3-D modeling of ignition and combustion processes in combustion chamber of automobile airbag

A. Rychkov\textsuperscript{1}, N. Shokina\textsuperscript{2} & H. Miloshevich\textsuperscript{1}
\textsuperscript{1}Institute of Computational Technologies SB RAS, Russia
\textsuperscript{2}Rechenzentrum Universität Stuttgart, High Performance Computing Center, Germany

Abstract

The paper is devoted to the mathematical modelling of the processes in the model of airbag combustion chamber with two slots as output nozzles. The influence of 3-D flow field structure on the ignition and combustion of fuel granules is investigated. The numerical algorithm is developed, which takes into account the non-uniform distribution of temperature in the bulk of every granule and allows the calculation of the combustion product movement in the regime of percolation. Thus, there is the possibility to predict the dynamic of the ignition process in the framework of the combustion model with a constant surface temperature. 3-D algorithm is based on the second-order upwind LU difference scheme with TVD-properties. The numerical results are presented for the concrete combustion chamber with two slots as output nozzles.

1 Introduction

Nowadays the numerical modeling is an effective tool for understanding in detail the complex physical-chemical processes in different technical devices. The airbag
becomes a very popular automotive individual safety device. It consists of the combustion chamber, which is filled with the granules of a solid monofuel with comparatively low combustion temperature, connected to the special elastic shell, which is made of a gas-proof fabric. In initial state the shell is rolled up into the compact roll. After collision of an automobile with an obstacle the system of solid fuel ignition responds. The combustion products fill the shell during 50–80 milliseconds, transforming it into the elastic bag.

The paper is devoted to the mathematical modelling of the processes in the model of airbag combustion chamber with two slots as output nozzles. The influence of 3-D flow field structure on the ignition and combustion of fuel granules is investigated. The numerical algorithm is developed, which takes into account the non-uniform distribution of temperature in the bulk of every granule and allows the calculation of the combustion product movement in the regime of percolation. Thus, there is the possibility to predict the dynamic of the ignition process in the framework of the combustion model with a constant surface temperature. 3-D algorithm is based on the second-order upwind LU difference scheme with TVD-properties.

The numerical results are presented for the concrete combustion chamber with two slots as output nozzles. The calculations are done for several variants of main initial data. The dynamics of development of the processes in the airbag combustion chamber is illustrated by the series of pictures. The numerical results show the good correspondence with full-scale tests. Therefore, the suggested method can be used for the forecast of processes in airbag combustion chambers at searching the optimal airbag construction.

2 Problem formulation

Fig.1 shows the scheme of the model of airbag combustion chamber with two slots as output nozzles. The symmetric configuration of the chamber allows to perform numerical modelling in a quarter of its volume only. The particles of the fuel (granules) have a cylindrical form. The particles of the igniter (booster) have a spherical form. The joint name for all particles is also used – the fuel elements. It is assumed that these fuel elements are distributed uniformly in the chamber and their location is not changed during the combustion process. Therefore, the fuel elements are assumed to be immovable and only their sizes decrease.

It is also assumed that the chemical reaction rates are large enough and the combustion processes come to an end near the surface of a fuel element. It allows to describe these processes with the help of source terms in the equations of mass balance and energy balance. The escaping gas is the combustion product of the fuel elements and their heat determine the gas temperature.
The following main assumptions are used in the mathematical formulation.

1. The flow is three-dimensional and non-stationary.

2. The continual model of two interpenetrating mediums is used. These mediums are the gas (combustion products) and burning hard material (fuel elements) as porous medium.

3. The work of the friction force and the pressure force are not taken into account in the energy equation at the description of the gas flow due to the small flow velocity. The heat transfer between the fuel elements caused by the heat conduction is also neglected.

4. The particles of the booster have a spherical form. A cylindrical form of the fuel granules is brought into accord with the equivalent diameter of a sphere.

5. The material of the fuel elements is homogeneous. The booster particles contain incombustible part (fine-dyspersated particles, for example \( KBO_2, K_2CO_3 \), which usually occupy 40-50% of the whole capacity). The incombustible part plays important role in the process of a fuel element ignition, therefore the mathematical model has to take it into account.

6. The combustion products consist of a mixture of the perfect gas with the constant adiabatic exponent and the incombustible small solid particles, which are the booster combustion product. These particles move with the same velocity as the gas stream. It is assumed that the size of the particles is small enough, therefore the carrying gas can be considered as two-phase equilibrium medium.

7. The initiation of the booster particle ignition occurs due to the heat exchange between the ignitor combustion products, which enter the left boundary of the booster combustion chamber at the time moment \( t = 0 \). It is assumed that the composition of the ignitor combustion products is identical to the composition of the carrying gas, but does not contain solid phase.

8. The temperature distribution inside a fuel element is described approximately by a spline function.

9. The combustion of a fuel element starts when its surface temperature reaches
the given value $T_v$. The combustion rate depends on a local static pressure and is calculated by the formula $r_b = r_{b0} (p/p_0)^\nu$ with different constants $r_{b0}$ and $\nu$ for the booster particles and the fuel particles.

The equations, which describe three-dimensional non-stationary processes in the airbag combustion chamber, is written in the cylindrical coordinate system:

\[
\frac{\partial p}{\partial t} + \frac{\partial}{\partial x} (\rho u) + \frac{1}{r} \frac{\partial}{\partial r} (r \rho v) + \frac{1}{\partial \varphi} (\rho w) = J_p, \tag{1}
\]

\[
\frac{\partial \rho u}{\partial t} + \frac{\partial}{\partial x} (\rho u^2) + \frac{1}{r} \frac{\partial}{\partial r} (r \rho uv) + \frac{1}{\partial \varphi} (\rho uw) + \varepsilon \frac{\partial p}{\partial x} = \tag{2}
\]

\[
= \frac{\partial}{\partial x} \left( 2\mu \frac{\partial u}{\partial x} \right) + \frac{1}{r} \frac{\partial}{\partial r} \left( r \mu \frac{\partial u}{\partial r} \right) + \frac{1}{r} \frac{\partial}{\partial \varphi} \left( \mu \frac{\partial u}{\partial \varphi} \right) - \rho_p \beta u, \tag{3}
\]

\[
\frac{\partial \rho v}{\partial t} + \frac{\partial}{\partial x} (\rho uv) + \frac{1}{r} \frac{\partial}{\partial r} (r \rho v^2) + \frac{1}{\partial \varphi} (\rho vw) + \varepsilon \frac{\partial p}{\partial \varphi} = \tag{4}
\]

\[
= \frac{\partial}{\partial x} \left( \mu \frac{\partial v}{\partial x} \right) + \frac{1}{r} \frac{\partial}{\partial r} \left( 2r \mu \frac{\partial v}{\partial r} \right) + \frac{\partial}{\partial x} \left( \mu \frac{\partial v}{\partial \varphi} \right) + \frac{1}{r} \frac{\partial}{\partial \varphi} \left( \mu \frac{\partial u}{\partial \varphi} \right) - 2 \mu \frac{\partial v}{\partial x} + \frac{\rho w^2}{r} - \rho_p \beta v, \tag{5}
\]

\[
\frac{\partial \rho w}{\partial t} + \frac{\partial}{\partial x} (\rho uw) + \frac{1}{r} \frac{\partial}{\partial r} (r \rho uv) + \frac{1}{\partial \varphi} (\rho w^2) + \varepsilon \frac{\partial p}{\partial \varphi} = \tag{6}
\]

\[
= \frac{\partial}{\partial x} \left( \mu \frac{\partial w}{\partial x} \right) + \frac{1}{r} \frac{\partial}{\partial r} \left( r \mu \frac{\partial w}{\partial r} \right) + \frac{\partial}{\partial x} \left( \mu \frac{\partial w}{\partial \varphi} \right) + \frac{1}{r} \frac{\partial}{\partial \varphi} \left( \frac{2 \mu \partial w}{r} \right) - \frac{\rho vw}{r} - w \frac{\partial}{\partial r} (r \mu) - \rho_p \beta w, \tag{7}
\]

\[
\frac{\partial \rho h}{\partial t} + \frac{\partial}{\partial x} (\rho uh) + \frac{1}{r} \frac{\partial}{\partial r} (r \rho vh) + \frac{1}{\partial \varphi} (\rho wh) = \tag{8}
\]

\[
= \frac{\partial}{\partial x} \left( \frac{\lambda \partial h}{C_p \partial x} \right) + \frac{1}{r} \frac{\partial}{\partial r} \left( \frac{\lambda \partial h}{C_p \partial r} \right) + \frac{1}{r} \frac{\partial}{\partial \varphi} \left( \frac{1}{r} \frac{\lambda \partial h}{C_p \partial \varphi} \right) + \rho_p \gamma (T - T_s) - n_p \frac{\pi}{4} d_p^2 A_{sed} C_z (T - T_s), \tag{9}
\]

\[
p = \rho_s R_0 \frac{T}{M_g}, \tag{10}
\]
\[
\frac{\partial \xi}{\partial t} + u \frac{\partial \xi}{\partial x} + v \frac{\partial \xi}{\partial y} + w \frac{\partial \xi}{\partial z} + \frac{1}{r} \frac{\partial}{\partial r} \left( \rho \frac{\partial \xi}{\partial r} \right) = \frac{(\alpha_s - \xi) J_{gr}}{\rho},
\]
(15)

\[
\frac{\partial \rho_p}{\partial t} = -J_p,
\]
(16)

\[
\rho = \varepsilon \rho_1, \quad \rho_1 = \rho_g + \rho_z, \quad \xi = \frac{\rho_z}{\rho}, \quad \rho_p = (1 - \varepsilon) \rho_p^0
\]
(17)

\[
h = C_p T, \quad C_p = (1 - \xi) C_{pg} + \xi C_z.
\]
(18)

The index \( g \) refers to the gas, \( p \) to the fuel elements, \( z \) to the fine-dispersed particles. The following notations are used:
- \( t \) – time;
- \( u, v, w \) – components of the gas velocity vector \( \vec{U} \) along axes \( Ox, Or \), \( O\varphi \) of the cylindrical coordinate system;
- \( J_p \) – income of the gas due to the particle combustion;
- \( \varepsilon \) – porosity;
- \( p \) – gas pressure; \( \mu \) – gas viscosity coefficient; \( \lambda = \varepsilon \lambda_g \) – heat conductivity coefficient; \( M_g \) – molecular weight of the gas;
- \( \rho_g, \rho_z, \rho_p^0, C_{pg}, C_z, C_{pp} \) – density and specific heat of gas, fine-dispersed particles and fuel elements;
- \( \beta, \gamma \) – interphase coefficients of resistance and heat exchange;
- \( A_{sed} = K_{sd} |\vec{U}| \rho_z \) – sedimentation coefficient of the fine-dispersed particles on a fuel element surface; \( K_{sd}, n_p, d_p \) – empirical sedimentation coefficient (0 \( \leq K_{sd} \leq 1 \)), concentration of fuel elements and their diameter;
- \( \alpha_s \) – quota of the fine-dispersed particles;
- \( T \) – gas temperature; \( T_s \) – surface temperature of a fuel element; \( T_v \) – ignition temperature of a fuel element.

The interphase resistance coefficient \( \beta \) is calculated for \( \varepsilon_g \leq 0.8 \) by Ergun formula:
\[
\beta = 150 \frac{\varepsilon \mu_g}{(\varepsilon d_p \varphi_p)^2 \rho_p^0} + 1.75 \frac{\rho_p |\vec{U}|}{\varepsilon d_p \varphi_p \rho_p^0}.
\]
(19)

For \( \varepsilon_g > 0.8 \) the resistance coefficient is calculated as the resistance coefficient of a sphere with a mass, which is equivalent to a real non-spherical granule mass:
\[
\beta = \frac{3}{4} C_D \frac{\rho_g |\vec{U}|}{d_p \varphi_p \rho_p^0} e^{-2.65},
\]
(20)

\[
C_D = \begin{cases} 
\frac{24}{\Re_p} & (1 + 0.15 \Re_p^{0.687}), \quad \Re_p \leq 10^3, \\
0.44 & \Re_p > 10^3,
\end{cases}
\]
(21)

where \( \varphi_p \) is the form coefficient.
The heat exchange coefficient is calculated by the following formulas:

\[ \gamma = \frac{6 \text{Nu} \varepsilon \lambda_g}{d_p^2 \rho_p^2}, \quad \text{Nu} = \text{Re}_p \text{Pr}^{0.33} 0.56 \varepsilon_g / 4, \quad \text{Pr} = \frac{C_g \mu_g}{\lambda_g}. \tag{22} \]

The changes of a mass and size of fuel elements during the combustion and the gas mass generation due to particle combustion are calculated as follows:

\[ \frac{d(d_p)}{dt} = -2r_p, \quad r_p = r_{p0} \left( \frac{P}{P_0} \right)^\nu, \quad J_p = \rho_p^0 \pi d_p^2 r_p n_p. \tag{23} \]

3 Initial and boundary conditions

3.1 Initial conditions.

At \( t = 0 \) the gas phase velocity is equal to zero everywhere in the computational domain; the gas phase pressure is equal to the pressure \( p_0 \) of the surrounding environment; the temperature of each phase is equal to the temperature \( T_0 \) of the surrounding environment:

\[ u = v = w = 0, \quad p = p_0, \quad T_g = T_p = T_z = T_0. \tag{24} \]

3.2 Boundary conditions for the gas phase.

At the inlet:
- at \( t < t_{ig} \) the ignitor work is modelled as the hot gas input thorough the left boundary of booster combustion chamber:

\[ \rho u = J_{ig}(t), \quad v = w = 0, \quad T_1 = T_{ig}, \tag{25} \]

where \( J_{ig} \) is the ignitor mass generation rate, \( t_{ig} \) is the ignitor work time;
- at \( t \geq t_{ig} \) the following conditions are specified:

\[ u = v = w = 0, \quad \frac{\partial T}{\partial x} = 0. \tag{26} \]

At the impermeable surfaces of the computational domain the no-slip conditions (the gas velocity vector is equal to zero) and the heat insulation condition for the temperature are set:

\[ u = v = w = 0, \quad \frac{\partial T}{\partial n} = 0. \tag{27} \]

If at the outlet (the slot) the gas is subsonic, then the non-reflective boundary condition is specified. If the gas flow is supersonic, then no conditions are set.
4 Solution methods

The surface temperature $T_s$ of the fuel element is determined from the approximate solution of the heat conduction equation for the sphere with the radius $R_p$ in the spherical coordinate system:

$$\frac{\partial T_p}{\partial t} = \frac{\lambda_p}{\rho_p C_{pp} R_p} \frac{1}{y^2} \frac{\partial}{\partial y} \left( y^2 \frac{\partial T_p}{\partial y} \right), \quad y = \frac{r}{R_p},$$

(28)

in the domain $D\{t \geq 0, y_0 \leq y \leq 1\}$, where $T_p$ is the temperature of a fuel element, $y_0 = 1 - \alpha_p r/(l_p)$, $l_p = \lambda_p/(r_0 C_{pp} \rho_p)$ is the thickness of the heat layer in a fuel element at the stationary combustion [1]. $\lambda_p$ is the heat conductivity coefficient of a fuel element, $\alpha_p = 5$. The value of the coefficient $\alpha_p$ is selected during the calculations so that the condition on the boundary $y = y_0$ does not influence significantly the defined temperature value $T_s$.

The following boundary conditions are set for (28):

$$T_p (0, y) = T_0, \quad T_p (t, y_0) = T_0, \quad \frac{\partial T_p}{\partial y} \bigg|_{y=1} = \alpha (T_s - T),$$

(29)

where $\alpha = \frac{R_E}{\lambda} \left( \frac{N_0 \lambda_E}{d_p} + A_{sed} C_z \right)$.

The numerical solution is obtained using the implicit difference Cranck–Nicolson scheme of the second order. When the temperature $T_s$ reaches the value $T_v$, the combustion of the particle begins. Therefore, the condition $T_s = T_v$ is used instead of the equation (28), and the value of the heat exchange coefficient 28 is set equal to zero.

The second-order upwind LU difference scheme with TVD-properties is used for solving the system of equations (1) – (18). This scheme is close to the scheme [3]. Unfortunately, there are some iteration convergence problems at the calculation of the flow field in the places of the particle concentration sudden changes. Therefore, the following conditions on the discontinuity surface are used in the mass and energy conservation laws [2]:

$$[\rho u_n] = 0, \quad \left[ C_{pp} T + \frac{u_n^2}{2} \right] = 0, \quad [\rho u_t] = 0,$$

(30)

where $u_n$, $u_t$ are the normal and tangential (to the discontinuity surface) components of the gas velocity vector. The flow parameters are defined from the conditions on the discontinuity surface:

$$[p] = j^2 \left[ \frac{1}{\rho_g} (1 - \varepsilon)^{-1} - \frac{1}{\rho_g+} \right], \quad \left[ \frac{1}{\rho} \right] = \frac{1}{\rho_g+} - \frac{1}{\rho_g} (1 - \varepsilon)^{-1}, \quad j = \rho u_n,$$

(31)
with using of the one of the flow parameters.

The values with the sign "+" are taken in the two-phase flow domain, the values with the sign "−" are taken in the pure gas domain. For example, at $u_n \geq 0$ the pressure value $p_+$ is specified behind the discontinuity surface, and the value $p_-$ is given at $u_n < 0$ respectively. This approach allows to avoid the appearance of the parasitic oscillations of the numerical solution.

5 Calculation results

The certain problem for the successful application of the numerical modelling to the airbag working processes is the lack of the reliable experimental data on the thermal and physical characteristics of the fuel elements and the sedimentation coefficient $K_{sd}$. These data influence significantly the ignition and combustion processes. Therefore, the numerical calculations are done for the four variants of these main initial data:

- **Variant 1:** $T_v = 500$ K, $\lambda_p = 3.25 \frac{W_1}{m_1 K}$, $K_{sd} = 0$.
- **Variant 2:** $T_v = 500$ K, $\lambda_p = 13 \frac{W_1}{m_1 K}$, $K_{sd} = 0.25$.
- **Variant 3:** $T_v = 500$ K, $\lambda_p = 3.25 \frac{W_1}{m_1 K}$, $K_{sd} = 0.25$.
- **Variant 4:** $T_v = 650$ K, $\lambda_p = 3.25 \frac{W_1}{m_1 K}$, $K_{sd} = 0$.

$T_v$ is the fuel ignition temperature. The booster mass is $0.002$ kg, the fuel mass is $0.06$ kg ($\varepsilon = 0.218$ and $\varepsilon = 0.408$ respectively). The parameter values are taken as follows: $r_{b0} = 0.0001$ m/s, $\nu = 1.0$ for the booster; $r_{b0} = 0.002$ m/s, $\nu = 0.6$ for the fuel. The booster ignition temperature is $T_{vb} = 550$ K.

Fig. 2 shows the pressure curves, depending on the time, at the point on the symmetry axis on the left boundary of the booster combustion chamber (here and further the curve numbers correspond to the calculation variant numbers).

The sharp pressure peaks point to the fast heating and subsequent ignition of the booster particles because of their small sizes. The further "stratification" of the curves is due to the back influence of the pressure change in the fuel granule combustion chamber after their ignition and burning. The back pressure waves are better seen on the gas temperature distribution curves (Fig. 3) at the same point.

Fig. 4–6 show the parameter changes on the left boundary of the fuel combustion chamber. The character of the pressure change (Fig. 4) and the gas temperature change (Fig. 5) indicate the significant influence of the parameters $\lambda_p$, $T_{vp}$ and $K_{sd}$ on the gas-dynamic processes in the airbag.

The fuel granule ignition dynamics is shown in Fig. 6. The influence of the additional heat transport is clearly seen. This heat transport is caused by the sedimentation of the hot fine-dispersated particles, which appear due to the booster combustion, on the granule surface. The increase of the heat conductivity coefficient leads the significant "retardation" of the ignition and to the non-uniform
Figure 2: The gas pressure on the left boundary of the booster combustion chamber

Figure 3: The gas temperature on the left boundary of the booster combustion chamber

Figure 4: The gas pressure on the left boundary of the fuel combustion chamber

Figure 5: The gas temperature on the left boundary of the fuel combustion chamber

combustion of the fuel granules in the combustion chamber.

Figure 6: The temperature of fuel granule surface on the left boundary of the fuel combustion chamber

Fig. 7 – 10 show the dynamics of the processes in airbag for the basic data variant 3 at the time moments $t = 1$ ms, $2$ ms, $3$ ms and $5$ ms. The spatial character of the flow at the initial stage of the device work is clear seen. The distribution rate of the ignition zone in the booster combustion chamber is significantly higher than in the fuel combustion chamber. The reason is in the difference between the sizes of these fuel elements.
Further, as the ignition process and the combustion develop, the adjustment of the temperature field and the pressure field occurs. It allows the passage to the simpler modelling level, i.e. to the one-dimensional or even zero-dimensional level.

Figure 7: The gas temperature at $t = 1$ ms
Figure 8: The gas temperature at $t = 2$ ms

Figure 9: The gas temperature at $t = 3$ ms
Figure 10: The gas temperature at $t = 5$ ms

References