

Processing superposed measurement data by regression algorithms

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Abstract

The structure of physical phenomena usually results from interrelated influences. Assuming a corresponding decomposition model is known, the problem of measurement data analysis will often be reduced to fitting the data by superposed single influence functions with respect to a fixed period of time or a common spatial region. But since in many cases the influences have unknown shifts of time and position, additional relations with respect to the domain have to be taken into account.

We present a modular system for fitting one or two-dimensional measurement data, using arbitrarily superposed model functions. The desired intrinsic shape parameters of the model functions and as the parameters of the transition of time and position are determined by special constraint minimisation procedures.

There are many applications of such regression models, but because of its non-convex basic structure the efficiency of the approach strongly depends on the availability of suitable start estimations. We present a fitting approach that is in a certain sense universal. However, a satisfactory solution can be found only if good conditions prevail for the application.

We give examples of time series processing and a particular case of surface measurement.

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1 Introduction

The characteristic information about observed physical phenomena is generally fairly well hidden behind crowds of raw measurement data. To extract this information we have to process the measurement data in such a way that the properties of interest of the phenomenon are explicitly represented. Correspondingly, two complementary kinds of processing procedures are applied in order to achieve this objective :

- a general reduction of effects induced from outside the phenomenon
- an estimation of specific parameters of the underlying model describing the phenomenon

Treating the data in either way or the other each has its own advantages. The first emphasizes the approximation aspect, whereas in the second the primary aspect is that of data interpretation. Both ways appear to be relevant for a high quality data representation and, if possible, both should be considered as a whole.

Nevertheless the following is dominated by the second way. The phenomena and objects our institute mainly has to deal with are chosen from industrial manufacturing and technological processes. These are usually effected by selective technical activities and the measurement data are collected in order to confirm that the aim has been achieved and how the effects observed correspond with the engineer's expectation. Our measurement data processing therefore normally is a recovery and prove of known structures.

Assuming a superposition model, we have to fit the data by single pattern functions with respect to a fixed period of time or a common spatial region. But in many cases it turns out that the corresponding parts have independent shifts of time and position, making it necessary for additional relations with respect to the scope to be taken into account.

Of course, this extension of the usual superposition model makes the problem of analysing measurement data more complex. But in some cases of application our optimisation-based approach delivers amazingly satisfactory solutions. This encourages us to design and investigate a modular system of data fitting procedures that operate with an application-sensitive basic pattern and that should be nearly as flexible as enhanced spline approximation systems. Let us introduce the main ideas in a summarised form.

2 Mathematical model formulation

Just for convenience we restrict our general discussion to the one-dimensional case of time series. The corresponding extension to a two (or more)-dimensional context seems to be self-evident.

The signal $Z(t)$ should ideally be composed of $i = 1, \dots, M$ components :

$$Z(t) = \sum_i f_i(t)$$

where $f_i(t) = f_i(t, p_i) \quad t \in [T_{iS}, T_{iE}]$

and $p_i = (p_{i1}, \dots, p_{in(i)})$ vector of intrinsic form parameters of the component i

$[T_{iS}, T_{iE}]$ scope of the component i

To ensure continuity and possibly also smoothness it may be necessary to introduce specific parameter restrictions like

$$T_{iE} = T_{i+1S} \quad ; \quad f_i(T_{iE}; p_i) = f_{i+1}(T_{i+1S}; p_{i+1})$$

for some concatenated components, and

$$f_i(T_{iS}; p_i) = f_i(T_{iE}; p_i) = 0$$

for some others.

The objective now is to tailor the superposition, i.e. both the form parameters and the scope of the single components, so that it satisfies the least-squares criterion subject to the given measurement data set $z_j \quad (j = 1, \dots, N)$

$$\sum_j [z_j - Z(t_j)]^2 = \sum_j [z_j - \sum_i f_i(t_j; p_i)]^2 \rightarrow \min$$

Our main intention is not the general substitution of formal approximation models like splines. We will only prepare some additional macro-like patterns corresponding to the specific interpretation model and include them into the general approximation process wherever it seems to be expedient. The new aspect indeed is the variable extent of the time of such parts. It will therefore be possible that the uncertain beginning or the moment of change from one time phase of the process to another becomes an ordinary variable of optimisation.

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Of course, this complex approach requires much more numerical and computational effort than a standard spline approximation procedure. Because of the non-orthogonality of the basic functions f_i , the character of the mathematical fitting model changes from approximation to non-linear regression. The structure of the underlying optimisation problem is generally a non-convex one and therefore we have to deal with local solutions rather often. The pros and cons of such an approach should be weighed very carefully and justified only by the specific needs of a single case of application.

Our test implementations are based on the IMSL Fortran library routines [1] running on HP 9000. The regression problem is solved by a modified Levenberg - Marquardt method. The optimisation uses auxiliary intercepts and straffunctionals to control the model-induced restrictions where necessary. To summarize it not quite seriously, it works well if it works like the final machining of previously well-chosen blanks.

3 Example 1 : Analysing high voltage impulses

The area of application of the first example is chosen from high voltage engineering [2]. It is selected to illustrate the fitting principle described above and does not serve for a discussion of the referenced hv model itself, therefore the physical background should be explained only briefly:

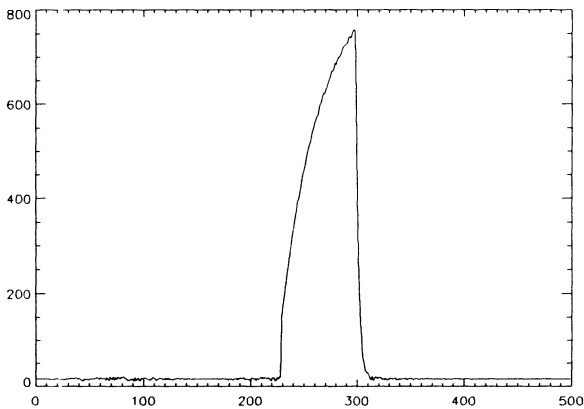


Figure 1: Chopped lightning impulse response

The recorded measurement data reflect the behaviour of hv-transformers released by lightning impulses. The impulse triggers diverse reactions of the transformer, especially the activation of protection facilities, so that the response is divided into single periods of time due to well-known reasons. The slightly simplified response signal presented in Fig. 1 consists of an ascending and a descending exponential part, embedded in a nearly constant surrounding. We develop the formal signal description rather schematically with offsets d_i included for each component.

$$u(t) = \sum_i u_i(t) + d_i$$

where

$$u_1(t) = u_0, \quad d_1 := 0$$

$$u_2(t) = u_1 \exp(-(t - T_s) / \tau_1) - u_2 \exp(-(t - T_s) / \tau_2) \quad t \in [T_s, T_c]$$

$$u_3(t) = u_3 \exp(-(t - T_c) / \tau_3) \quad t \geq T_c$$

Ensuring a continuous concatenation here we get :

$$u_0 = u_2(T_s) + d_2 \Rightarrow d_2 := u_0 - u_2(T_s) \quad \text{and likewise}$$

$$u_2(T_c) + d_2 = u_3(T_c) + d_3 \Rightarrow d_3 := u_2(T_c) - u_3(T_c) + d_2$$

It is a simple task for the skilled hv engineer to estimate very precisely the initial parameters for entering the regression procedure. In particular, the variable moments of start T_s and breakdown T_c are directly available from the measurement curve. It only remains to implement a subroutine that returns the residual *res* of the regression function at the given parameter vector with respect to a single measurement value $z_j(t_j)$. This task turns out to be very simple here:

```
d1 := 0 ;
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d2 := u1(Ts) - u2(Ts) + d1 ;
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d3 := u2(Tc) - u3(Tc) + d2 ;
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if tj < Ts      then res := zj - u1(tj) - d1 ;
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if Ts ≤ tj ≤ Tc then res := zj - u2(tj) - d2 ;
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if tj > Tc      then res := zj - u3(tj) - d3 ;
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.
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With reference to more complex examples, it may be difficult to reduce the interdependence of intrinsic and auxiliary parameters, which is a reasonable condition for optimisation.

The residuals with respect to the discrete time t_j of observation give an instructive impression if the correspondence between signal and regression function is a reasonable one or if further components like oscillations, for instance, should be added to the model. The data are ideally fitted if the residuals represent a white noise process.

Depending on a sufficient number of observations (high sampling rate) the regression procedure is proof against existing noise. Speaking generally, the pre-processing of data (filtering) is useful to alleviate the problem of locality but it may cause a loss of information. Tests in which the Euclidean distance serves as the metric of approximation instead of the residuals (a rather informal concession to how the human eye is used to fit) do not produce seriously different results but take considerably longer processing times.

4 Example 2 : Analysing surfaces

The final section is devoted to special problems of industrial measurement techniques. The manufacture and testing of micro-geometrical surfaces plays a significant role in electronics, for instance.

We have successfully adopted our regression principle in micro-mechanics to analyse measurement data of the surface of roughness specimens [3]. As such surfaces have a characteristic wave structure, we used the following formal regression functions to fit them.

$$f_k(x, y) = A_k \cdot \cos(u_k x + v_k y) + B_k \cdot \sin(u_k x + v_k y)$$
$$u_k, v_k \in R, \quad k = 1, \dots, M$$

The total number of regression functions M is a priori unknown here. A reasonable value of M depends on the progress of reducing the residuals with respect to the measurements $(x_i, y_j; z_{ij})$ taken from an equidistant grid.

$$\sum_{ij} [z_{ij} - \sum_k f_k(x_i, y_j)]^2 \rightarrow \min \quad k = 1, \dots, M, \quad M \rightarrow$$

This approach seems to be very similar to the Fourier transformation. Indeed, the FFT substituted for our regression approach could avoid much expense if we were able to determine a reasonable fundamental interval. But it seems to be impossible in most cases that the expected regression functions represent a harmonic system of waves with respect to any certain interval.

The immense advantage of the regression approach is the effect of data compression compared with standard approximation procedures. Only a small number of representative parameters are necessary to reproduce the surface sufficiently well. Even in cases where the structure of the surface is less distinct the data compression effect is considerably related to results obtained by FFT, for instance.

Perhaps the advised local adaptation of the waves may be missed in the example. Indeed, the model contains only a variable phase dependency which could be made explicit by a simple trigonometric transformation, but begin/end interrelations are irrelevant in this specific case of application. However, we expect an interesting area for testing our approach in micro-surface treatment and manufacturing, where analytically described profiles of thin layers locally cut or bounded by simple contours play an outstanding role. It is just here that we see the advantages of the above regression approach, but as yet this has no solid basis in experimental experience.

References

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2. Schon, K. & Schulte, R. Intercomparison of high voltage impulse measurements , *BCR Report EUR 13682 EN* , Brussels , 1991 .
3. Moeck, H., Richter, D. & Walzel, M. Reference algorithms for the approximation of shape measurement data using non-orthogonal periodic functions , *Measurement* , 1994 , **13** , 59 - 65 .