Vibration analysis of free isotropic cracked plates

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Abstract

Cracks occurring in structural elements of machines cause a local reduction in the stiffness and consequently lead to a change in the modal parameters, such as the resonant frequencies. The measurement of the degree of variation in resonant frequencies can provide a suitable tool for identifying the location and the extension of damage. For this reason the variation of the natural frequencies in the cracked elements has been the subject of many investigations. In the present paper, the dynamic behaviour of free thin isotropic square plates, with a trough crack emanating from one edge, were investigated by means of an extensive series of finite element analysis carried out by a commercial code. The variations of both the natural frequencies and the modal shapes have been pointed out in terms of a dimensionless frequency factor depending on the length of the crack and on Poisson’s ratio. Numerical results have been verified by experimental measurement.

Keywords: FEM, cracked plates, modal analysis.

1 Introduction

Cracks occurring in structural elements of machines cause a local reduction in the stiffness and consequently lead to a change in the modal parameters (e.g., natural frequencies). The degree of change depends on the location, the nature and the extension of the damage. Frequency measurement can be cheaply carried out from the measured time response of a dynamically excited structure, therefore a frequency based approach could be a simple and economic mean to evaluate the structural damage [1-3]. For this reason the free vibration of cracked elements has been a subject of many investigations. Several papers deal with the flexural vibration of clamped and simply supported plates. Byrne [4] has
examined the growth rate of an edge crack in acoustically excited panels representing parts of an aircraft. A single panel with an edge crack is modelled as a flat plate clamped on three edge and part of the fourth. The crack is represented by the unclamped part of the fourth edge. Natural frequencies and modal shapes have been obtained by using the Rayleigh method. Stahl and Keer [5] have solved the eigenproblem of simply supported cracked plates by using the homogeneous Fredholm integral equation of the first kind. Solecki [6] has considered the vibration of simply supported rectangular plates with a symmetrical crack parallel to one edge. The problem is analyzed by the finite Fourier transformation of discontinuities function representing the displacements and the slopes across the crack faces. Qian et al [7] have investigated the vibration behaviour of simply supported and cantilever square plates with a trough crack. In particular the eigenfrequencies were determined for different crack length by means of a finite element model of the plate. McGee et al [8] analysed the vibration of circular plates with clamped V-notch or rigidly constrained radial crack by means of the classical Ritz method and the choice of a decomposed displacements field representing two sets of admissible functions assumed for the transverse vibratory displacements. Huang et al [9,10] have used the FEM and a full field technique (electronic speckle pattern interferometry, ESPI) in order to investigate the influence of the crack length on the resonant frequencies of free circular and clamped square cracked plates. In the present paper, the flexural vibrations of free thin isotropic square plates, with a crack emanating from one edge, were investigated by an extensive series of numerical analysis carried out by a commercial finite element code. In particular the variation of both the natural frequencies and the modal shapes have been pointed out in terms of a frequency factor depending on the a-dimensional crack length (i.e., the ratio between the crack length and the edge of the plate) and on the Poisson’s ratio. In order to verify the numerical results an inexpensive computerized equipment [11] for measuring the resonant frequencies is also proposed.

2 Approximate frequency equation

The present paper is focused on the study of the influence of a trough crack on the dynamic behavior of a free square thin plate and in particular on its frequencies of natural vibration. To this end, the natural frequency $f$ of the plate has been expressed as [12]

$$f = \frac{\pi}{2} \sqrt{\frac{D}{\rho t}} \frac{\lambda}{l^2},$$

where $D=Et^3/[12(1-\nu^2)]$ is the flexural stiffness of the plate, $E$ and $\nu$ are the Young’s modulus and Poisson’s ratio, respectively, $\rho$ is the density of the material, $t$ is the thickness, $l$ is the width of the side of the square plate and $\lambda$ is a dimensionless frequency factor depending only on $\nu$ for an un-cracked plate and also on the a-dimensional crack length ($a/l$) for a cracked one. In order to
establish the relationships between frequency factors $\lambda$, $\nu$ and $a/l$, the finite element method (FEM) has been adopted. The overall dynamic behavior of plates with trough cracks at different positions $b$ from the side of the plate was examined. A schematic representation of the cracked plate is reported in Fig.1. For sake of brevity only the results for two crack positions ($b=l/2, l/4$) are reported in the paper.

Figure 1:  Schematic representation of the cracked plates.

In each case the a-dimensional crack length assumes values between 0 (no crack) and 0.4. 2-D finite element models of the plate have been developed. The basic FE model in this study is a square plate 100mm by 100mm in dimension and 1mm in thickness. The boundary conditions for the models were all edge free. Quadratic eight node QUAD elements were used and both the effects of the transverse shear deformation and of the rotary inertia were neglected. The normal mode solution for predicting the natural frequencies of the plates was carried out by using the general-purpose finite element code MSC/NASTRAN (SOL 103), with MSC/PATRAN as the pre- and post-processor. The Lanczos extraction method [13] was adopted. The crack has been obtained duplicating the nodes along the crack lines, besides, in order to capture the square root singularity, a refined mesh around the crack tip was used [14]. However, an analysis conducted by the authors, not reported in the present paper, shown that the difference between the natural frequencies evaluated with a refined mesh and those obtained by means of a roughly ones was negligible, therefore, because a qualitative analysis is the aim of the paper, the second ones was adopted. The values of the first ten resonant frequencies have been calculated for various values of $\nu$ and for each $a/l$, and for fixed values of material density and Young’s modulus. Then, $\lambda$ was computed by mathematically inverting Eq. (1). In order to make less tedious the calculation of the dimensionless factors an automatic procedure was implemented in MATLAB environment. The procedure allows one to change in turn the values of the Poisson’s ratio for each model of the cracked plates simply by rewriting the card (MAT 1) containing the values of the elastic properties. The results obtained by means of the aforementioned procedure are reported in the next section.

3 Finite element results

The effect of the Poisson’s ratio on the behavior of a cracked plate has been examined. In particular all the model properties are kept the same except for the
Poisson’s ratio, which has been increased from zero to 0.5 by steps of 0.02. The values of the dimensionless frequency parameters $\lambda$ as a function of $\nu$ are reported in Figs. 2 and 3. For sake of clarity only two cases per graph has been reported: the un-cracked case (continuous lines) and the cracked case (dotted lines) for $a/l=0.4$.

The presence of a crack causes a reduction in the stiffness and therefore a decrease of the resonant frequencies for each mode of vibration.

Besides in both cases ($b=l/4$ and $b=l/2$) a decoupling between the fourth and the fifth and the sixth and the seventh natural frequencies can be observed. This
effect is more evident for $b=l/4$. The same effect was observed for the intermediate values of the a-dimensional crack length. The values of the dimensionless frequency factors $\lambda$ as a function of $a/l$, for $\nu=0.3$, and the corresponding modal shapes are reported in Figs. 4 and 5. As it can be observed from these graphs the crack growth causes a decrease of the resonant frequencies.

![Figure 4: Frequency factor $\lambda$ versus $a/l$ (b=l/4).](image1)

![Figure 5: Frequency factor $\lambda$ versus $a/l$ (b=l/2).](image2)
In particular, it can be pointed out that there isn’t a sensitivity to neither the location or the extension of the crack for the first three modes of vibration, from this point of view similar finding are obtained in [9-10]. For \( b = l/4 \) a decrease in both the 6th and the 7th resonant frequency around the same value of the crack length. Another interesting feature for \( b = l/4 \) model is the mode reversal that took place when \( a/l \approx 0.12 \), between the 7th and the 8th resonant modes. For \( b = l/2 \), the 6th resonant frequency undergoes a great decrease for \( a/l > 0.2 \). From a fracture mechanics point of view the more critical modes are the 1st, the 2nd and the 4th modes for \( b = l/2 \) (the correspondent modal shape are enclose in the black box), because they introduce an out of phase displacement along the crack line and therefore can induce a mode III fracture (i.e., tearing mode). Similar conclusion can be made for \( b = l/4 \), in this case the more critical modes are the 4th, the 6th and the 7th.

4 Experimental equipment

The test apparatus used for the measurement of the natural frequencies is shown in Fig. 6.

![Figure 6: Experimental set-up.](image)

It consists of an impulser, a pickup transducer to convert the mechanical vibration into an electrical signal, an ordinary personal computer provided with a data acquisition card and dedicated software for measuring the fundamental resonant frequencies. The exciting impulse is imparted by lightly striking the square plate with the impulser, the operator can choose to use a simple implement or an impulse force test hammer. In the latter case the electrical signal, the input signal, provided by the transducer will be used to improve the accuracy of the measurement. The size and geometry of the impulser depends on the size and weight of the specimen and the force needed to produce vibration. The plate should be tested with all its edges free. This condition can be approximated by supporting the specimen on soft materials (e.g. cotton pad, soft sponge). The dynamic response of the plate is detected by an accelerometer attached to the plate and, after having been amplified, it is addressed in the form...
of an electrical signal, the output signal, to either the NI DAQ card (PCI-M-
IO/16-E-1) or the sound card (SB 16Bit/44100Hz). Alternatively, in the case of
light or small plates, a non contacting acoustic microphone could be used [11].
The signals are then analyzed and processed by a proper Virtual Instrument (VI)
implemented in LabView environment. It transforms the sampled time functions,
into a frequency spectrum by a Fast Fourier Transform algorithm and computes
the ratio between the output and the input spectra.

5 Comparison between numerical and experimental results

A steel (38NiCrMoK) square plate, with a crack emanating from the center of
one edge (b=l/2), has been used in this study for experimental investigations. The
crack has been machined to the chosen dimension by using a jeweller saw. The
a-dimensional crack length assumes values between 0 (no crack) and 0.4. In
order to simulate the free boundary condition the specimen was placed on some
soft sponge support. Frequency measurement has been carried out by mean of
the test apparatus described in the previous section. The frequency factors
obtained numerically superimposed on to the experimental measurements are
shown in Fig. 7.

![Figure 7: Experimental results.](image)

It can be observed that a good agreement exists, especially for the lower
values of the a-dimensional crack length. As expected according to the numerical
results, as the crack advances the 6\textsuperscript{th} frequency drastically decrease. Besides,
increasing a/l the difference increase in particular the max difference is less to
about 2%. This difference is probably due to an imperfect machining of the
crack, in particular it is difficult to accurately monitor the crack length and to
maintain a straight crack line as a/l increase from 0 (no crack) to 0.4.
6 Conclusion

In the paper numerical calculation of the resonant frequencies and simulation of the modal shapes of thin isotropic cracked square plates with free boundary condition have been performed. In particular the overall effect of the crack length, the crack position and the Poisson ratio on both the resonant frequencies and the modal shape have been discussed. It has been observed that the presence of a crack causes a reduction in the stiffness and therefore a decrease in the resonant frequencies and a modification in the modal shapes. In both cases examined the more affected natural frequencies were the 6th and the 7th; these are the only ones that can provide a suitable mean to detect the crack dimension from a simple frequency measurement. On the other hand if a full field technique is available the variation of the modal shape can provide more quantitative data.

References


