The combination of numerical simulations and experiments in dynamic material research

P. Verleysen, J. Degrieck
Faculty of Engineering, Department of Mechanical Construction and Production, Ghent University, Belgium

Abstract

In civil engineering structures quasi brittle materials, such as concrete and some composites, are often used. In most applications these materials are loaded dynamically. Consequently, the study of the dynamic behaviour of quasi brittle materials is of major importance. Based on a series of experiments, a material model to describe the dynamic tensile behaviour of quasi brittle materials was developed within the framework of damage mechanics. The constitutive equations describing the dynamic material behaviour have been implemented in a finite element program. The equations contain some material parameters. These material parameters are determined by means of a combined experimental-numerical technique. The necessary experimental results are obtained by means of split Hopkinson bar tests. Although, the test technique has some limitations, certainly when testing quasi brittle materials, Hopkinson tests are used very often in high strain rate material testing, because practical test execution is rather simple, and because Hopkinson type tests give rise to relatively easy interpretation of test results. An automatic procedure has been worked out. With guess-values of the parameters a numerical simulation of a Hopkinson experiment is performed. Well-chosen results of the simulation are compared with experimental signals, a new set of parameters is proposed, a new simulation performed, the results are again compared with the experiment. The parameters are adapted just until sufficient agreement between the experiment and the simulation is obtained.

It has been found that the combination of experiments and numerical simulations of the complex stress states and time histories involved in Hopkinson tests is an adequate and essential tool for material modelling.
1 Introduction

In medium to high strain rate (strain rates from 1 to 5000 s\(^{-1}\)) material testing (Davies [1]) split Hopkinson bar setups (Hopkinson [2]) (or Kolsky apparatus (Kolsky [3][4])) are often used. This is because practical test execution is relatively simple and because Hopkinson tests give rise to straightforward interpretation of the test results (Ruiz [5], Zhao [6]).

The test method was originally designed for metallic materials; materials showing large deformations upon rupture. Under some conditions the stress and the deformation in metallic specimens during an experiment are assumed to be uniaxial and homogenous. As can be proved, in that case the history of stress, strain and strain rate in the specimen during an experiment can be obtained very easily. However, for non-metallic materials the test technique has some limitations. When used for (quasi) brittle materials the experimental setup and the extraction of the signals have to be adapted. One reason for this is the low accuracy of the test method in the region of small strains (Verleysen [7]). Another reason is a consequence of the softening behaviour exhibited by quasi brittle materials; the deformation will not be homogenous, but will be highly localised in a small zone of the specimen. Information concerning the actual distribution of stress and deformation along the specimen can in that case be obtained by measuring on the specimen itself to obtain the deformation over the length of the specimen or by performing a wave analysis in the specimen by means of numerical simulations. We chose the latter method. By means of the finite element method we simulated the deformations and stresses in the specimen. Time histories of stresses and strains at different places in a specimen were obtained by means of dynamic simulations with an appropriate material model and well-determined material constants.

In this contribution a short review of the split Hopkinson bar test technique is given. Then the constitutive equations used to describe the dynamic tensile behaviour of quasi brittle materials are given. Subsequently, attention is focussed on the extraction of the material constants.

2 Test method

2.1 Experimental setup

Figure 1 provides a schematic representation of a typical split Hopkinson bar setup for tensile testing. The setup consists of two long bars, an input bar and an output bar, between which a specimen is sandwiched. In the setup used for our experiments, the input bar has a length of 2.25 m, the output bar is 1.45 m long. Both Hopkinson bars are aluminium bars with a diameter of 25 mm. The choice
of the Hopkinson bar material and dimensions is based on numerical simulations (Verleysen [8]).

The anvil at the outer end of the input bar is hit by an impactor, which is generally pneumatically accelerated. A tensile strain wave $\varepsilon_i$, the so-called incident wave, is thus generated and propagates along the input bar towards the specimen. Upon reaching the specimen the wave is partly reflected back to form the reflected wave $\varepsilon_r$ and is partly transmitted to form the transmitted wave $\varepsilon_t$. The strains associated with the waves $\varepsilon_i$, $\varepsilon_r$ and $\varepsilon_t$ are most often measured by means of strain gauges. These strain gauges are located at well chosen points on the bars, away from the specimen.

2.2 Calculation of the stress, strain and strain rate in the specimen

The strain signals have to be shifted, forward or backward, towards the interface planes with the specimen, in order to obtain forces and displacements at both ends of the specimen. It is very common to assume that the stresses and deformations are uniaxial and homogenous in the specimen (Kolsky [3][4]). In that case the history of the stress $\sigma$, the strain $\varepsilon$ and the strain rate $\dot{\varepsilon}$ in the specimen can be obtained by:

$$\sigma(t) = \frac{A_s E_s}{A_p} \varepsilon_i(t), \quad \varepsilon(t) = -\frac{2 C_s}{L_p} \int_0^t \varepsilon_r(\tau) d\tau, \quad \dot{\varepsilon}(t) = -\frac{2 C_s}{L_p} \varepsilon_r(t)$$

with $A_s$ ($A_p$) the area of a section of the Hopkinson bars (specimen), $E_s$ the modulus of elasticity of the Hopkinson bars, $C_s$ the longitudinal wave propagation velocity in the Hopkinson bars, $L_p$ the specimen length.
The validity of formulas (1) is limited by test conditions, such as the specimen geometry and the length of the incident wave. In some cases the assumptions of homogenous stress and deformation in the specimen are never fulfilled. Such is often the case when testing heterogenous materials as concrete or fibre reinforced materials; after all, the specimen dimensions can’t be arbitrarily reduced, they have to be large compared to the granulate or fibre size.

3 Numerical modelling

In this section an overview of the constitutive equations used to describe the dynamic behaviour of quasi brittle materials is given. More details concerning the material model can be found in [7].

The material model is developed within the framework of damage mechanics (Kachanov [9], Krajcinovic [10], Rice [11]). A damage parameter $D$ is introduced; $D$ describes the gradual degradation of the stiffness the material:

$$E = E_0 (1 - D) , \quad D = 1 - \frac{E}{E_0}$$ (2)

with $E_0$ the stiffness of the undamaged material and $E$ the stiffness reduced by damage of the material. The value of $D$ varies from 0 for the undamaged material to 1 when the stiffness of the material is completely lost. The stress can be calculated from:

$$d\sigma = E_0 (1 - D) d\varepsilon - E_0 \varepsilon d(D)$$ (3)

where $d\varepsilon$ is the strain increment and $d(D)$ the increase of the damage. The damage evolution, described by the damage law, is splitted into two parts: a part describing the nucleation of damage and a part describing the growth of existing damage. For the nucleation of the damage as a function of the stress $\sigma$ the following function is proposed:

$$\dot{D}_{\text{nucleate}} = \frac{\partial D_{\text{nucleate}}}{\partial\varepsilon} = C1 \left[ \left( \frac{\sigma}{\sigma_{\text{max}}} \right)^2 - \left( \frac{\sigma_{\text{stat}}}{\sigma_{\text{max}}} \right)^2 \right] > 0$$ (4)

$C1$ is a material constant. $\sigma_{\text{stat}}$ is the value of the so-called static curve. The static curve gives the relation between the stress and damage during a static tensile test. $\sigma_{\text{max}}$ is the maximum value of the stress reached in a static experiment. During a dynamic experiment the value of the stress $\sigma$ according with a certain damage can be greater than the value of $\sigma_{\text{stat}}$ of the static curve; the point $(D, \sigma)$ is above the static curve. The damage development can’t follow the stress; in that case the damage will grow. The higher the strain rate, the higher the difference between $\sigma_{\text{stat}}$ and $\sigma$, so the higher will be the damage growth. Thus,
although the strain rate does not appear explicitly in the constitutive equations
the proposed material model is strain rate dependent.

The propagation term is quite similar to the nucleation term:

\[
\dot{D}_{\text{propagate}} = \frac{\partial D_{\text{propagate}}}{\partial t} = C7 \cdot \left[ \left( \frac{\sigma}{\sigma_{\text{max}}} \right)^2 - \left( \frac{\sigma_{\text{stat}}}{\sigma_{\text{max}}} \right)^2 \right] \cdot D > 0
\] (5)

C7 is a dynamic material parameter. The total damage growth consists of the
sum of the nucleation (formula (4)) and propagation (formula (5)) term:

\[
\dot{D} = \dot{D}_{\text{nucleation}} + \dot{D}_{\text{propagate}}
\] (6)

The constitutive equations (3) and (6) describing the dynamic material
behaviour have been implemented in a finite element program called
"IMPACT". IMPACT is a one dimensional program using an unconditionally
stable algorithm for the time integration (Verleysen [7]). The program allows
simulation of one-dimensional wave propagation problems as arise in the
Hopkinson experiment: the two Hopkinson bars and the specimen are modelled
and the incident wave \(q(t)\) is applied to the beginning of the input bar.

4 Combined numerical-experimental technique

With numerical simulations knowledge of the dynamic characteristics of the
specimen can only be obtained in an indirect way. Indeed, to simulate the wave
propagation in the specimen during a Hopkinson test, knowledge of the
constitutive equations is necessary. But, in most cases the experiments are
performed to derive these constitutive equations.

This problem can be solved by a so-called "numerical-experimental" method
(Zukas [12], De Wilde [13], Sol [14]). This alternative procedure for the
interpretation of the test results consists generally of the following steps:

1. suitable constitutive equations are assumed with well-chosen start-
   values of the constants,
2. during the experiment as much information as possible is gathered; velocities at the surface, histories of strain, .... Also the boundary conditions have to be precisely known,
3. with the assumed constitutive equations and constants a numerical
   simulation of the experiment is performed,
4. the numerical results are compared with the experimental
   observations. When the agreement is insufficient, step 3 and 4
   have to be repeated with improved values of the constants used in
   the constitutive equations,
5. to verify the obtained constants and constitutive equations, other experimental configurations or other experimental observations can be simulated with the same material law.

The constitutive equations presented in section 3 contain two unknown constants: C1 and C7. With guess-values of those constants a numerical simulation of a Hopkinson experiment is performed. In the simulation the measured incident wave is applied on the beginning of the 1.45 m-long input bar with diameter 25 mm. These dimensions correspond with the real dimensions of the bar. With the considered length the extraction of the reflected wave is straightforward. In line of the first bar a specimen (with a length of 10 mm and diameter of 25 mm) and the output Hopkinson bar is modelled. The length of the output bar in the simulation is 1 m, this is sufficiently long: during the time the incident tensile wave interacts with the specimen no other waves reach the specimen. A length of 1 m also allows the transmitted wave to be extracted from the simulation results in a direct way, without interference with other waves.

The incident wave, the wave reflected by and the wave transmitted through the specimen are measured during a Hopkinson experiment. All information (the strain rate, the strain and the stress in the specimen) derived directly from these signals using formulas (1) is based on assumptions concerning the homogeneity of stresses and deformations in the specimen and thus subject to errors. Therefore it is recommended not to use the stress, the strain or strain rate obtained by these formulas to implement the combined numerical-experimental method, but use directly the transmitted or reflected wave.

The information in the reflected wave is directly connected with the deformation of the specimen (see formulas (1)). Together with the incident wave, the reflected wave allows the extraction of the stress history in the specimen. After all when the stress is homogeneous in the specimen, the following equation gives the same results as equation (1):

\[ \sigma(t) = \frac{A_s E}{A_p} (\varepsilon_i(t) + \varepsilon_r(t)) \] (7)

Since the reflected wave is connected with the deformation and stress in the specimen, we used the reflected wave to optimize the values of the material model parameters. The use of the reflected wave has another important advantage, explained in the next paragraph.

When testing plain microconcrete the process of the degradation of the specimen until fracture consumes very little time (± 50 µsec). After the specimen is broken the incident wave (which has a duration of = 200 µsec) is completely reflected on the fracture surface. Only the part of the reflected wave corresponding to the time during which the specimen is not completely broken
was used to optimise the parameters. The part of the wave after fracture of the specimen can be used to synchronise the simulated and experimentally measured waves. Figure 2 shows those two waves; when \( t > 0.52 \) ms the specimen is broken, between \( t = 0.47 \) ms and \( t = 0.52 \) ms the specimen is degrading. The simulation is performed with start-values of the material parameters. This explains the bad correspondence between the simulated and experimental wave in the beginning.

Figure 3 gives the part of reflected wave used for optimisation of the material constants. The experimental wave, the wave simulated with start values of the parameters, and the result after optimisation of the parameters is presented in this figure. The optimisation of the constants is performed automatically; initial values of the constants are optimised without intervention of the operator. Therefore the program IMPACT for the simulation of the Hopkinson tests is incorporated in ODRPACK (Boggs [15]). ODRPACK is a package of routines written for the iterative optimisation of constants and is based on the least squares method. After a first simulation with guess-values of the material constants a so-called goal function is calculated. This goal function is here the quadratic deviation between the simulated and the experimentally measured reflected wave. Subsequent simulations with adapted values of the constants are performed, every time the goal function is calculated, new values for the constants are proposed, ... that until the goal function reaches an acceptable minimum and thus the agreement between the experiment and the simulation is as good as possible.
Figure 3: Part of the measured reflected wave taken into account for the extraction of the material parameters. Also simulations of that part of the reflected wave are presented; one is performed with start values of the parameter, the other with the values of the parameters after optimisation.

Afterwards, with the finally obtained values of the constants, the simulated transmitted wave is compared with the transmitted wave measured during the experiment.

5 Results

5.1 Material

There are indications that the material model presented in section 3 can be used for quite a lot of quasi brittle materials. It is sufficient to use the static curve for the considered material in equation (4) and (5). Also the combined numerical-experimental technique presented in section 4 is not bounded with a specific material. The material considered for the results of the technique presented in this contribution is microconcrete. Microconcrete is a concrete on scale: the size of the granulates is limited (in our case < 3 mm).

5.2 Parameters

Table 1 gives the values of the constants C1 and C7 extracted from four experiments with the above described automatic procedure. The experiments 1 to 4 were executed with different histories of the strain rates during the
Figure 4:
Simulated and experimental history of the stress as a function of the deformation in a specimen during a split Hopkinson bar tensile experiment.

As can be seen the value for C1 and C7 does not vary much from experiment to experiment. The final simulations to obtain the stresses, strains, strain rates, damage, ... in the specimen were performed with mean values of C1 and C7.

<table>
<thead>
<tr>
<th>experiment</th>
<th>C1</th>
<th>C7</th>
</tr>
</thead>
<tbody>
<tr>
<td>exp1</td>
<td>11777</td>
<td>45358</td>
</tr>
<tr>
<td>exp2</td>
<td>10676</td>
<td>48321</td>
</tr>
<tr>
<td>exp3</td>
<td>13727</td>
<td>37461</td>
</tr>
<tr>
<td>exp4</td>
<td>14727</td>
<td>53823</td>
</tr>
<tr>
<td>mean value</td>
<td>12726</td>
<td>46240</td>
</tr>
</tbody>
</table>

Table 1:
Values for C1 and C7 obtained with the automatic extraction procedure for four experiments.

Figure 4 gives the stress as a function of strain obtained with the equations (1) and the simulated curve. As can be seen, the agreement between the numerical simulation and the experimental result is very good. More simulations and more details of these simulations are presented in (Verleysen [7]).
6 Conclusions

In this contribution a combined numerical-experimental method is presented. It has been found that the combination of experiments and numerical simulations is an adequate and essential tool for material modelling. The technique allows an optimal use of experimental data.

The necessary experimental results are here provided by split Hopkinson bar tests. After a short description of the test method, the constitutive equations needed to model the dynamic behaviour of some quasi brittle materials are given.

The connection between experiments and simulations allows determination of material model parameters. An algorithm was worked out to optimise the parameters automatically. The results of this algorithm are given for a microconcrete. With the so-determined values of the parameters an excellent agreement between experiments and simulations is obtained. So, it can be concluded that the presented constitutive equations with the material parameters give a valid description of the material behaviour for the considered range of strain rates.

Once the parameters are determined the numerical simulations provide a valuable source of information; information of the material behaviour can be obtained in other circumstances than the experiment.

7 References


