Overshooting error in inelastic pseudodynamic tests on stiff specimens

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Abstract

The results of pseudodynamic (PD) tests, those concerning stiff specimens especially, may be seriously impaired by systematic errors. The paper deals with the error analysis of PD seismic tests on relatively stiff half-size-scale infilled frames, modelled as single-degree-of-freedom systems. Several performance indices are reported and applied to a first group of experiments. The nature and degree of the error inherent in the laboratory system is deduced. A simple correction for that error is statistically identified and used in the subsequent tests. The improvement is assessed by applying the aforementioned indices again. It is shown that one can mitigate the error by taking account of the stiffness deterioration of the specimens.

1 Introduction

The PD method allows one to simulate the effect of earthquakes on physical models of the structures by combining quasi-static experimental and dynamic numerical techniques. In fact, a PD test may be thought as a step-by-step inelastic dynamic analysis where inertia and damping is numerically modelled, while the restoring force is measured on a specimen at each (discrete) instant of the response, and entered into the equation of motion.

Such a method has a number of advantages. Compared to full numeric analysis, the real hysteretic behaviour is considered. Compared to quasi-static cyclic testing, the dynamic nature of the seismic response is taken into account, within limits. Compared to shaking-table testing, a dynamic experimental system is not needed, since dynamics is handled by computer software governing and interacting with the instruments.
However, the systematic experimental errors may seriously impair the results [1, 2]. If the accuracy in displacing the specimen under investigation is poor, incorrect measures of the restoring force come, and wrong displacements to be imposed next are calculated. These errors propagate through the time steps, they accumulate, and possibly become uncontrolled.

The paper deals with the error analysis of PD seismic tests on single-bay single-storey half-size-scale reinforced-concrete (RC) frames infilled with nonstructural masonry, modelled as single-degree-of-freedom systems. In the case of stiff specimens, as the infilled frames are, attention must be paid to the issue of experimental errors, since the restoring force is sensitive to the displacement, at least in the elastic stage. The performance of the laboratory system briefly described in the following section is appraised.

2 PD testing setup

The present system is based on endless-screw jacks as displacement actuators. The rotating shaft of an electric motor, orthogonal to the axis of the screw, drives the jack by revolving in both ways for loading/unloading, or pushing/pulling movement (fig 1). An anti-turn device and inverter for frequency adjustment complete the jack, fully operated by remote control through a personal-computer digital-to-analog-converter board and related software. Jack’s capacity is 500 kN transient, 350 kN stationary; its stroke is ±20 cm. This jack was found to be not as expensive as comparable hydraulic equipment, nonetheless it is quite slower as well: its maximum speed is 1.45 cm/minute. In addition, as the jack approaches the target position, it is proper to lessen its speed more and more in order to improve the accuracy. When the difference between the present and target position becomes 0.05 mm or less, the speed is set to about one tenth of the maximum (precisely, 400 digital units of 4096). Each PD test on infilled frames, comprising a thousand time steps, lasts for several hours.

The displacement of the specimen is measured in real time with accurate resolution: 0.5 μm over ±25 cm (fig 1). PD testing is step-wise: as soon as

Figure 1: Jack (on the left) and displacement transducer (on the right).
the target displacement is reached, the halting command is sent to the jack motor for data acquisition and processing during the hold period. Then the jack should instantly stop. However, the inertia of the rotating shaft is found to cause some additional motion. A systematic overshooting error recurs, against undershooting reported for hydraulic jacks [3].

Special inaccuracy follows such kinds of error in PD tests [1]. One can easily see in the case of linear behaviour that overshooting yields spurious energy dissipation (fig 2). If the displacement $d_{m_{i}}$ is imposed on the specimen at the step $i$, instead of the correct one $d_{c_{i}}$, the restoring force $r_{i}^{*}$ greater than $r_{i}$ is measured. This applies to every step where velocity is positive. Contrarily, overshooting yields smaller force when velocity is negative (step $i + 1$ in fig 2). Apparent hysteresis results. In like manner, undershooting progressively adds energy to the response, which may even diverge.

The PD system was calibrated with cantilever-type steel columns whose stiffness was 1.5 kN/mm, preliminary to the infilled frames. The mean overshooting error $d_{e} = 0.01$ mm (fig 2) was statistically identified. There is a simple remedy for reducing this error, once known. It consists in properly advancing the halting command to the jack: as soon as the difference between the target displacement $d_{c}$ and the present, in-real-time measured displacement equals the error $d_{e}$, the halting command has to be sent.

3 Specimens of infilled frame

Two sorts of infilled RC frames were tested, those designed to withstand the seismic action (weak-beam strong-column frame), and those designed for gravity loading only (strong-beam weak-column frame). Most of the frames were infilled with nonstructural masonry, made of hollow bricks and
cement mortar, which was disregarded in the design (tab 1). Some specimens are shown in fig 3. Each one was subjected twice to the EW accelerogram recorded in Tolmezzo (Friuli, Italy) on May 6th 1976, being the second time damaged owing to the first PD test. The explicit integration algorithm proposed by Shing & Mahin [4] was adopted. Additional information and details about the seismic response are reported elsewhere [5, 6].

Perhaps unexpectedly, preliminary elastic tests on the infilled frames did not show any appreciable systematic error. This is due to their noticeable stiffness, compared to that of the steel columns used to calibrate the PD system (sec 2). $k_{11}$ in tab 2 is the initial stiffness of the virgin specimens in the first PD test; such a stiffness proved to be able to prevent overshooting. Nevertheless, table 2 also shows how much the stiffness decreases as the

<table>
<thead>
<tr>
<th>Spec. name</th>
<th>Weak member</th>
<th>Span (cm)</th>
<th>Rebar type</th>
<th>Beam reinforcement</th>
<th>Column reinforcement</th>
<th>Frame infilled</th>
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†Not in the second PD test.

Figure 3: Specimens before (on the left) and under (on the right) PD testing.
Table 2: Stiffness of the specimens (kN/mm).

<table>
<thead>
<tr>
<th></th>
<th>C1</th>
<th>C2</th>
<th>L1</th>
<th>L2</th>
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<th>N2</th>
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<td>142</td>
<td>144</td>
<td>123</td>
<td>186</td>
<td>154</td>
<td>185</td>
<td>206</td>
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<tr>
<td>$k_{cr}$</td>
<td>61</td>
<td>44</td>
<td>45</td>
<td>57</td>
<td>41</td>
<td>49</td>
<td>22</td>
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<td>27</td>
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<tr>
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<td>5.8*</td>
<td>11</td>
<td>5.9</td>
<td>10</td>
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<td>6.9*</td>
<td>5.7</td>
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<tr>
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<td>4.2*</td>
<td>5.3</td>
<td>3.8</td>
<td>5.0</td>
<td>2.1</td>
<td>2.9</td>
<td>4.1*</td>
<td>4.2</td>
<td>3.8*</td>
<td>4.1</td>
<td>4.7</td>
</tr>
</tbody>
</table>

*Pertaining to the frame tested bare.

Seismic damage occurs. $k_{cr}$ is the stiffness after cracking of the infill during the first PD test; $k_{2i}$ is the initial stiffness of the damaged specimens in the second PD test; $k_{2f}$ is the final stiffness at the end of the same test, not so greater than that of the aforementioned steel columns.

Information about stiffness decrease was not known before. The PD tests performed first, those on the weak-beam specimens (C1 to N2, tab 1), were carried out disregarding any systematic error. Then data for assessing the overshooting in the case of infilled frames were obtained. The proper correction was statistically calibrated and applied to the subsequent PD tests on the weak-column specimens. This is discussed in the next sections.

4 Performance indices

Several parameters have been proposed to detect and assess the errors in the PD tests [7]. They can be readily used with any integration algorithm, provided both the target displacement $d_c$ and the true one $d_{tn}$ (fig 2) are recorded. Such indices are reported here and illustrated for the N2 and V22 specimens. These specimens exhibited a similar seismic response, apart from the noticeable strength deterioration of the latter (fig 4) due to

![Figure 4: Load-displacement loops of the N2 and V22 specimens.](image-url)
its weaker columns (tab 1). Hence the effect of the overshooting correction, applied to the latter specimen but not to the former, can be seen. The so-called position error at time step \( i \) of the response is defined as:

\[
d_{ci} = d_{mi} - d_{ci}
\]  

(1)

Its time history may convey both the size (e.g., based on the peak-to-peak amplitude) and the nature, random or systematic, of the error. In fact, a square wave denotes systematic, overshooting or undershooting errors, whose values are almost constant and signs depend on the velocity (fig 2). This is what is seen for the N2 specimen only in the second half of the initial PD test (fig 5), and from beginning to end in the second test, not shown here for space limitation. It follows that as the stiffness decreases due to seismic damage (tab 2), the overshooting error arises. On the face of it, the error in fig 5 seems to be at least not greater than that reported with the use of hydraulic jacks [3, 7].

Some systematic error in the PD test of the N2 specimen is confirmed by a frequency analysis through the power spectral density function (PSD):

\[
PSD\{\cdot\} = \frac{|\text{FFT}\{\cdot\}|^2}{T}
\]  

(2)

![Figure 5: Time history of the position error in the first PD test.](image)

![Figure 6: PSDs of the displacement (on the left) and position error (on the right) vs. period in the first PD test.](image)
where $\text{FFT}\{\cdot\}$ denotes the discrete Fourier transform and $T$ is the duration of the seismic response. It is shown in fig 6 that, given an almost equal period in the displacement $d_c$ of the specimens under consideration, the same period is clearly found in the position error $d_e$ of the N2 specimen only, in fact resembling a square wave as written before.

The position error and PSD do not make the presence of systematic overshooting, rather than undershooting, so clear as the cumulative error does. This is defined as follows:

$$S_{e_i} = \sum_{j=1}^{i} d_{e_j} \text{sign}\{d_{c_j}\}$$  \hspace{1cm} (3)

where $d_{c_j}$ is the velocity at time step $j$. On the basis of eqn (1), a positive growing cumulative error has to be associated with systematic overshooting. That applies to the infilled-frame specimens (fig 7). One notes how overshooting starts at about two seconds in the first PD test, when the infill cracked and the stiffness largely decreased. Differently, overshooting is present from the beginning in the second PD test on the N2 specimen, being in effect damaged and less stiff from the start.

Figure 7: Time history of the cumulative error.

Figure 8: Time history of the energy error (on the left) and absorbed energy according to the target displacement (on the right).
A performance index more suitable than the previous ones for quantifying the effect of the error on the results is the energy error, herein defined as:

\[ E_{ei} = E_{mi} - E_{ci} \approx \sum_{j=1}^{i} \frac{r_{j-1}^* + r_{j}^*}{2} (d_{e,j} - d_{e,j-1}) \] (4)

where \( E_m \) (\( E_c \)) is the absorbed energy calculated with the actual (target) displacement, and measured restoring force. The comparison of this index with \( E_m \) or \( E_c \) suggests how much the error weighs on the results (fig 8). Obviously, the exact assessment is not possible, as the absorbed energy in the absence of the error propagation remains unknown. In the case of elastic tests, one can conveniently refer to the maximum deformation energy.

5 Error assessment

As regards the weak-beam specimens, the mean \( m\{\cdot\} \) of the position error in absolute value is separately plotted in fig 9 for each specimen, test, and stage of the tests. The lowest values are observed within two seconds of the first PD test, i.e. with undamaged specimens. The systematic overshooting error relevant to the damaged infilled specimens in the first test is identified with 0.0075 mm on average (see the horizontal dashed line in fig 9). Such value was assumed for halting the jack early during all the subsequent tests on the weak-column specimens, provided the infill, if any, had cracked.

The mean and standard deviation \( s\{\cdot\} \) of the position error pertaining to all the specimens, over entire tests now, is shown in fig 10. The dashed lines denote the averages over the infilled specimens with weak beam or column, separately, in the first PD test. The effect of the correction adopted for the latter specimens is seen as a smaller mean with a similar deviation. The dotted lines denote the averages irrespective of infill and test. The correction adjusted with the infilled specimens in the first test only is found not to impair the whole results, as it yields a very like mean, and greater deviation meaning weaker systematic nature of the error.

![Figure 9: Mean of the position error in the weak-beam specimens.](image-url)
Figure 10: Statistics of the position error.

In order to appraise the weight of overshooting, the energy error at the end of each PD test is normalised by both the fictitious deformation energy associated with the maximum displacement undergone and related restoring force:

$$E^*_c = \frac{1}{2} \cdot |d_c|_{\text{max}} \cdot |r(|d_c|_{\text{max}})|$$  \hspace{1cm} (5)

and the final absorbed energy $E_c$. Such ratios are plotted in figs 11 and 12, respectively. The dotted lines denote the averages over all tests without or with the error correction, as before. The improvement in the latter case is evident. Nonetheless, the results are acceptable in any case: the percentages on $E^*_c$ are a few units (fig 11), those on $E_c$ are less than one (fig 12). The numerical simulation of the seismic response of the weak-beam specimens otherwise confirms the accuracy of the results [5].

6 Conclusions

The error connected with the use of electric screw jacks in PD testing on infilled RC frames has been examined. The relatively high stiffness does not pose a problem, being the displacement measured with accurate resolution. However, it has been found that a systematic overshooting error arises as the stiffness decreases owing to seismic damage. This error has been quantified on a statistical basis. Constantly advancing the halting command to the jack simply mitigates its effect, as several performance indices show. More rationally, one should consider that overshooting varies with the stiffness; the proper correction should progressively increase during the test.
Figure 11: Energy error normalised by the maximum-displacement energy.

Figure 12: Energy error normalised by the absorbed energy.

References


