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Internal shape factor of straight wires sheathed electrical heating elements

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Abstract

This paper focuses on the study of the internal shape factor of sheathed electrical heating elements. In the first of the study, the geometries are presented and three dimensionless parameters are introduced. Then examples of temperature fields are given. The analysis of finite elements simulations leads to the proposition of an analytical expression of the dimensionless internal shape factor. The use of shape factors avoid confidentiality problems. During this analysis a forth dimensionless shape factor is introduced. Taking advantage of the simulations, a non uniformity factor is proposed and it is shown that this non uniformity is in all cases inferior to 16%, and in most cases inferior to 2%.

In the second part of the study, considering that experimental results have led to the determination of the thermal conductivity of the insulation material, the proposed relation is applied to two examples, one determining the maximum heat flux for a given temperature, the second one determining the maximum temperature of the heating wire for a given heat flux.

These two examples show that standard coiled wires heating elements are more powerful than straight wires heating elements when standard insulation material is used.

1 Introduction

Sheathed electrical heating elements are widely used in many processes (liquid heating, gas heating, infrared heating, ...) as they are robust in a large temperature range. When the final user is not directly concerned by the internal temperature distribution, he is looking for a long life of the heating element. This can be achieved by the manufacturer of the element by carefully choosing the diameter of the heating wire. This becomes important when the service temperature of the sheathed element becomes high (about 1,000 °C). The two main techniques that are used to manufacture sheathed electrical heating elements are presented in figure 1.

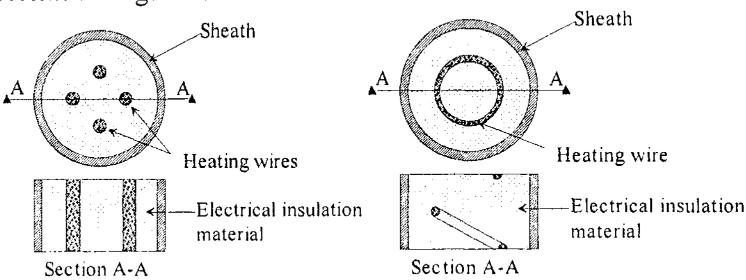

Fig. 1: sheathed heating element with straight or coiled wires

The most used wires are the coiled wires. But when the heat fluxes are high, it is believed that one has to move to straight wires. Only few papers deal with the temperature distribution in electrical heating elements [1] [2] [3] [4], and none of them deal with straight wires heating elements. This can be due to the fact that this distribution strongly depends on the value of the thermal conductivity of the electrical insulation material. This value depends on the final value of the density of the material, and so depends on the way this material is compacted. It is possible to measure the thermal conductivity by comparing experimental data (e.g. the temperature difference between the axis of the element and its surface for a known heat flux) and computed results. This has been done for few different types of elements and at different locations (straight parts and bends), but the results are confidential. To avoid such confidentiality problems, instead of giving the internal thermal resistance, a dimensionless internal shape factor will be computed. This dimensionless shape factor is defined as the ratio of the actual shape factor to the shape factor of a heating element that would consist of a continuous cylindrical heating part.

In the case of straight wires, 2D numerical simulations can accurately represent the actual phenomenon. In this study, the geometries will not include the sheath; the thermal resistance of the cylinder being well known. But the applications will take it into account.

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2 Theoretical development

2.1 Description of the tested geometries

Figure 2 shows the different geometrical parameters that influence the temperature field in the sheathed element.

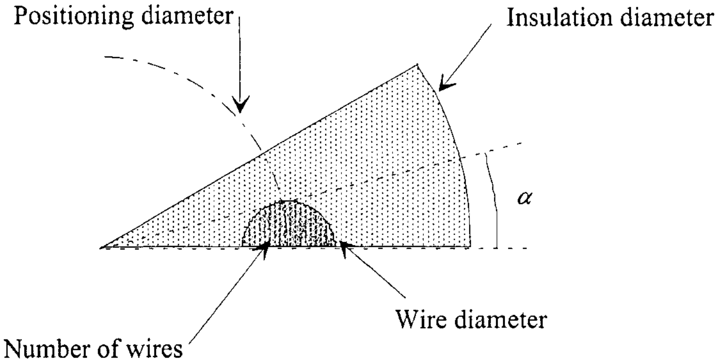

Fig.2 : Influential geometrical parameters

Other parameters such as the thermal conductivity of the insulation material and the thermal conductivity of the sheath influence the temperature field. But as only the internal shape factor is looked for, these parameters will not be studied. They are taken into account in the examples that are presented below.

Figure 3 summarizes the values that have been studied.

Positioning diameter (mm)	4						
Diameter of the insulation (mm)	8	10	12	14	16	18	20
Wire diameter (mm)	0.2	0.4	0.6	0.8	1		
Number of wires	2	3	4	6			

Fig. 3 : Numerical values of the geometrical parameters

Combining all the values of the geometrical parameters makes a set of 140 geometries. All these geometries have been simulated using the finite element software Systus+ [5]. The geometries and their resolution have been prepared using a Visual Basic program. Figure 4 shows examples of temperature fields.

Fig. 4 : Examples of temperature fields in a sheathed element

It is legitimate to think that the dimensionless shape factor will not directly depend on the parameters, but will only depend on geometrical ratios. This explains that only one positioning diameter has been studied.

It is possible to introduce 3 dimensionless geometrical parameters:

$$G1 = \frac{\textit{diameter of the insulation material}}{\textit{positioning diameter}} = \frac{D_i}{D_p},$$

$$G2 = \frac{\textit{diameter of the insulation material}}{\textit{wire diameter}} = \frac{D_i}{D_w},$$

$$G3 = \textit{dimensionless angle} = \frac{\cos^{-1}\left(1 - \frac{1}{2}\left(\frac{D_w}{D_p}\right)^2\right)}{\frac{\pi}{n_w}} = \frac{\alpha}{\frac{\pi}{n_w}}.$$

The aim of this study is to give an analytical expression of the internal shape factor as a function of these 3 dimensionless geometrical parameters.

2.2 Analysis of the simulations

As it has been previously said, 140 simulations have been carried out. For all these simulations, 4 temperatures have been extracted from the results:

- the mean temperature on the outer side of the insulation material $T_{mean\ outer\ side} = \frac{1}{n_{nodes\ outer\ side}} \sum T_{node\ outer\ side}$,
- the maximum temperature on the outer side of the insulation material $T_{max\ outer\ side} = Max(T_{node\ outer\ side})$,
- the minimum temperature on the outer diameter of the insulation material $T_{min\ outer\ side} = Min(T_{node\ outer\ side})$,
- the maximum temperature of the wire $T_{max\ wire} = Max(T_{node\ wire})$.

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Knowing the heat Q dissipated by the wire (normally by Joule effect), it is easy to compute a shape factor:

$$S = \frac{Q}{k_i\left(T_{max\ wire} - T_{mean\ outer\ side}\right)}. \tag{1}$$

The mean temperature on the outer diameter has been chosen because it is the one that is easy to calculate during service. Usually, the convection coefficient between the sheathed element and the fluid to be heated is known. Knowing the temperature of the fluid and the amount of heat per unit surface to be transferred, it is easy to calculate the mean surface temperature of the element. Knowing the thickness of the sheath and its thermal conductivity, it is then possible to calculate the mean temperature of the outer diameter of the insulation material. The maximum temperature of the wire has been chosen because this is this temperature that determines if there is any danger that the wire melts. Then, this shape factor is compared to the shape factor of a sheathed element that would consists of a continuous heating cylinder:

$$S_0 = \frac{2\pi}{Ln\left(\frac{D_i}{D_p}\right)} = \frac{2\pi}{Ln(G1)}. \tag{2}$$

The comparison is made through the calculation of the dimensionless shape factor defined as follows:

$$\bar{S} = \frac{S}{S_0} = \frac{Q\,Ln(G1)}{2\pi\,k_i\left(T_{max\ wire} - T_{mean\ outer\ side}\right)}. \tag{3}$$

The first idea is to look for a dependence of the dimensionless shape factor on the ratio of the diameters $G1$ (Fig. 5). It can clearly be seen that there is no straight dependence and that the dimensionless shape factor surely depends more on the other dimensionless parameters.

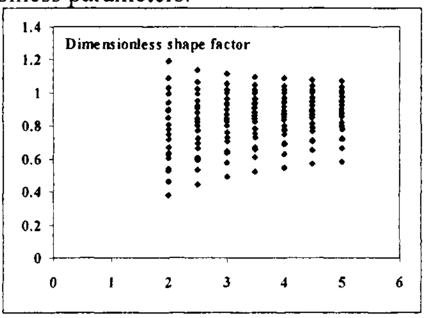

Fig. 5 : Evolution of the dimensionless shape factor versus $G1$

It is then necessary to find another dimensionless parameter. The dimensionless angle could be the next parameter. But figure 6 suggests that it could not be sufficient; the wire being to close to the outer side of the insulation.

Fig. 6 : Configurations with the same dimensionless angle

The idea is to plot the evolution of the dimensionless shape factor versus the dimensionless angle corrected by the parameter G1 (Figure 7). This introduces a forth dimensionless factor:

$G4 = G3\,G1$.

For clarity reasons, the results are presented separately for the 4 values of the number of wires.

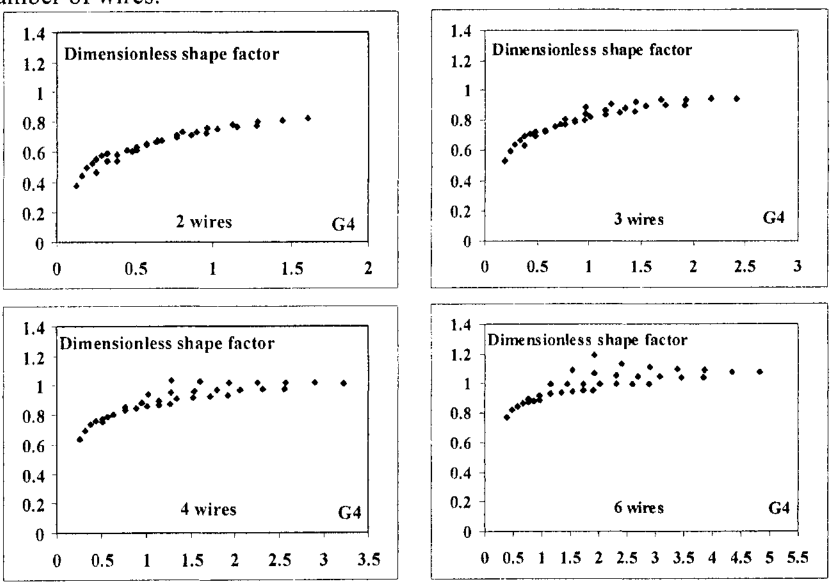

Fig. 7 : Evolution of the dimensionless shape factor versus parameter G4

It can be seen that there is a large difference between the actual shape factor and the one that could be calculated assuming that the heating element is a continuous cylinder.

To analytically represent the results the following equation is proposed (solid lines in Figure 9):

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$$\bar{S} = (a\,G4)^{b} + c\,, \qquad (4)$$

with a, b and c, depending on the number of wires. Figure 8 gives the numerical values in the different cases.

Number of wires	a	b	c
2	0.5	0.22	-0.13
3	1	0.18	-0.2
4	2	0.14	-0.23
6	4	0.1	-0.25

Fig. 8 : Values of the 3 parameters of equation (4)

Figure 9 shows the comparison, depending on the number of wires, between the actual dimensionless shape factor and the dimensionless shape factor computed using equation (4). It can be seen that there is a good agreement between the proposed representation and the actual values of the dimensionless shape factor. It has to be noticed that when the dimensionless shape factor is equal to unity, the wires are equivalent to a continuous cylinder.

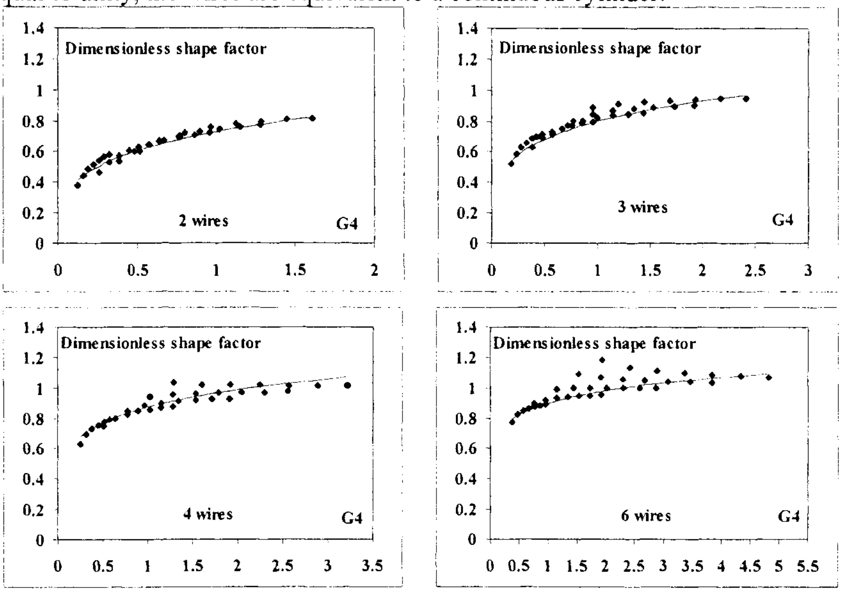

Fig. 9 : Comparison of the actual dimensionless shape factor with the proposed analytical representation

It is often assumed that the heat flux is constant at the surface of an electrical heating element. The simulations that have been carried out to compute the internal shape factor make the calculation of a non uniformity factor possible. It is proposed to define this non uniformity factor as follows:

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$$\delta = \frac{T_{max\ outer\ side} - T_{min\ outer\ side}}{T_{max\ wire} - T_{mean\ outer\ side}} .$$

This factor is null when the wire can be assimilated to a continuous cylinder, and should increase when the dimensionless shape factor decreases. So, it should informative to plot the evolution of this factor versus the dimensionless internal shape factor (Fig. 10).

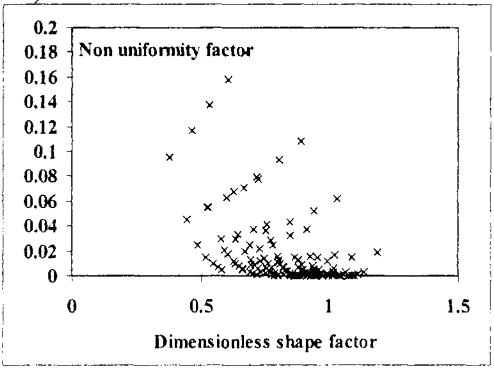

Fig. 10 : Evolution of the non uniformity factor versus the dimensionless internal shape factor

It can be seen that the non uniformity is limited to 16%, that is quite high, but that in most cases, is inferior to 2%, that is satisfactory. It is also informative to plot the evolution of the non uniformity factor versus parameter G4 (figure 11).

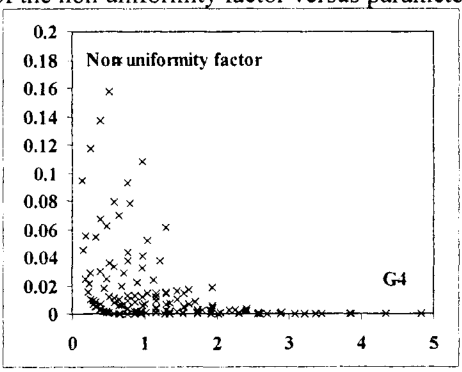

Fig. 11 : Evolution of the non uniformity factor versus G4

It can be seen that for all geometries that are characterized by G4>1.5, the non uniformity factor is inferior to 2%.

Now that the analytical expression of the dimensionless shape factor is established, it is possible to present examples.

3 Applications

3.1 High temperature air heater

In this example, it will be supposed that air has to be heated at 1,000°C and that the maximum wire temperature is 1,200°C. The application of the proposed equations will lead to the calculation of the maximum heat q that it will be possible to deliver per unit surface of sheathed heating element. It is necessary to assume other numerical values:

- convection coefficient: $h = 100\,W/m^2K$,
- element outer diameter: $D_e = 16\,mm$,
- sheath thickness: $t = 1\,mm$,
- sheath material thermal conductivity: $k_s = 15\,W/mK$,
- insulation material thermal conductivity: $k_i = 10\,W/mK$.

In that case, the dissipated heat flux may be written as follows:

$$q = \frac{T_{max\ wire} - T_{air}}{\frac{1}{S}\frac{D_e}{2\,k_i}Ln\left(\frac{D_e - 2t}{D_p}\right) + \frac{D_e}{2\,k_s}Ln\left(\frac{D_e}{D_e - 2\,t}\right) + \frac{1}{h}} . \qquad (5)$$

Introducing equation (4) in equation (5), leads to the following expression:

$$q = \frac{T_{max\ wire} - T_{air}}{\frac{1}{(a\,G4)^b + c}\frac{D_e}{2\,k_i}Ln\left(\frac{D_e - 2t}{D_p}\right) + \frac{D_e}{2\,k_s}Ln\left(\frac{D_e}{D_e - 2\,t}\right) + \frac{1}{h}} .$$

Then it can be said that the highest value has to be chosen for G4. This leads to choose 6 wires. The wire diameter is then 1 mm. The heat flux is then 18,205 W/m². In the same conditions, a standard coiled wire heating element would have dissipated about 18,650 W/m². This shows that in this case, a standard heating element is more powerful than a straight wire heating element.

3.2 High heat flux water heater

In this example, it will be supposed that water has to be heated at 100°C and the heat flux should be $q = 1{,}000\,kW/m^2$. The application of the proposed equations will lead to the calculation of the maximum temperature of the wire. Here again, it is necessary to assume other numerical values:

- convection coefficient: $h = 10{,}000\,W/m^2K$,
- element outer diameter: $D_e = 16\,mm$,
- sheath thickness: $t = 1\,mm$,
- sheath material thermal conductivity: $k_s = 15\,W/mK$,

- insulation material thermal conductivity: $k_i = 10\,W/mK$.

In that case, the maximum temperature of the wire may be written as follows:

$$T_{max\,wire} = q\left(\frac{1}{S}\frac{D_e}{2\,k_i}Ln\left(\frac{D_e-2\,t}{D_p}\right)+\frac{D_e}{2\,k_s}Ln\left(\frac{D_e}{D_e-2\,t}\right)+\frac{1}{h}\right)+T_{water}\,. \tag{6}$$

This leads to a temperature, for the straight wires of about 1185°C, and of about 925°C for the standard coiled heating element. This shows again that the standard geometry is better than the straight wires geometry.

It is necessary to say here that if bore nitride is used instead of magnesium oxide in straight wires elements, then for a temperature of 925°C in the wire, the heat flux goes up to 1,700 kW/m^2. This is much higher than for coiled heating elements in which it is impossible to use bore nitride.

In any case, it can be concluded that there is no risk of melting.

4 Conclusion

Simulations of the heat transfer in a sheathed electrical heating element has led to an analytical expression of the internal shape factor. This allows the prediction of the maximum temperature of the heating wire for a given heat flux, or the maximum heat flux for a given wire temperature. These simulations have allowed the calculation of a non uniformity factor. It has been shown that this non uniformity factor is, in most cases, inferior to 2%.

It has also been shown that the standard coiled wires heating elements are more powerful than straight wires elements when the same insulation material is used. It has to be noticed that they are also easier to manufacture.

Acknowledgments

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