). In particular, cases A to C refer to a linear assumption for the softening function, therefore the three available eqns. (5-7) have been used circularly to determine the two parameters of the model. In its turn, case D refers to a bilinear assumption for the softening function and the three eqns. (5-7) are no longer sufficient to determine the four material parameters involved. Therefore the $f_l$ value has been fixed in advance as given from table 2. Finally, using the identified values the structural response has been computed and compared with the measured one, figs. 3-4, not only in term of the $P-u$ curve, from which the data have been derived, but also in term of the $P$-cmob curve.

Table 3 - "Selective curve fitting"

<table>
<thead>
<tr>
<th>case</th>
<th>soft. funct.</th>
<th>from $P-u$ curve</th>
<th>identified parameters</th>
<th>avg. dev. (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>lin.</td>
<td>$G_f$, $\bar{w} G_f$</td>
<td>$f_l$ (MPa) $w_0$ (mm) $\alpha$ $\beta$</td>
<td>8.0</td>
</tr>
<tr>
<td>B</td>
<td>lin.</td>
<td>$\bar{w} G_f$, $H_f$</td>
<td>2.82 0.079 - -</td>
<td>16.5</td>
</tr>
<tr>
<td>C</td>
<td>lin.</td>
<td>$H_f$, $G_f$</td>
<td>3.10 0.049 - -</td>
<td>7.6</td>
</tr>
<tr>
<td>D</td>
<td>bilin.</td>
<td>$G_f$, $\bar{w} G_f$, $H_f$</td>
<td>4.14 0.189 0.106 0.090</td>
<td>2.9</td>
</tr>
</tbody>
</table>

In the "simultaneous" procedure all of the four parameters have been identified using eqn. (8). In the E case the error function $\Delta$ is evaluated only from the $P-u$ curve, whereas in the F case both curves $P-u$ and $P$-cmob have been introduced in $\Delta$, see fig. 5.

Table 4 - "Simultaneous curve fitting" (bilinear softening)

<table>
<thead>
<tr>
<th>case</th>
<th>error function</th>
<th>identified parameters</th>
<th>avg. dev. (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>$P-u$</td>
<td>$f_l$ (MPa) $w_0$ (mm) $\alpha$ $\beta$</td>
<td>2.1</td>
</tr>
<tr>
<td>F</td>
<td>$P-u + P$-cmob</td>
<td>3.81 0.116 0.182 0.162</td>
<td>3.5</td>
</tr>
</tbody>
</table>

A large spread in the identified values is observable for cases A, B and C, that show a poor fit whatever the response considered, fig. 3. On the contrary, case D gives a good fit, fig. 4, but the identified parameter values do not compare strictly with the same values for the E and F cases, although the average percent deviation is quite similar. Moreover, the insertion of the $P$-cmob curve in the
error function, case F, leads to a significative ameliorament in the fit of the same curve, fig. 5b, to a slight worsening of the P-u curve-fit, fig. 5a, and to a nonneglectable change in the parameter values. Finally, it is worth noting that the estimated tensile strength, cases E, and F, is close to the measured one.

Figure 3: Results of the "selective" fitting - Linear softening assumption (a) curve P-u, (b) curve P-cmod

Figure 4: Results of the "selective" fitting - Bilinear softening assumption (a) curve P-u, (b) curve P-cmod

Figure 5: Results of the "simultaneous" fitting - Bilinear softening assumption (a) curve P-u, (b) curve P-cmod
6 Conclusions

The errors involved by two different approaches, commonly used for the indirect determination of the material parameters of the mode I cohesive crack model, have been discussed and applied to the results of conventional 3PB tests carried out on notched HSC specimens. The "selective curve fitting" procedure, based on a priori assumptions of the actual behaviour, leads to immediate results that are sufficiently accurate only if the bilinear softening assumption is made. On the other hand the "simultaneous curve fitting" procedure, based on minimization techniques, shows very time demanding if compared to the increase of precision involved. In either case the equilibrium path can be reproduced fairly well, nevertheless the identified parameter values can show undesirable deviations from the actual ones. It can be concluded that to improve the results accuracy, further studies are required to provide reliable bounds for the expected deviations.

7 References