Comparison of predictive and experimental data for combined convection in horizontal duct flow

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ABSTRACT

The buoyancy effects on laminar flow of water in a uniformly heated circular duct are investigated experimentally over the range $9 \times 10^4 - 5 \times 10^6$ of the modified Rayleigh Number, $Ra_q$. Preliminary results of a full numerical simulation of the experiment are presented, using the finite-volume FLOW3D code.

INTRODUCTION

For many internal flow situations the effect of buoyancy is to cause significant modifications of the internal flow field and heat transfer rate. In horizontal duct flows, the buoyancy forces act orthogonally to the main flow direction, giving rise to secondary circulations over the duct section. The resulting flow and temperature fields are very complex and three-dimensional in nature. The most prominent effect of buoyancy is a substantial increase in the heat transfer rates, in respect to the case of pure forced convection.

The present study focuses on combined forced and free convection under developing, and fully developed laminar flow conditions in a horizontal cylindrical duct under the boundary condition of uniform heating at the pipe wall, condition $H$. In circular ducts the secondary motion is a pair of horizontal helical circulations. With condition $H$ these continue to be sustained by the heat supply all along the heat transfer length. This is contrary to the case of uniform wall temperature, condition $T$, where buoyancy effects subside in the fully developed region. This specific feature of the uniform heating case is attractive when investigating combined convection flows. Indeed, most of the present knowledge on the nature and the effects of buoyancy in ducts derives from experimental and theoretical studies.
on this case. An excellent review of experimental and theoretical work on combined convection in round tubes was given by Bergles [1,2].

Due to space limitations only the literature strictly related to this work will be mentioned, while outlining the present state of the art.

In the thermal inlet region, the expected trend of the circumferentially averaged Nusselt number, Nu₂, with the non-dimensional axial distance, z*, is as follows: up to a certain axial distance, Nu₂ decreases as in a forced convection flow. In correspondence with the onset of significant buoyancy, Nu₂ detaches from the constant property prediction, then approaches a fully developed value, Nu₉fd. This can be several times higher than the asymptotic constant property value, 4.364. The dominant role of the Rayleigh number, Ra, has been firmly established: increasing Ra, produces more intense secondary circulations, and higher Nu₉fd-values, while the onset of buoyancy and the thermal development are anticipated. The Prandtl number, Pr, also affects the buoyancy-induced flow independently; Pr has a relatively minor influence on the heat transfer rate, but a dominant role in determining the friction factor; this increases substantially for reducing Pr [3].

Newell and Bergles [4] were the first to point out the importance of the duct wall conductivity. Two limiting conditions were recognized: infinite wall conductivity, case H1, where the interfacial wall temperature is circumferentially uniform; and zero wall conductivity, case H2, where the interfacial wall heat flux density is circumferentially uniform. In the latter case, the fluid temperature over the duct cross-section is subject to a less intense vertical stratification, and secondary circulation. Consistently, lower Nu-values are obtained. Experimental data, which are obtained with ducts of different materials, are expected to give results intermediate between the above limiting conditions. Most of the experimental data for fully developed combined convection have been successfully correlated by Morcos and Bergles [5], by introducing an appropriate wall parameter, Pₗw, and considering the independent effect of Pr. A recent numerical investigation by Chen and Hwang [6] offered an alternative correlation for the effect of wall conduction, based on the same parameters. It is very difficult to account for the effects of axial wall conduction in a combined convection circumstance. However, there is some need for appreciating those effects, when highly conductive materials are used for the tube wall. In fact, Barozzi and Pagliarini [7, 8] demonstrated that axial wall conduction can have a definite effect on heat transfer results in forced convection.

The asymptotic region was investigated theoretically, either using analytical [9-13] or numerical techniques [4, 14-16]. The developing region problem was approached numerically, either using marching procedures [6, 17-20], or a fully elliptic 3D approach [21, 22]. In these solutions, the occurrence of a fully developed condition was always predicted. This agrees with several experimental findings such as the Petukhov and Polyakov [23] results for water. However, it is a
frequent occurrence in experiments that \( \text{Nu}_z \) progressively increases in the downstream direction, without assuming a distinguishable asymptotic trend. Such a behaviour is clearly demonstrated by the data of Hussain and McComas [24] for air, and Shannon and Depew [25, 26], for water and ethylene-glycol. Also a low-Re run in the Barozzi et al. [27] experiment showed a similar increasing trend of \( \text{Nu}_z \) with \( z^* \). Under these circumstances, \( \text{Nu}_z \) finishes well above the fully developed prediction for the same \( \text{Ra-Pr} \) values. This behaviour is not convincingly explained. Possible causes could be: experimental errors of a systematic nature, the onset of instabilities and transition to turbulent flow, the occurrence of backflow, and the effect of temperature-dependent properties.

The influence of viscosity variation with temperature was investigated by Hong and Bergles [17, 34] and by Shannon and Depew [26] for fully developed, and developing buoyancy affected flows, respectively. Martin and Fargie [29] considered the effect of temperature-dependent viscosity for oils; Allen et al. [30] showed the effects of temperature-dependent properties in full. These latter investigations, however, apply only in the initial part of the thermal entrance length, up to the onset of secondary circulations.

The flow stability of horizontal flows under heating conditions was investigated experimentally by El-Havary [31]. Theoretical stability analyses on this specific case have not been reported. However, Nandakumar et al. [16] predicted double (bifurcated) solutions in the fully developed region for \( \text{Ra}_q \geq 1.24 \times 10^6 \), and \( \text{Pr} = 5 \). This could actually be an indication of the possible onset of instabilities in the secondary flow, at relatively low \( \text{Ra} \)-values. In this connection, the numerical results of Mahaney et al. [32] and Nonino and Del Giudice [33] for a rectangular duct are also worthy of mention: for some of the heating conditions considered, \( \text{Nu}_z \) was found to assume a spatially oscillating behaviour, and no asymptotic trend could be observed above some critical \( \text{Ra} \)-value.

The literature survey then reveals that several questions still remain open, even for the well studied case of laminar combined convection in uniformly heated horizontal ducts. This gave motivation to the present study, which is a part of an extensive experimental and numerical investigation intended to explore the laminar and transitional regimes, as well as the effects of inclination. To this aim it was decided to accompany the collection of fresh experimental data with a thorough numerical simulation of the test section, including the convective part, and the wall effects. Research activities being in progress, only a limited amount of preliminary results are available at the time of writing, and are presented here.
EXPERIMENTATION

A schematic diagram of the experimental setup is shown in Figure 1. The heat transfer test section was a horizontal copper pipe ($k_w = 334.3 \text{ W/mK}$), 16 mm i.d., and 18 mm o.d. The length of the test section is 2.80 m, providing a maximum length to diameter ratio ($L_{ht}/D$) of 175. The heat transfer length was preceded by a 1.36 m long hydrodynamic development section, made with acrylic plastic material. This length is expected to provide a fully developed parabolic profile at the inlet of the heat transfer section, for Re up to 1500. A constant heat flux boundary condition was maintained all along the heated section. Heating was provided through six constantan ($\Theta 0.36 \text{ mm}$) wires, spirally wound on the outer surface of the tube, with an axial pitch of 5 mm, and AC powered in parallel. The test section was thermally insulated with a 40 mm layer of cellular rubber. Thin constantan wires ($\Theta 0.15 \text{ mm}$) were directly welded on the outer surface of the tube wall, to produce 22 thermocouples. These were used to measure the wall temperature, $T_w$, at ten axial positions, as indicated in Figure 2. Note that the downstream sections were provided with at least two thermocouples, placed at the upper ($\theta=0$) and lower ($\theta=\pi$) positions; some stations were provided with additional thermocouples at $\theta= \pm \pi/2$. The temperature of the fluid was measured by K-type thermocouples, at the duct inlet and in a mixing chamber at the end of the heated length. All thermocouple signals were measured with a high sensitivity HP3458A voltmeter and automatically collected and processed, with the method described in [42]. The overall accuracy of temperature measurements was estimated to be better than 0.1 °C. Flow rate measurements were performed by a stop-watch and a calibrated bottle. The electric power
supply, was determined by measuring the current and the voltage drop through the heating wires. Distilled water was used as the working fluid.

\[ \text{Nu}_z = \frac{q_w}{T_{w,av} - T_b} \frac{D}{k} \]

where, \( q_w \) is the local heat flux density, as locally corrected for radial heat losses through the insulation; \( T_{w,av} \) is the average wall temperature at the axial location \( z \). The local bulk fluid temperature, \( T_b \), was obtained from the enthalpy balance, as:

\[ T_b = T_o + \int_0^z \frac{4q_w}{\rho c_p w_m D} d\zeta \]

As for the averaging of the wall temperature, the simple arithmetic mean of the temperature readings was used. This is justified for all runs where the maximum top to bottom temperature difference was less than 0.2 °C. However in the highest Ra-run (\( Ra_q=5\times10^6 \)) this temperature difference increased up to 2 °C. This indicates the need for modelling conduction in the wall in future data processing.

A provisional estimate of the measuring errors led to an overall accuracy of ± 15% on \( \text{Nu}_z \). However, more accurate error analysis and data reduction procedures are under development, and will be used in the continuation of the research.

Three runs were carried out at \( Re \sim 1000 \), and two in the low-Re range (\( Re \sim 150 \)). \( \text{Nu}_z \) distributions are presented in Figure 3.
Figure 3. Local mean Nusselt numbers vs. non-dimensional distance

Experimental data:

a. \( \text{Re} = 904 \div 1244; \ Ra_q = 1.04 \times 10^5 \div 8.45 \times 10^6; \ \Pr = 7.1 \div 5.4; \)
b. \( \text{Re} = 151 \div 189; \ Ra_q = 8.90 \times 10^4 \div 1.50 \times 10^6; \ \Pr = 7.1 \div 5.5. \)
It is seen that in the initial part of the duct, $Nu_z$ is in very good agreement with Shah and London's [28] analytical predictions for constant property fluids. As expected, an earlier detachment from the reference curve occurs for increasing $Ra_q$. However, a clear asymptotic trend is only achieved for two of the runs in Figure 3a, while the two low-Re runs in Figure 3b and the high $Ra_q$ run in Figure 3a, show a monotonically increasing trend in the downstream region. When compared with the correlations for fully developed flow conditions, these latter results are clearly underpredicted. Such a behaviour was not totally unexpected, in view of the comments given in the introduction. Data are clearly insufficient for attempting any further interpretation of those effects.

**NUMERICAL ANALYSIS**

Numerical methods which have been used in the context of combined convection in duct flows, were reviewed by Barozzi and Collins [35]. The main conclusion of that survey was that accurate predictions of combined convection duct flows could only be achieved by treating the inherently three-dimensional, and elliptic nature of the problem. The adoption of the computer program FLOW3D Release 3, from Harwell Laboratory, U.K., then followed as a direct and natural consequence. The program allows the treatment of many individual aspects that a specific CFD problem may present, such as variable fluid properties, turbulent regimes, and conduction in solid regions. FLOW3D has been widely used in the prediction of very complex thermal-fluid circumstances (see e.g. [36,37]); however, it has not been applied previously to the case of present interest.

The HARWELL-FLOW3D algorithm is based on a Finite Volume Method - FVM - discretization of the governing p.d.e.s', and is particularly suitable for computing fluid flows in three dimensional geometries [38]. The method uses a general non-orthogonal boundary-fitted coordinate system, and, in contrast with most CFD codes, makes use of co-located, i.e. non-staggered, grids, for either the velocity components, or pressure and other scalar variables. The code has a poly-algorithmic structure, in that different solution algorithms can be specified such as SIMPLE [39], and SIMPLE-derived schemes. Various alternative linear algebra solvers are also available.

The final use of all the computational work is the direct numerical simulation of the experiment, including wall conduction effects. For the time being, however, only preliminary data are available, referring to either thermally developing convection with constant properties, or developing combined convection under uniform heating at the wall. These case-sudies are part of the code-validation process, presently on course.

Due to the symmetry of the problem towards the vertical midplane, only one half of the duct cross-section was considered. The grid was
obtained by patching a uniform orthogonal grid with an external annular grid (O-grid), as shown in Figure 4.

Figure 4. Multi-block and control-volume arrangement over the duct cross-section

This multi-block arrangement was preferred to the more conventional cylindrical grid, since it allows the grid to be easily fitted to the specific features of the secondary flow, while removing the singularity at the duct axis. The present results were obtained using 50 square volumes in the central block, Block 1 in the figure, and 10 rings in the O-grid, for a total of 192 elements. As an initial guess, 50 variable-size steps were used in the axial direction, to cover a length of 300 tube diameters. Overall, 9600 control volumes were used. This was considered sufficient for obtaining reasonable accuracy with affordable computational effort, in the starting phase of the simulation.

For the sake of brevity, the governing equations are not reported. The differential forms of the continuity, momentum, and energy equations are given in [40]. All fluid properties were assumed constant, except density in the buoyancy term of the momentum equations. In this case, a linear density-temperature dependence was used, according to the Boussinesq approximation.

Boundary conditions for the preliminary assessment were: i. uniform temperature, and fully developed (Poiseuille) flow at the entrance; ii. non-slip condition, and uniform heat flux density at the wall; iii. zero and constant axial derivatives for the velocity components, and
temperature, respectively, at the end section; iv. symmetry conditions at the vertical mid-plane.

Convective terms were discretized according to the hybrid scheme [39]; central differences were used for diffusive terms. The SIMPLEC method of Van Doormal and Raithby [41] was enforced to attach the velocity-pressure coupling. Under-relaxation is usually necessary to stabilize the convergence of iterative processes. Here, the relaxation factor was fixed at 0.65 for the velocity components. Pressure and temperature were not under-relaxed.

The discretized momentum and energy equations were solved with the Stone's method. The Incomplete Cholewsky Conjugate Gradient (ICCG) method was employed in the solution of the Poisson-like pressure-correction equation.

To estimate the adequacy of the computational grid, and the performance of the method, the program was run with input data corresponding to one of the experimental cases (Re = 951; Pr = 7.09; Ra_q = 8.83x10^5). For either forced and combined convection, a rather peaceful convergent behaviour was observed; however, about 2000 outer cycles were necessary to obtain a convergence level of 3x10^-4 on the total mass residual. The corresponding computer times were 2.2x10^4, and 1.2x10^3, with SC IRIS Indigo WS, and CRAY YMP/432, respectively. Actions are on course to accelerate convergence, such

![Graph](image)

Figure 5. Local mean Nusselt numbers vs. non-dimensional distance
Numerical results, and comparison with experimental data
as the optimization of relaxation factor, the use of alternative
schemes and solvers, and the application of multigrid techniques.

Preliminary results are presented in Figure 5. It can be noted that
forced convection results compare rather well with the reference
solution from Shah and London [28]. In terms of Nu_z, the deviation
from the reference values is inside 3% for z* > 1x10^3. However, the
numerical solution becomes inaccurate in the proximity of the inlet
section and in the asymptotic region. In both cases, the use of smaller
axial steps is expected to cure the effect. Results for the combined
convection case are also presented in Figure 5. Direct comparison
with experimental data is clearly inappropriate, since the numerical
results refer to boundary condition H2, while the experiment
approximates the infinite conductivity case, H1. Even so, the onset of
buoyancy is well predicted, and the agreement remains quite
satisfactory up to z* = 0.01. Further downstream, the experimental
data are underpredicted; this is consistent with the expected effect of
wall conduction.

Overall, the results are encouraging, in that they demonstrate the
feasibility of the proposed program, based on the cross-validation of
experimental data, and numerical predictions.

CONCLUDING REMARKS

In this phase, it is shown that the prediction of mixed convection
flows is a difficult task. Despite this, there is a great practical need for
it. This consideration is at the origin of the great amount of research
effort which has been performed through the world, aimed to obtain
reliable theoretical predictions of these thermo-fluid problems. It may
also be pointed out that no real progress can be achieved on the
predictive (numerical) side, without concurrent experimental
validation and experience-led direction. So, in this research area, the
development of new numerical tools does not tend to offset, but rather
to stress the importance of experimentation.

NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Units</th>
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<tbody>
<tr>
<td>c_p</td>
<td>Specific heat at constant pressure</td>
<td>(J/kgK)</td>
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<tr>
<td>D</td>
<td>Duct diameter</td>
<td>(m)</td>
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<tr>
<td>g</td>
<td>Gravitational acceleration</td>
<td>(m/s^2)</td>
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<tr>
<td>h_z</td>
<td>Mean heat transfer coefficient</td>
<td>(W/Km^2)</td>
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<tr>
<td>k</td>
<td>Fluid thermal conductivity</td>
<td>(W/mK)</td>
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<tr>
<td>k_w</td>
<td>Wall thermal conductivity</td>
<td>(W/mK)</td>
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<tr>
<td>L_{ht}</td>
<td>Heat transfer length</td>
<td>(m)</td>
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<tr>
<td>Nu_z</td>
<td>Mean Nusselt number (at z)</td>
<td></td>
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<tr>
<td>Pr</td>
<td>Prandtl number</td>
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<tr>
<td>q_w</td>
<td>Mean heat flux density (at z)</td>
<td>(W/m^2)</td>
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<tr>
<td>Ra_q</td>
<td>Modified Rayleigh number</td>
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<tr>
<td>Re</td>
<td>Reynolds number</td>
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</table>
T<sub>b</sub> Bulk temperature of the fluid (at z) (K)
T<sub>o</sub> Inlet temperature of the fluid (K)
T<sub>w,av</sub> Mean wall temperature (at z) (K)
w<sub>fd</sub> Fully developed axial velocity (m/s)
w<sub>m</sub> Mean axial velocity (m/s)
x Horizontal coordinate (m)
y Vertical coordinate (parallel to g) (m)
z Axial coordinate (m)

z* = z/(D Re Pr) Non-dimensional axial coordinate

α = k/CP ρ Fluid thermal diffusivity (m<sup>2</sup>/s)
β Thermal expansion coefficient (K<sup>-1</sup>)
θ Circumferential coordinate (rad)
μ Dynamic fluid viscosity (kg/ms)
ν Kinematic fluid viscosity (m<sup>2</sup>/s)
ρ Fluid density (kg/m<sup>3</sup>)
o (suffix) calculated at T<sub>o</sub>

REFERENCES


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