Investigation on dynamic properties of linear systems with subspace identification method

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Abstract

A subspace identification method is proposed as inverse dynamic analysis on examination of real structure behaviour with actual loads and noise-contaminated input/output data. The change of dynamic parameters (stiffness, damping) may be related to the structural history (erosion, friction, fatigue, damage, cracks) and cause the decrease of reliability and serviceability or result in structural collapse. When structural damage is small or hidden inside the structure, its detection cannot be done visually. Vibration monitoring is proposed as a useful and non-destructive dynamic parameter evaluation tool. Some tests on real, impulse loaded models have been conducted and data are procured. It is assayed how damage occurrence or decrease of structural system integrity will lead to the change of the dynamic characteristics. Special software is developed for experiment monitoring, determination of relevant mechanical characteristics and damage localization. Experimental results are used in the software both as a complete data set (number of the acceleration sensors equal to the dof) and as an incomplete data set (number is less than dof). In the latter case missing data are iteratively computed. Benefits and drawbacks of the subspace iteration method are emphasized. However, some unsolved topics are left for investigation.

Keywords: subspace identification method, dynamic properties, structural damage, vibration monitoring, linear system.

1 Introduction

Erosion, friction, fatigue, internal damages and cracks cause gradual degradation of structural performances during a long term service: the stiffness of the system
is weakening, whereas the damping of the system is strengthening. When the structural damage is small or it is in the interior of the system, it cannot be visually detected. A useful non-destructive evaluation tool is the vibration monitoring, based on the fact that the occurrence of damage or loss of integrity in a structural system leads to the changes in the dynamic properties of the structure. In recent years, various analytical and experimental techniques have been proposed and developed to deal with the issue of damage or fault detection in structural systems. One of them is a simple method called the *subspace identification method for identification of state space models* introduced by Ho and Kalman (1966). The main idea of this work was to test and prove this method and detect the damage position in the structure.

2 Introduction to subspace identification method

2.1 Structural analysis and FEM model

The dynamic behaviour of complex structures is often modelled by a system of second order linear ordinary differential equation,

\[ M \ddot{w}(t) + D \dot{w}(t) + Sw(t) = f(t) \]  

(1)

where \( M, D \) and \( S \) are *mass*, *damping* and *stiffness matrices* of the structure respectively and \( w(t) \) and \( f(t) \) are the *displacement vector* and the *force vector*. Let \( n \) be the number of degrees of freedom of the system. Then,

\[ M, S, D \in \mathbb{R}^{nxn}; w(t), f(t) \in \mathbb{R}^{nx1}. \]

Having in mind various influences on the structure during its course of service, it is understandable that the dynamic properties of the system are changing. There can be significant differences between initial values \( D_0 \) and \( S_0 \) compared to current values \( D \) and \( S \). An evaluation of the current values can be a very difficult task. If the traditional methods known from damage mechanics and other relevant fields are used it is necessary to model the evolution of a system property, obtain system property parameters, trace the history of motion and loading, carry out complicated analysis and computation under prescribed initial and boundary conditions, and finally derive the degraded property and responses of the system of interest.

Another way, which will be considered in this work, is to use an “inverse” method, i.e. extract information about system properties from experimental input/output data; hence, it does not require such a costly foregoing procedure. This way is based on the following procedures.

(i) Given is the structural system. It is possible to arrange some actuators at some locations in the structure which will produce excitation of the system. Let \( m \) be the number of actuators. *The force vector* \( f(t) \) can be replaced by an input \( Gu(t) \), i.e.,

\[ f(t) = Gu(t), \]  

(2)
where \( u(t) \in \mathbb{R}^{m \times 1} \) is an \textit{m-input force vector}; and \( G \in \mathbb{R}^{m \times m} \) represents the \textit{input location influence matrix}.

(ii) At the same time, \( l \) sensors arranged at some locations in the system measure the response of the structure under foregoing excitation and the \textit{output} can be expressed as:

\[
y(t) = C_\eta w(t) + C_\nu \dot{w}(t) + C_\sigma \ddot{w}(t) + \overline{D}'u(t)
\]

where \( y(t) \in \mathbb{R}^{l \times 1} \) is an \textit{l-sensor output vector}; \( C_d \in \mathbb{R}^{l \times n} \), \( C_v \in \mathbb{R}^{l \times n} \) and \( C_a \in \mathbb{R}^{l \times n} \) represent \textit{output displacement, velocity and acceleration location influence matrices}, respectively, and \( \overline{D}' \in \mathbb{R}^{l \times m} \) is the \textit{direct transmission matrix} corresponding to direct input/output feedthrough. Note that there is no transmission matrix \( \overline{D}' \) if the position is measured. If the measured data are displacements (resp. velocities, accelerations), they will be referred to as \textit{displacement (resp. velocity, acceleration) sensing}. In these three cases, two of the matrices \( C_d, C_v \) or \( C_a \) vanish, respectively.

(iii) With foregoing data it is possible to find \( M, S, D \) so that the dynamic system modelled by eqn. (1) exactly supplies the output data (3) measured with input data (2).

The subspace identification method for identification of state space models as one of the latest among the several approaches for the mass, stiffness and damping identification is proposed here. It can be said that this method has a property of a black-box system because the full information about the stiffness and damping matrices are not available. This comes from the fact that the structural model equations identified either by the modal analysis based on FFT or by the subspace identification method for state space models or by other known identification techniques, are not really the second order dynamic differential equations (1). They are only a form of eqn. (1) under an unknown coordinate transformation. Generally, it is difficult to transform, in complete and unique sense, the former into the eqn. (1).

The main goal of this method is to evaluate three structural property matrices \( M, S \) and \( D \). The well known principle of mass conservation can be applied here; so, the mass matrix of the system is constant given by its initial value \( M_0 \). With information about the construction of the stiffness matrix it is possible to detect the location of the damaged or faulty elements, if any, in the structural system.

2.2 Subspace identification method for state-space models

Linear and quasi-linear ordinary differential equations of any given order with input/output, including the second order differential equations (1) with (2)-(3), may be equivalently expressed in a form of state space model. The equations below are known as state space model with multi-inputs and multi-outputs.

\[
\dot{x}(t) = \overline{A}x(t) + \overline{B}u(t),
\]

\[
y(t) = \overline{C}x(t) + \overline{D}u(t)
\]
where \( x(t) \in \mathbb{R}^{N \times 1} \) is the state vector, \( u(t) \in \mathbb{R}^{m \times 1} \) is the input vector, \( y(t) \in \mathbb{R}^{l \times 1} \) is the output vector, \( A \in \mathbb{R}^{N \times N} \) is the system matrix, \( B \in \mathbb{R}^{N \times m} \) is the control matrix, \( C \in \mathbb{R}^{l \times N} \) is the observer matrix, and \( D \in \mathbb{R}^{l \times m} \) is the direct transmission matrix.

In the theory of the system identification and realisation the only available information is the input, i.e., system excitation, and output, i.e., the system response on the given excitation; hence initial behavior of the system is totally unknown. Mathematically, the main problem is to find such a state-space model (eqs. (4)-(5)) of a minimal dimension for the given experimental input/output data, so that input and output are satisfied.

Let us consider the structural system with the impulse input excitation and let the response be measured as
\[
y(t) = y(i \Delta t)
\]
Response is measured in equidistant time steps \( \Delta t \) that have to be “very” small. Our job is to calculate system matrices \((A, B, C, D)\) from eqs. (4) – (5) for given \( y(t), \Delta t \) and \( u(t) \). The next step will be the evaluation of the dynamic properties \( S \) and \( D \) from the system matrices using a special algorithm, given below.

Thus the system realisation problem may be reformulated as follows: for the given impulse response functions of the system, i.e., a set of Markov parameters,
\[
\{Y(s)\} = \begin{cases} 
D, & s = 0 \\
CA^{-1}B, & s > 0 
\end{cases}
\]  
(6)

find a triplet \( \{A, B, C\} \), called realisation of a state space model (4) – (5).

A standard algorithm based on a subspace identification method, called Eigen-system Realisation Algorithm (ERA) is a widely used method for solving the latter problem. One of the main steps in ERA algorithm is calculating of the Hankel matrix. For the derivation of Hankel matrix and for more details see [1].

### 2.3 Iteration algorithm for the case of incomplete output data

The number of sensors which is required depends on the total number of DOF of the system under consideration [1]. That means that \( l \) (number of sensors) has to be equal to \( n \) (number of DOF) for uniquely evaluating the stiffness and damping matrices. For the system which has a very large \( n \) it is difficult to put so many sensors on the structure. Moreover, due to the fact that the singular value decomposition of the Hankel matrix involves the dependence on the system eigenvalues, an increased number of eigenvalues generates eigenvalues with higher values. That causes the choice of shorter time steps \( \Delta t \). Usually, for the system with a large number of DOF only a set of incomplete data may be available. If \( l < n \), we can use an iteration algorithm to provide a set of lacking data using following procedure.

We can arrange \( l \) sensors at \( l \) locations in structural system. The missing \( n-l \) number of real sensors can be replaced with the fictive sensors which are
properly arranged. If we know the state vector (response) $\mathbf{x}(t)$ for input $\mathbf{u}(t)$ we can gain the missing output data. The initial values for stiffness and mass matrices $\mathbf{S}_0, \mathbf{M}_0$ are calculated using structural analysis for discretized system or from discretization methods for continuous systems. The evaluation of initial damping matrix is a difficult task. Hence $\mathbf{D}_0 = 0$ can be used as initial value and the current values of $\mathbf{D}$ as well as $\mathbf{S}$ can be computed through the algorithm.

$$
\mathbf{A}_a = \begin{bmatrix}
0 & \mathbf{I}_n \\
-\mathbf{M}_0^{-1}\mathbf{S}_a & -\mathbf{M}_0^{-1}\mathbf{D}_a
\end{bmatrix},
$$

$$
\mathbf{B}_a = \begin{bmatrix}
0 \\
\mathbf{M}_0^{-1}\mathbf{G}_a
\end{bmatrix},
$$

$$
\mathbf{C}_a = \begin{bmatrix}
0 & \mathbf{I}_{n-1}
\end{bmatrix}
\begin{bmatrix}
\hat{\mathbf{C}}_d & \hat{\mathbf{C}}_v
\end{bmatrix}
\begin{bmatrix}
\mathbf{C}_a & \mathbf{M}_0^{-1} \begin{bmatrix}
\mathbf{S}_a & \mathbf{D}_a
\end{bmatrix}
\end{bmatrix},
$$

$$
\dot{\mathbf{x}}_a(t) = \mathbf{A}_a \mathbf{x}_a(t) + \mathbf{B}_a \mathbf{u}(t),
$$

$$
\tilde{\mathbf{y}}_a(t) = \tilde{\mathbf{C}}_a \mathbf{x}_a(t) + \tilde{\mathbf{D}}_a \mathbf{u}(t),
$$

$$
\hat{\mathbf{y}}_a(t) = \begin{bmatrix}
\mathbf{y}(t) \\
\hat{\mathbf{y}}_a(t)
\end{bmatrix},
$$

$$
\begin{bmatrix}
\bar{\mathbf{A}}_{a+1} \\
\bar{\mathbf{B}}_{a+1} \\
\bar{\mathbf{C}}_{a+1}
\end{bmatrix} = \text{ERA} \left( \hat{\mathbf{y}}_a(t) \right),
$$

$$
\begin{bmatrix}
\mathbf{S}_{a+1} \\
\mathbf{D}_{a+1} \\
\mathbf{G}_{a+1}
\end{bmatrix} = \Phi \left( \bar{\mathbf{A}}_{a+1}, \bar{\mathbf{B}}_{a+1}, \bar{\mathbf{C}}_{a+1} \right),
$$

The procedure stops when the algorithm provides satisfactory values for matrices $\mathbf{S}, \mathbf{D}$ and $\mathbf{G}$. In the previous equations $\overline{\mathbf{C}} \in \mathbb{R}^{l \times n}$ is the observer matrix for the measured data, $\tilde{\mathbf{C}} \in \mathbb{R}^{(n-1) \times n}$ - observer matrix for the missing data, $\bar{\mathbf{D}} \in \mathbb{R}^{l \times n}$ - direct transmission matrix for the measured data, $\tilde{\mathbf{D}} \in \mathbb{R}^{(n-1) \times n}$ - direct transmission matrix for the missing data, $\mathbf{0}$ is zero matrix of size $l \times (n-1)$, $\mathbf{y}(t) \in \mathbb{R}^{l \times 1}$ - measured output, $\tilde{\mathbf{y}}(t) \in \mathbb{R}^{(n-1) \times 1}$ - the output of a missing data, $\hat{\mathbf{y}}(t) \in \mathbb{R}^{n \times 1}$ - complete output.

3 Experimental equipment

3.1 Physical properties of the tested structural systems

In the scope of testing and proving the theoretical and numerical part of this work some experiments have been done. Two steel bars (IPB 100 profile), both of 4m length, with different way of supporting were tested in laboratory. First of them was hanging on the rubber ropes which were placed at a distance of 0.5m from both edges (in further text: BarI). The second bar was simply supported at both ends on steel rollers and clamped with a special mechanism and rubber rolls which allowed deflections, but vertical and horizontal displacement at the ends were constrained (Bar II).
3.2 Equipment

The structure was instrumented using the piezoelectric accelerometers. 16 sensors were screwed in the beam equidistantly on the upper side. Locations were chosen so sensors were not at the supports for both bars; indeed, this would be pointless due to the boundary conditions. The task was to measure the response of the structure at the sensor locations due to the impulse excitation. The sensors were connected to a **SPIDER 8** which had two devices and both of them had 8 inputs. The instrument was transferring the measured data in mV/V to a PC where all data were converted into acceleration values. The software which was used for measurement monitoring and data converting to the desired form and graphs was **Catman**\(^\text{®} 31\) (Hottinger Baldwin Messtechnik GmbH). A digital FFT analyser was also used for testing the eigenfrequencies of the bars which were expected during the measuring.

3.3 Simulation of the damage

In both experiments the bars were tested without and with simulated damage. A steel clamp was placed but not tightened at some position of the beam, fig.2. The response of the beam with this additional mass was measured. Then the clamp was tightened and the response was measured again. In the second case one part of the beam was clamped and it was expected that it would be enough to show that some damage occurred in that part. The difference between responses of the beams in both cases can be seen through diagrams in the section 4.1.
4 Experimental procedure

4.1 Load details and experimental results

The excitation of the bars was an impulse load applied by hammer. It was checked that the duration of acting force was really "short" - less then 0.2 ms. This assured that acting force could be considered as an impulse load; indeed, the sample frequency was 4800Hz, i.e., the time step was \( \approx 0.2\text{ms} \). In each case the duration of the structure response measuring was 3 sec. The whole set of data was supplying information from 16 locations of the beam. Data were stored in such a way that they could be used as input data for the written program in Scilab as well as for a finite element program.

Although a large amount of data was collected, only few characteristic cases have been selected for presentation in this section. In the first four diagrams the whole sets of data for one measurement for one characteristic sensor are shown and the influence of damping can be clearly seen, fig. 3.-fig. 6. Instrument was not set to trigger, i.e., measurement does not start when some impact is applied, because the intention was recording of initial noise as well as the noise after finishing of the beam vibration. First three diagrams represent the response of the Bar I. Because of the elastic supports the influence of damping is less strong as in the case of the Bar II where the supports were stiffer and located at the edges. Then, damping is acting very strongly, fig. 6. Due to the soft supports in Bar I, it can be said that the rigid body motion played an important role. This leads to the conclusion that Bar II was a more acceptable structural system for this purpose.

In the last diagram data were collected from different measurements and, therefore, they were rejected from the moment when the response of structure occurred; the time axis was chosen to start from zero at that point, fig.7. The sample frequency in each measurement was the same, so, the rejection was allowed. This diagram represents the response of the structure at the same place of the bar but in three different cases: first, called free, is without additional mass of clamp which will later simulate damage; second, named undamaged is case where a clamp was placed on the structure but it was only an additional mass to a new structure, i.e., it was strongly tightened and the wave could pass trough the beam without delay; and third, damaged, where clamp was not tightened strongly; hence the structure was damaged in the clamped part, compared to the foregoing case two. Comparing the results it can be concluded that this kind of damage simulation can be applied because it evidently causes the expected differences in output data.

In fig. 7. the delay of the response of the system can be seen in an undamaged and damaged case in comparison with the free case. The greater mass in the first two cases produces greater inertia and the answer of the structure is delayed. The stiffness of the beam in undamaged case is grater in comparison with damaged case. That causes the greater frequency of the beam oscillation (the smaller period of oscillation). The amplitudes cannot be compared because in each test random intensity of the impulse force was applied. The amplitudes of
acceleration are related to the load and hence different in each measurement. But the shape of the diagrams can be compared.

4.2 Presentation of the measurement results

For more measurement results see [2].
5 Numerical procedure

As mentioned before, the necessary number of sensors has to be equal to the total number of DOF. Hence the beams were considered as a finite DOF discretized structure with 8 constitutive elements (i.e., 9 nodes) and in each node 2 DOF were allowed: lateral deformation and bending angle. Although in two nodes at supports lateral deformation is restrained the total number of DOF was considered to be 16. So, the number of sensors (16) was equal to the number of DOF (16). Hence the theoretical result of the stiffness and damping matrices $S$ and $D$ had to be unique. In order to check the case when we have the set of incomplete data (section 2.3.) the idea was to ignore measured results from some of sensors which could be considered as fictive and hence their data have to be calculated through algorithm (7)-(14). Further, the results from algorithm could be compared with measured values for the same sensors. This comparison could give the information about validity of the algorithm.

The program was written in Scilab, a free MatLab R clone. Scilab has been designed for engineering and scientific applications and it is a user-friendly environment such as MatLab R. A state space realization is also one of the powerful algorithms that are implemented in this software package (routine imrep2ss).
The program has been written to make the full investigation of the system identification problem (for the complete code see [2]). In the first part of the program the user is defining the characteristics of the structural system under consideration, and which are necessary for the further evaluation. In evaluation part the Scilab incorporated routines for the system realization are also recommended. It has been seen that the numerical procedure, though ready to be used, stuck at the output of the subspace method. The main reason for this behaviour is the fact that this method is still not developed for use of automatic execution. Indeed, for each single set of data, the right subset of data to be used in the subspace method has to be selected by hand. For this purpose the subset data have to be analyzed by FFT in order to see if the eigenvalues are comparable to eigenvalues of a similar beam. Also, the sample time step plays a prominent role for the quality of subspace identification. The routines developed so far may be used with any other identification method or for subspace methods that are less sensible on the data window or the used time step. Whenever such a method and/or routine are available the program may be of valuable use.

6 Conclusion

Based on the theoretical proposals from [1] and through the experimental and numerical work, this paper shows that subspace identification algorithm is a tool for the automatic identification of system parameters, that is not yet fully elaborated.

Further investigation and development in this field will be the topic of the future work, especially in overcoming the noticed inadequacies. Also, it should be considered if the linear system description is sufficient for the detection of damage. Alternately, more sophisticated methods should be used to account for nonlinear effects. The noise and other disturbing influences that may substantially change the input data should be particularly taken into account.

References


