Finite element and experimental vibration analysis of viscoelastic composite structures

E. Barkanov¹, A. Chate¹, E. Skukis¹, O. Täger² & H. Kolsters³
¹Institute of Materials and Structures, Riga Technical University, Latvia
²Institute of Lightweight Structures and Polymer Technology, Technical University of Dresden, Germany
³Department of Aeronautics, Royal Institute of Technology, Sweden

Abstract

Finite element and experimental vibration analyses of viscoelastic composite structures are performed. The present implementation gives the possibility to preserve the frequency dependence for the storage and loss moduli of viscoelastic materials and to use the same hypothesis - complex modulus model in the free vibration, frequency and transient response analyses. Dynamic characteristics of viscoelastic composite structures are evaluated by the energy method, the method of complex eigenvalues, from the resonant peaks of the frequency response function and using the steady state vibrations. To verify numerical results, the dynamic characteristics of different sandwich and laminated composite panels have been determined by the pulse and non-contact laser techniques.

Keywords: composite structures, viscoelastic material, complex modulus model, frequency dependence, free vibration, frequency and transient responses.

1 Introduction

Composites are widely used now in different structural applications requiring high stiffness-to-weight and strength-to-weight ratios, and high damping properties. Among these applications considerable part is aeronautical, ship and automobile structures and their components made from sandwich and laminated composites with viscoelastic layers. In this case it is necessary to develop effective and convenient engineering tools to model and analyse the vibration behaviour of viscoelastic structures.
Previously, the complex modulus model was widely used to describe the rheological behaviour of viscoelastic materials Alam et al. [2]. But engineering practice shows that the storage and loss moduli of real viscoelastic materials are strongly dependent from the frequency and temperature, fig. 1. Due to increasing application of high damped and lightweight structures, some progress has been recently achieved in the vibration analysis of composite structures with the frequency-dependent viscoelastic damping Zapfe et al. [3].

The objective of the present study is development of methods and algorithms for vibration analysis of viscoelastic composite structures using the same hypothesis - complex modulus module for the frequency and time domains, and preserving the frequency dependence for the material storage and loss moduli.

2 Finite element vibration analysis

The forced vibration equation of a structure made from the frequency-dependent viscoelastic material described by the complex modulus model Nashif et al. [1] appears as follows in a matrix form:

$$\mathbf{M}\ddot{\mathbf{X}}^* + \mathbf{K}^*(\omega)\mathbf{X}^* = \mathbf{F}(t)$$

(1)

where $\mathbf{M}$ is the mass matrix of a structure, $\mathbf{K}^*(\omega) = \mathbf{K}(\omega) + i\mathbf{K}'(\omega)$ is the complex stiffness matrix of a structure, $\mathbf{X}^*$ and $\dot{\mathbf{X}}^*$ are the complex vectors of displacements and accelerations, and $\mathbf{F}(t)$ is the load vector. The matrix $\mathbf{K}(\omega)$ is determined using the storage moduli $E(\omega)$ and $G(\omega)$, while $\mathbf{K}'(\omega)$ is found using the imaginary parts of the complex moduli $E''(\omega) = \eta_E(\omega)E(\omega)$ and $G''(\omega) = \eta_G(\omega)G(\omega)$, where $\eta_E(\omega)$ and $\eta_G(\omega)$ are the material loss factors and $\omega$ is a frequency. In the present paper three methods are used in order to calculate the dynamic characteristics of a structure: the method of complex eigenvalues, the energy method and the method evaluating the dynamic parameters from the frequency response. Additionally the dynamic properties of
structures under loads applied are estimated by the transient response analysis using the steady state vibrations.

2.1 Method of complex eigenvalues

Damped eigenfrequencies and corresponding loss factors in this case are determined from the free vibration analysis of a structure

\[ [\mathbf{K}'(\omega) - \omega'^2\mathbf{M}]\mathbf{X}' = 0 \]  \hspace{1cm} (2)

where \( \omega' = \omega + i\omega'' \) is the complex eigenfrequency. The real part \( \omega \) represents the damped eigenfrequency of a structure and the imaginary part \( \omega'' \) specifies the rate of decay of the dynamic process.

Eqn (2) can be written as the non-linear generalised eigenvalue problem

\[ \mathbf{K}'(\omega)\mathbf{X}' = \lambda\mathbf{M}\mathbf{X}' \]  \hspace{1cm} (3)

where \( \lambda' = \omega'^2 \) is the complex eigenvalue and \( \mathbf{X}' \) is the complex eigenvector. Solution of eqn (3) starts with a constant frequency (const = \( \omega \)). Then at each step the linear generalised eigenvalue problem with \( \mathbf{K}'(\omega) = \text{const} \) is solved by the Lanczos method Barkanov [4], which is programmed in a truncated version, where the generalised eigenvalue problem is transformed into a standard eigenvalue problem with a reduced order symmetric tridiagonal matrix. Orthogonal projection operations are employed with greater economy and elegance using elementary reflection matrices. The iteration process terminates, when the following condition is satisfied

\[ \frac{|\omega_{i+1} - \omega_i|}{\omega_i} \times 100\% \leq \xi \]  \hspace{1cm} (4)

where \( \xi \) is a desired precision and \( \omega_{i} \) is the real part of eigenfrequency of a structure calculated from the linear generalised eigenvalue problem with the storage and loss moduli for the frequency \( \omega_i \), which was obtained from the same equation in the previous step. The following relation determines the modal loss factors of a structure for each vibration mode

\[ \eta_n = \frac{\lambda''}{\lambda_n} \]  \hspace{1cm} (5)

This approach gives the possibility to preserve the frequency dependence of viscoelastic materials and to calculate structures with high damping.
2.2 Energy method

In the energy method it is assumed, that for structures with slight damping, the dynamic characteristics can be calculated by the equation of natural vibrations of the corresponding undamped structure. In this case the eigenvalues \( \lambda = \omega^2 \) and corresponding eigenvectors \( \mathbf{X} \) for an elastic structure are determined from the non-linear generalised eigenvalue problem with real matrices

\[
\mathbf{K}(\omega)\mathbf{X} = \lambda \mathbf{M}\mathbf{X}
\]  

Eqn (6) is solved like in the previous method, but at each step the subspace iteration algorithm Bathe and Wilson [5] is used. Then the following relation calculates the modal loss factors

\[
\eta_n = \frac{\mathbf{X}_n^T \mathbf{K} \mathbf{X}_n}{\mathbf{X}_n^T \mathbf{K} \mathbf{X}_n}
\]  

It should be noted that this method gives the possibility to preserve the frequency dependence only for the storage moduli and can be used only in the case of structures with slight damping.

2.3 Frequency response analysis

In the case of harmonic vibrations \( \mathbf{F}(t) = \mathbf{F} e^{i\omega t} \) a solution of eqn (1) is found in the form \( \mathbf{X}^*(t) = \mathbf{X}^* e^{i\omega t} \) and the system of complex linear equations is obtained

\[
[\mathbf{K}^*(\omega) - \omega^2 \mathbf{M}]\mathbf{X}^* = \mathbf{F}
\]  

where \( \omega \) is the frequency for which the response of a structure is calculated, \( \mathbf{X}^* \) is the complex amplitude of displacements and \( \mathbf{F} \) is the amplitude of applied force. Eqn (8) is solved by the Gauss algorithm Bathe and Wilson [5] for each frequency. Dynamic characteristics of a structure (eigenfrequencies and corresponding loss factors) can be easily obtained from the frequency response, fig. 2. The eigenfrequencies \( f_n = \frac{\omega_n}{2\pi} \) of a structure present the points of the real part of the response spectrum, where the amplitude of the displacements is zero, but the corresponding loss factors can be determined by analysing the resonant peaks at frequencies \( f_a \) and \( f_b \) for a particular mode as follows

\[
\eta_n = \frac{1 - (f_b/f_a)^2}{1 + (f_b/f_a)^2}
\]
This method takes a considerable computing time, since the dynamic stiffness matrix $[K'\omega - \omega^2 M]$ must be recalculated, decomposed and stored at each frequency step. On this reason for structures modelled by a great number of degrees of freedom and in the case of a great number of desired dynamic characteristics to be calculated, it is more efficient to use the results of the free vibration analysis. The frequency response analysis may be successfully applied in the case, when it is necessary to determine a small number of desired dynamic characteristics, or when the eigenfrequency of the undamped structure is already known and only it recalculation and determination of the corresponding loss factor for the damped structure is necessary. This approach also gives the possibility to preserve the frequency dependence of viscoelastic materials and to calculate structures with high damping.

![Figure 2: Frequency response.](image)

### 2.4 Transient response analysis

The transient response of a structure made from frequency dependent viscoelastic material cannot be obtained effectively applying direct integration methods or modal superposition method, because in this case it is not possible to determine a variation of the material properties $E'(\omega)$ and $G'(\omega)$ with respect to time. Time domain behaviour of a structure may be obtained from the frequency domain response by the Fourier transform technique.

The method proposed is based on the assumption that any complex input signal can be interpolated by trigonometric polynomials. It is more convenient for this purpose to use the Fourier transform to find the frequency spectra of excitation

$$F'(\omega_j) = F[F(t_k)]$$

where $t_k$ is a set of discrete times for the excitation $F(t)$ and for the response $X'(t)$, $\omega_j$ is a set of discrete frequencies for the frequency spectra of excitation $F'(\omega)$ and for the frequency response $X'(\omega)$. The response of a structure for
each trigonometric component is calculated exactly using the matrix of transfer functions. Incidentally, it is necessary to solve the following system of complex linear equations:

\[
\left[ K^*(\omega_j) - \omega_j^2 M \right] X^*(\omega_j) = F^*(\omega_j)
\]

Displacements of a structure in the time domain can be obtained by the inverse Fourier transform

\[
X^*(t_k) = F^{-1}\left[ X^*(\omega_j) \right]
\]

Numerical realisation of the Fourier transform is performed by the routine [6] using a variant of the fast Fourier transform algorithm known as the Stockham self-sorting algorithm, which takes advantage of the cyclic repetition of the complex exponentials in the discrete Fourier transform and drastically reduces the number of calculations required. Before using the program, several calibration functions Brigham [7], having theoretically exact Fourier transforms, were used to develop a confidence in the results. To produce equivalence between the continuous and discrete transforms and working in the real-world time and frequency domains, \(0 < t, \omega < \infty\), the scale factors have been changed. Then the discrete Fourier transform pair for a separate sequence can be written in the following form:

\[
F^*(\omega_j) = F\left[ F(t_k) \right] = a_1 \sum_{k=0}^{N-1} F^*(t_k) e^{-\frac{2\pi i k}{N}}
\]

\[
X^*(t_k) = F^{-1}\left[ X^*(\omega_j) \right] = a_2 \sum_{j=0}^{\infty} X^*(\omega_j) e^{\frac{2\pi i j}{N}}
\]

where \(N\) is a number of samples, \(a_1\) and \(a_2\) are the scale factors and in the general case of loading: \(a_1 = 2\sqrt{N}\Delta t\), \(a_2 = \sqrt{N} \frac{\Delta\omega}{2\pi}\). Obviously, an accuracy of the discrete Fourier transform depends on the number of samples \(N\) and the sampling interval \(\Delta t\). The choice of \(\Delta\omega\) and \(N\) depends on the frequency response shape, the accuracy needed and the computing capacity available. The frequency interval \(\Delta\omega\) for the inverse transform must be reciprocal of the total time record length and equals to \(\Delta\omega = 2\pi / N\Delta t\).

It is necessary to note that the value of function at a discontinuity must be defined as the midvalue if the inverse Fourier transform is to hold. Moreover, using discrete Fourier transform, it is necessary to remember that it is based on the assumption about periodicity of the load applied. For periodic functions with known periods, it is necessary to choose \(N\Delta t\) interval equal to a period or integer multiple of a period. For those cases, where the period of a periodic
function is not known, the concept of a data-weighting function or data window
must be employed. For the non-periodical loads, the period of load can be
expanded by addition of long interval for a zero loading.

To characterise the transient response, fig. 3, the value of maximum
displacement in a time of load action and logarithmic decrement showing the
velocity of vibration fading are taken into consideration. The value of
logarithmic decrement is introduced as

$$\delta = \frac{1}{N} \ln \frac{X_n}{X_{n+N}}$$  \hspace{1cm} (15)

where $X_n$ and $X_{n+N}$ are the amplitudes of $n$ and $n+N$ cycles respectively, and $N$
is an arbitrary integer. It is necessary to note, that logarithmic decrements are
calculated from the steady state vibrations.

![Transient response](image)

Figure 3: Transient response.

3 Experimental vibration analysis

An experimental vibration analysis has been made for different sandwich and
laminated composite plates and laser-welded sandwich panels. In most cases the
free-free boundary conditions have been applied for the experimental samples,
excluding the case connected with the vibration analysis of large sandwich and
laminated plates fixed from all sides and excited by load speakers. The dynamic
characteristics (eigenfrequencies, eigenmodes, modal loss factors) have been
determined using three different experimental set-ups:

- **ISI-SYS laser vibrograph** operating on the interferometry and correlation
  principles and using the forced normal mode excitation method for vibration
  measurements,
- **POLYTEC laser vibrometer** operating on the Doppler principle and
  measuring back-scattered laser light from a vibrating structure to determine
  its vibration velocity and displacement,
- **pulse technique** using accelerometers for measurements and impulse
  hummer for an excitation, and presented in fig. 4.
4 Results and discussion

To calibrate an experimental set-up consisting of POLYTEC laser vibrometer and loadspeaker for excitation, the analytical Whitney [8] and I-DEAS finite element solutions have been obtained for clamped from all sides CF/EP composite plate [+45/-45/90/0/0/90/-45/+45], with the following geometrical parameters: length 0.9 m, width 0.6 m and thickness 0.0041 m. The material properties of lamina are taken as follows: $E_x = 84.76$ GPa, $E_y = 5.00$ GPa, $G_{xy} = 4.12$ GPa, $\nu_{xy} = 0.30$, $\rho = 1444$ kg/m$^3$. CF/EP composite plate has been modelled in I-DEAS by 40x60 4-node shell finite elements based on the classical lamination theory. Table 1 shows a good correlation between theoretical and experimental eigenfrequencies for six first modes.

Table 1: Eigenfrequencies $f_n$ (Hz) of CF/EP composite plate.

<table>
<thead>
<tr>
<th>Mode $n$</th>
<th>Analytical</th>
<th>I-DEAS</th>
<th>Experimental</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/1</td>
<td>80.5</td>
<td>80.4</td>
<td>76.8</td>
</tr>
<tr>
<td>2/1</td>
<td>122.6</td>
<td>122.1</td>
<td>124.3</td>
</tr>
<tr>
<td>3/1</td>
<td>191.9</td>
<td>191.2</td>
<td>191.9</td>
</tr>
<tr>
<td>1/2</td>
<td>198.1</td>
<td>197.8</td>
<td>216.3</td>
</tr>
<tr>
<td>2/2</td>
<td>241.0</td>
<td>238.9</td>
<td>239.2</td>
</tr>
<tr>
<td>3/2</td>
<td>309.6</td>
<td>306.7</td>
<td>316.7</td>
</tr>
</tbody>
</table>
To estimate correctness of the approaches developed, the pulse technique has been used for more complicated case to carry out vibration analysis of the laser-welded sandwich panel with I-core longitudinal webs and rigid polyurethane foam (PU145). This sandwich beam has the following geometrical parameters: length $L=2$ m, width $B=0.18$ m, height $H=0.044$ m, and the following cross-section dimensions, fig. 5: $t_f = 0.002$ m, $t_c = 0.04$ m, $t_w = 0.004$ m, $2p = 0.12$ m. The material density and mechanical properties are taken as follows:

- steel: $E_j = E_w = 210$ GPa, $\nu_j = \nu_w = 0.3$, $\rho_j = \rho_w = 7800$ kg/m$^3$,
- PU145: $E_{foam} = 48$ MPa, $G_{foam} = 10$ MPa, $\nu_{foam} = 0.3$, $\rho_{foam} = 145$ kg/m$^3$.

![Figure 5: Geometry of laser-welded sandwich panel.](image1)

![Figure 6: Experimental frequency response of laser-welded sandwich panel.](image2)

To make measurements, laser-welded sandwich panel is suspended in two long lightweight strings for the achievement of free-free boundary conditions. An accelerometer is attached to the end of sandwich panel and the impulse hammer is used for excitation at the opposite end. Upon impact the time histories of the accelerometer and force transducer in the hammer’s head were recorded for ten seconds and then Fourier transformed after data reduction and signal conditioning. Using visual inspection of both time histories as well as the frequency response, this specimen was tested until ten good measurements were obtained. To determine the resonance frequencies and corresponding loss factors from the frequency response plot, MATLAB code has been developed using a Nyquist plot. In order to capture resonance frequencies of up to 1000 Hz and to obtain a frequency resolution of about 0.125 Hz, the sampling frequency of the data acquisition board was set to 12800 Hz and a lowpass filter with a cut-off frequency of 3200 Hz was used.

The laser-welded sandwich beam has been modeled by 8 broken line sandwich beam finite elements with two types of complex sandwich core material properties homogenisation Barkanov et al [9] to get convergence for ten first eigenfrequencies. The first eigenfrequencies obtained from experiments and approaches developed are given in table 2, where a good agreement between different theoretical results and experiment is observed. It is necessary to note that the large number of local modes makes difficult considerably the realization
of experiment. On this reason, some experimental eigenmodes have been lost but some local modes can be presented in the spectrum. An absence of some theoretical eigenfrequencies in table 2 is connected with an impossibility to obtain the torsion modes using sandwich beam finite element that is clearly seen from fig. 6. Unfortunately, it was not possible to obtain a good correlation between theoretical and experimental modal loss factors since additional mechanisms of energy dissipation have been presented in the construction in time of experiment. But theoretical model includes only the dissipation mechanism connected with the viscoelastic material properties of foam.

Table 2: Eigenfrequencies $f_n$ (Hz) of laser-welded sandwich panel.

<table>
<thead>
<tr>
<th>Experimental</th>
<th></th>
<th>Theoretical</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Free vibration analysis</td>
<td>Frequency response analysis</td>
</tr>
<tr>
<td></td>
<td>By equivalent stiffness</td>
<td>By role of mixture</td>
</tr>
<tr>
<td>79.9</td>
<td>81.5</td>
<td>81.5</td>
</tr>
<tr>
<td>213.1</td>
<td>219.2</td>
<td>219.7</td>
</tr>
<tr>
<td>319.9</td>
<td>-</td>
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<td>394.6</td>
<td>414.4</td>
<td>416.2</td>
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<td>587.9</td>
<td>-</td>
<td>-</td>
</tr>
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<td>-</td>
<td>656.5</td>
<td>661.1</td>
</tr>
<tr>
<td>746.4</td>
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<td>-</td>
</tr>
<tr>
<td>913.0</td>
<td>931.3</td>
<td>940.3</td>
</tr>
</tbody>
</table>

5 Conclusions

The present approaches have been developed with the aim to use them as universal tools in the finite element analysis of viscoelastic composite structures applied widely in the aeronautical, ship and automobile industry. This technique gives the possibility to preserve an exact mathematical formulation for the viscoelastic frequency-dependent damping model and to calculate structures with high damping. Numerical and experimental results demonstrate a validity of the present implementation.

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References


