Sensitivity analysis and optimisation of thermo-elasto-plastic process with applications to welding side heater design

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Abstract

A computational scheme is presented to optimize the design variables of the quasi-static weakly coupled thermo-elasto-plastic process in the three dimensional Lagrangian reference frame. Sensitivity formulations are developed using direct differentiation method. These formulations are used to optimize the dimensions of side heaters in the transient thermal tensioning welding process for minimum residual stress. The results of the direct sensitivity analysis are validated by comparing with those of the finite difference sensitivity analysis. Optimization is performed using the BFGS line search method.

1 Introduction

Numerical optimization methods require the evaluation of sensitivities of the solutions with respect to each design variable. Sensitivity analysis has been widely used for many design optimization problems [1]. Sensitivity analysis can be performed by analytical or by finite difference techniques [2]. The analytical methods give more accurate results and are computationally more efficient than the finite difference method. The analytical sensitivities can be computed either by direct differentiation or by the adjoint method [1]. The direct differentiation method is computationally more efficient than the adjoint method if the problem
has more constraints than design variables.

Sensitivity analysis for coupled systems is presented in reference [3]. In coupled problems, the sensitivity analysis for the thermal problem is performed initially, and the results are imported to the mechanical sensitivity analysis. Sensitivity analysis for the thermo-elastic-plastic process in two dimensional frame with the assumptions of generalized plane strain has been implemented in minimizing welding residual stress and distortion [4]. Sensitivity analysis for the thermo-elastic-plastic process in Eulerian reference frame has been developed in optimizing laser forming process [5]. These two approaches have critical assumptions in their formulations. The two dimensional approach is limited in explaining three dimensional effects, in spite of the generalized plane strain assumption. The Eulerian approach is applicable only for the steady-state process. Therefore, it is necessary to develop sensitivity analysis of thermo-elastic-plastic process in three dimensional Lagrangian reference frame.

In this paper, conventional finite element formulations for the thermo-elastic-plastic analysis in the three dimensional Lagrangian reference frame are reviewed. Then, thermo-elastic-plastic sensitivity equations are developed using the direct differentiation method. The sensitivity formulations are verified by comparing with the results obtained by the finite difference sensitivity analysis. The sensitivity equations are implemented to a welding optimization example. The design variables of side heaters in the fillet welding process are optimized for the minimum residual stress.

2 Analytical formulation

Finite element formulations for quasi-static thermo-elastic-plastic processes in Lagrangian reference frames have been widely used [6, 7, 8, 9, 10]. The thermal analysis is assumed to be transient while the elastic-plastic quasi-static Thermoplastic processes are typically assumed to be weakly coupled, that is, the temperature profile is assumed to be independent of stresses and strains. Thus a heat transfer analysis is performed initially, and the results are imported for the mechanical analysis.

2.1 Transient thermal analysis

For a spatial frame $x$ fixed to the body, and time $t$, the governing equation for transient heat conduction analysis is given by the following:

$$\rho C_p \frac{dT}{dt} = \nabla \cdot [k \nabla T] + Q \quad \text{in the entire volume } V \text{ of the material} \quad (1)$$
where $\rho$ is the density of the flowing body, $C_p$ is the specific heat capacity, $T$ is the temperature, $k$ is the temperature-dependent thermal conductivity matrix, $Q$ is the internal heat generation rate and $\nabla$ is the spatial gradient operator.

The initial temperature field $T^0$ is prescribed in the entire volume $V$. The boundary conditions on the surface are $T = T^p$ on the surface $A_T$ and $q^s = q^p$ on the surface $A_q$. The surface flux $q^s$ is evaluated by taking the component of the heat flux $q$, normal to the surface, therefore, $q^s = q \cdot \hat{n}$ on any surface $A$ where $\hat{n}$ is the unit outward normal to the surface $A$.

### 2.2 Mechanical analysis

\[
\nabla \cdot S + b = 0 \quad \text{in } V
\]

where $S$ is the second-order stress tensor, and $b$ the body force vector. The boundary conditions are:

\[
\begin{align*}
\mathbf{u} &= \mathbf{\bar{u}} & \text{on surface } A^u \\
S \mathbf{n} &= \mathbf{\bar{t}} & \text{on surface } A^t
\end{align*}
\]

where $\mathbf{\bar{u}}$ are the prescribed displacements on surface $A^u$, $\mathbf{\bar{t}}$ are the prescribed tractions on surface $A^t$, and $\mathbf{n}$ is the unit outward normal to the surface $A^t$. The total strain is the Green’s strain:

\[
E = \frac{1}{2} \left\{ \nabla \mathbf{u} + [\nabla \mathbf{u}]^T \right\}
\]

Thanks for the symmetry, the stress tensor $S$ and strain tensor $E$ are commonly expressed in the vector form $\mathbf{\sigma}$ and $\mathbf{\varepsilon}$ (usually called engineering stress and strain) for the computational efficiency. Then the initial conditions are:

\[
\begin{align*}
\mathbf{u} &= \mathbf{u}^0 \\
\mathbf{\varepsilon}_p &= \mathbf{\varepsilon}_p^0 \\
\mathbf{\varepsilon}_q &= \mathbf{\varepsilon}_q^0
\end{align*}
\]

where $\mathbf{\varepsilon}_p$ is the plastic strain vector and $\mathbf{\varepsilon}_q$ is the equivalent plastic strain.

Assuming small deformation thermo-elasticity, the total strain vector $\mathbf{\varepsilon}$ is decomposed into the elastic strain vector $\mathbf{\varepsilon}_e$, $\mathbf{\varepsilon}_p$ and thermal strain vector $\mathbf{\varepsilon}_t$:

\[
\mathbf{\varepsilon} = \mathbf{\varepsilon}_e + \mathbf{\varepsilon}_p + \mathbf{\varepsilon}_t
\]
The stress strain relationship is:

$$\sigma = C \epsilon_r = C [\epsilon - \epsilon_p - \epsilon_I] \quad (10)$$

where $C$ is the temperature-dependent material stiffness tensor.

Through the finite element formulations, the element residual $R$ is obtained as follows

$$R(\mathbf{U}) = \sum_{V} \left[ B^T \mathcal{N} \sigma - N^T b \right] W J - \sum_{A^c} N^T \bar{t} w_j \quad (11)$$

where

$$n \sigma = n^{-1} \sigma + \Delta \sigma \quad (12)$$

Differentiating equation (10) using incremental expression yields following equations

$$\Delta \sigma = n^{-1} \sigma \left[ \Delta \epsilon - \Delta \epsilon_p - \Delta \epsilon_I \right] + \Delta C n^{-1} \epsilon_c \quad (13)$$

The elastic predictor $\sigma_B$ and corresponding elastic strain $\epsilon_{Be}$ are defined as follows

$$\sigma_B = n^{-1} \sigma + n^{-1} C \left[ \Delta \epsilon - \Delta \epsilon_I \right] + \Delta C n^{-1} \epsilon_c \quad (14)$$

$$\epsilon_{Be} = n^{-1} \epsilon_c + \left[ \Delta \epsilon - \Delta \epsilon_I \right] \quad (15)$$

Using the associative $J_2$ plasticity [11], the yield function $f$ is:

$$f = \sigma_m - \sigma_Y \quad (16)$$

where $\sigma_m$ and $\sigma_Y$ are the Mises stress and yield stress. Active yielding occurs when $f \geq 0$. In case of active yielding, the evolution of $\Delta \epsilon_q$ can be evaluated by the radial return algorithm [6].

$$\Delta \epsilon_p = \Delta \epsilon_q a \quad (17)$$

where

$$a = \frac{3}{2} L m \quad (18)$$

$$m = \frac{1}{\sigma_{Bn}} s_B \quad (19)$$

$$L = \text{diag}(\ 1\ 1\ 1\ 2\ 2\ 2\ ) \quad (20)$$

where $\sigma_{Bn}$ and $s_B$ are the Mises stress and the deviatoric stress of the elastic predictor $\sigma_B$. 
3 Sensitivity analysis from the method of direct differentiation

3.1 Thermal sensitivity

The formulation of the thermal analysis sensitivities follows the derivation in [9] and is not repeated here for conciseness.

3.2 Mechanical sensitivity

Differentiating the global residual equation with respect to each design variable \( \phi_i \) yields

\[
\frac{dU}{d\phi_i} = - \left\{ \frac{dR}{dU} \right\}^{-1}_{\text{constant } \phi_i} \frac{DR}{D\phi_i}
\]

where \( D \) stands for the variance due to all variables but \( U \). Similar to the thermal sensitivity formulation, the only term that needs to be evaluated for displacement sensitivity is \( \frac{DR}{D\phi_i} \), which is:

\[
\frac{DR}{D\phi_i} = \sum_{\nu} \left[ \frac{dB^T}{d\phi_i} \sigma w J + B^T \frac{D\sigma}{D\phi_i} w J + B^T \sigma w \frac{dJ}{d\phi_i} \right]
\]

In case of non-active yielding:

\[
\frac{D\sigma}{D\phi_i} = \frac{D\sigma_B}{D\phi_i}
\]

In case of active yielding, \( \sigma \) can be expressed as follows from the radial return algorithm

\[
\sigma = \sigma_h + \sigma_Y m
\]

where \( \sigma_h \) is the hydrostatic stress of \( \sigma \). Then \( \frac{D\sigma}{D\phi_i} \) is evaluated as follows

\[
\frac{D\sigma}{D\phi_i} = \frac{D\sigma_h}{D\phi_i} + \sigma_Y \frac{Dm}{D\phi_i} + m \frac{D\sigma_Y}{D\phi_i}
\]

4 Numerical implementation

The transient thermal tensioning also known as side heating technique can be used to control the welding distortion and residual stress without modifying design specification [12]. In this work, the heat source and positions of side heaters are optimized with other variables fixed for the minimum residual stress using the sensitivity equations developed in the previous section in a 3D Lagrangian reference frame. No constraints are considered in this example.
4.1 Welding conditions

The schematic welding configuration in this simulation is shown in the Fig 1. Side heaters are followed by two welding torches. Convection boundary conditions are assigned for all free surfaces. The internal heat generation rate by the welding torch, modeled with a "double ellipsoid” heat source model [13], is given as,

\[
Q = \frac{6\sqrt{3}Q_w\eta_w\bar{f}}{abc\pi} e^{-\left(\frac{3a^2}{a^2} + \frac{2b^2}{b^2} + \frac{2c^2}{c^2}\right)} \quad [W/mm^3] \quad (26)
\]

where \(Q_w\) (2680.35 W/mm\(^3\)) is the welding heat input; \(\eta_w\) (1.0) is the welding efficiency, \(x\), \(y\), and \(z\) are the local coordinates of the double ellipsoid model aligned with the weld fillet; \(a\) (5\(\sqrt{2}\) mm) is the weld width; \(b\) (5\(\sqrt{2}\) mm) is the weld penetration; \(c\) is the weld ellipsoid length; \(v\) (6.35 mm/s) is the torch travel speed. The side heat source is applied on the top surface of the plate as shown in Fig 1 and is defined as follows

\[
\bar{q}(x, z) = \frac{Q_w\eta_w}{2B_s L_s} M_x M_z \quad (27)
\]
<table>
<thead>
<tr>
<th>Constrained point</th>
<th>Displacement constrained direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>X Y Z</td>
</tr>
<tr>
<td>P2</td>
<td>X</td>
</tr>
<tr>
<td>P3</td>
<td>X Y</td>
</tr>
</tbody>
</table>

Table 1: Boundary conditions for the mechanical analysis (see Figure 1 for P1, P2 and P3).

\[
M_x = \left\{ \tanh(S_{x2}[x + \phi_2 + B_s/2]) - \tanh(S_{x1}[x + \phi_2 - B_s/2]) \right. \\
+ \left. \tanh(S_{x1}[x - \phi_2 + B_s/2]) - \tanh(S_{x2}[x - \phi_2 - B_s/2]) \right\} / 2   \tag{28}
\]

\[
M_z = \left\{ \tanh(S_{z1}[z - L_s/2]) - \tanh(S_{z2}[z + L_s/2]) \right\} / 2 \tag{29}
\]

where \(x\) and \(z\) are the local coordinates from the center of the side heating; \(Q_s (W/mm^2)\) is the side heating input, \(\eta_s (1.0)\) is the side heating efficiency; \(B_s \) \( (6") \) and \(L_s (1") \) are the band width and length of the side heating, \(S_{x1} \) \( (0.2) \), \(S_{x2} \) \( (0.2) \), \(S_{z1} \) \( (0.2) \) and \(S_{z2} \) \( (0.2) \) are used to control the gradient of heat flux in the side heater edges. The numbers in the parentheses are the values which are used in this simulation. Material properties of A36 steel used in this simulation [14]. The isotropic hardening coefficient is assumed to be 8000 \( [MPa] \) at any temperature.

A finite element model is developed. The dimensions are \(12" \times 12" \times 1/8"\) for base plate and \(12" \times 2" \times 1/8"\) for the stiffener. This model has 13864 nodes and 2352 20-noded brick elements. Since high temperature gradients are prevalent at the welding region, the mesh is finer along the welding torch path and coarser away from it. The boundary conditions for the mechanical analysis are shown in Table 1.

4.2 Optimization

Since the residual longitudinal compressive stress away from the weld zone can be used as a criterion of welding induced buckling [12], the optimization problem is expressed as follows

\[
\min F = \min \sum_e (l_e^e \sigma_{zz}^e)^2 \tag{30}
\]

\[
(\phi_i)_{min} \leq \phi_i \leq (\phi_i)_{max} \tag{31}
\]
where $\sigma_{zz}^e$ is the centroid longitudinal residual stress at element $e$ in the objective region shown in Figure 1, and $l_e^e$ is the x-direction length of the element. The gradient of the objective function $F$ is obtained as follows.

$$\frac{\partial F}{\partial \phi_i} = 2 \sum_e (l_e^e)^2 \sigma_{zz}^e \frac{\partial \sigma_{zz}^e}{\partial \phi_i}$$

(32)

The Design variables are the side heat source $Q_s(=\phi_1)$, transverse position of the side heater $\phi_2$ and the distance between the side heater and the first welding torch $\phi_3$ as shown in Equation (27) and Figure 1.

The optimization loop is implemented using the BFGS line search method provided in the DOT package [15].

4.3 Results

The results of the numerical optimization are summarized in Table 2. The total analysis time for each side heating configuration is set up to 3000 seconds for both the thermal and mechanical analyses because temperature distribution becomes uniform and the residual stress distribution shows no more change after that time.

Figure 2 shows the longitudinal residual stresses along the "Center Line" (see Figure 1) for three cases. The residual stress in the objective region is successfully reduced for the optimum side heater. The vertical dotted-line in the left side from the axis line indicates where the objective region start.

5 Conclusions

Direct sensitivity formulations for the thermo-elastic-plastic processes in three dimensional Lagrangian reference frame have been developed. The sensitivity formulations are successfully implemented in an optimization procedure to
determine optimal side heater dimensions for minimum welding residual stress in the transient thermal tensioning process.

References


