A numerical and experimental investigation of the dynamic behavior of a heat pump

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Abstract

A dynamic numerical model of a compression vapor cycle heat pump is presented. The model is based on mass and energy balances, thermodynamic and thermal properties of the working fluid and constructive characteristics of heat pump components. The model enables the prediction of temporal and spatial distribution of refrigerant temperatures, pressures and mass flows, as well as water temperatures for variable boundary conditions that occur during the heat pump operation. Experimentation has been performed on a laboratory test rig. Refrigerant pressures and temperatures, compressor electric power consumption, as well as water temperatures and mass flows were measured. The measurement of dynamic operating conditions was successfully performed using the data acquisition system controlled by personal computer. The use of graphic interface enabled analyses of working conditions at operating limits of the heat pump. Results of the dynamic heat pump model were confirmed by comparison with experimental data.

1 Introduction

Detailed dynamic models play an important role in investigations of processes taking place in compression refrigeration system during transients. Among numerous refrigeration system models and the computer programs derived from them, detailed transient models are the most complex. Some of those were given by Yasuda et al. [1], Chi and Didion [2], MacArthur and Grald [3], Upmeier [4], Ney [5] and Pavković [6]. In development of the model presented in [6], a physically based description of all processes taking place in a compression heat pump was respected. The aim was to build a flexible model, capable of predicting fully distributed data about refrigerant temperatures, pressures and mass flows as well as water temperatures for variable boundary conditions that occur during the heat pump operation.
Model descriptions

2.1 Mathematical model of a compressor

A mathematical model of a reciprocating semi hermetic compressor is based on the mathematical description of the motion of crankshaft mechanism, mass and energy conservation equations for refrigerant, equations for refrigerant flow through cylinder valves and valve dynamics. Heat exchange between refrigerant and cylinder wall as well as between electric motor and refrigerant has been considered.

\[
\frac{dM_R}{d\varphi} = \frac{dM_{RI}}{d\varphi} - \frac{dM_{RO}}{d\varphi} - \frac{dM_{RA}}{d\varphi} \quad (1)
\]

\[
dQ_R = \sum_i dQ_i + \sum_j dM_{Rj} h_{Rj} \quad (2)
\]

\[\sum_i dQ_i\] represents a heat exchange between the refrigerant and cylinder surroundings. With the first law expression for an open system

\[
dQ_R = M_R d i_R + i_R dM_R + p_R dV_R \quad (3)
\]

the energy equation (2) becomes:

\[
M_R d i_R + i_R dM_R = \sum_i dQ_{Ri} + \sum_j dM_{Rj} h_{Rj} - p_R dV_R \quad (4)
\]

Using equation (4) and equation of state for a refrigerant, a refrigerant temperature change for a time step can be calculated for a real fluid. A cylinder wall energy balance gives the change of the cylinder wall temperature. A heat
exchange between the cylinder wall and refrigerant in the cylinder and the refrigerant surrounding the cylinder has been considered. The friction heat of piston rings and electric motor heat (for a semi hermetic compressor) has been taken into account as well.

The multicylinder compressor model was established. Control volumes, such as suction pipe, compressor casing, electric motor, cylinders and discharge pipe were presented in the form of a control vector, where components of the vector are integers describing a sort of control volume. The control matrix of connection for mass and energy exchange contains integers representing the connection indexes describing the type of connection (heat exchange, mass exchange through pipe, mass exchange through the valve with variable cross section). Refrigerant mass flow between control volumes is driven by pressure difference, and can assume positive or negative value. The valve cross flow area is determined from the differential equation resulting from the force balance on a valve plate (figure 2). The vector of volume integers and matrixes of connections control the generation and solving of the equation system during the simulation of the dynamic process.

**Figure 3:** Heat and mass exchange paths for a four cylinder compressor

### 2.2 Mathematical model of an evaporator

The physical processes in the evaporator can be described by conservation equations for mass, momentum, and energy, which refer to a general flow case with heat transfer. Conservation equations have to be set for the refrigerant, water and pipe wall in order to describe the evaporator. A simultaneous solution results in velocity, temperature and pressure fields in time and space. All equations have to be solved, taking into consideration initial and boundary conditions. It has been assumed that refrigerant vapor and liquid are in equilibrium in a saturated state and that vapor and liquid pressures are equal for the considered cross section. The refrigerant flow has been considered one-dimensional. A bulk approach has been accepted. The void fraction model has been used to account for slip effects. It has also been assumed that refrigerant properties in a saturated state are temperature and pressure dependent. Potential and kinetic energy, kinetic energy of pressure waves, the work associated with rate of pressure change as well as heat conduction in refrigerant, have been
neglected. The effect of viscous dissipation has been taken into account by pressure drop calculation, which has been performed separately. With those approximations, the one-dimensional form of conservation equations for refrigerant, pipe wall and water gives:

\[
\frac{\partial \rho_R}{\partial t} + \frac{\partial}{\partial x} (\rho_R u_R) = 0.
\]  

(5)

\[
\frac{\partial (\rho_R h_R)}{\partial t} + \frac{\partial (\rho_R u_R h_R)}{\partial x} = \alpha_R \frac{O}{A} (T_S - T_R).
\]  

(6)

\[
\rho_S A_S c_S \frac{\partial T_S}{\partial t} + \alpha_R O_R (T_S - T_R) - \alpha_w O_w (T_w - T_S) = 0.
\]  

(7)

\[
\frac{\partial (\rho_w A_w c_w T_w)}{\partial t} + \frac{\partial (\rho_w A_w u_w c_w T_w)}{\partial x} - \sum \alpha_w O_w (T_S - T_w) = 0.
\]  

(8)

\[
\frac{\partial M_w}{\partial x} = 0.
\]  

(9)

The finite volume method [7] has been used to solve the equation system (5), (6), (7), (8) and (9). The evaporator has been divided into a set of control volumes including the refrigerant, pipe and water. In the center of the control volume \(i = 1...N\), scalar quantities are located. Vector quantities are defined at control volume interfaces \(i = 1...N + 1\).

![Figure 4: Control volumes of a refrigerant evaporator](image1)

![Figure 5: Arrangement of water and refrigerant control volumes in a cross flow evaporator](image2)
The conservation equations in discretized form have been set for all control volumes $i = 1,\ldots,N$ for the refrigerant, water and pipe walls. Interconnection between the refrigerant $i = 1,\ldots,N$ and water $j = 1,\ldots,N_W$ control volume indexes has been provided in the simulation program to ensure proper calculations. The system of equations is solved using TDMA algorithm [7]. To account for the different vapor and liquid refrigerant velocities through the pipe, the void fraction model of Rouhani was used [8]. Chisholm’s model [8] was used to evaluate the friction pressure drop. The flow pattern in horizontal tubes has been determined according to [8], and that information was used in heat transfer calculations.

2.3 Mathematical model of a condenser

The model describes a water-cooled shell and tube condenser. Inlet refrigerant mass flow $\dot{M}_{\text{Rei}}$ depends on condenser pressure and it is calculated in the compressor model. The mass flow of subcooled liquid $\dot{M}_{\text{Rco}}$ depends on pressure difference between the condenser and evaporator inlet, and the action of the thermostatic expansion valve (TEV).

Discretized mass and energy conservation equations for the condenser have been determined from mass and energy balances for control volumes of the condenser (Figure 6). Those control volumes are refrigerant vapor (1.1 and 1.2), refrigerant liquid on pipes (2.1), subcooled liquid refrigerant on the shell bottom (2.2), water in pipes (6.1 and 6.2) and water in deviating chambers (7.1, 7.2 and 7.3), pipes (3.1 and 3.2), plates (4.1, 4.2 and 4.3), shell in contact with refrigerant liquid (5.1) and shell in contact with refrigerant vapor (5.2).

Figure 6: Control volumes and heat and mass paths for condenser model
The refrigerant pressure drop in the condenser has been neglected. Heat transfer coefficients for condensation on a tube bundle and convective heat transfer between the subcooled liquid on the bottom of the shell and flooded pipes have been calculated according to [8].

2.4 Thermostatic expansion valve mathematical model

There was a lack of information about valve constructive data (spring forces, bulb temperature-pressure dependence, etc.), which are necessary to build a valve model similar to the valve model used for the compressor. Therefore, a simple thermostatic expansion valve model has been built. Valve needle displacement has been described by a single polynomial, whose coefficients have been obtained from experimental data of valve needle displacement, achieved for variable evaporator outlet pressures, bulb temperatures and adjustment of static superheat, using the least square method. Correlation between the needle displacement and refrigerant cross flow area has been set, and the refrigerant mass flow through the valve has been calculated with known pressure difference. Bulb temperature transient was taken into account through the exponential function, using the bulb time constant, which was also determined experimentally for an observed valve.

2.5 Refrigerant

Single-component refrigerant properties were achieved from tables and diagrams for the refrigerant saturated vapor, and presented in a form of regression polynomial with coefficients that were achieved by the least square method. Polynomials were chosen because of fast data exchange in a custom-built heat pump simulation program. That data exchange speed could not be met by using commercial refrigerant databases. In a superheated region, a Martin Hou equation of state was used according to [10].

2.6 Heat pump model

Component models have been merged into a heat pump model. Boundary conditions (Figure 7) are water mass flow through the evaporator \( \dot{M}_{w_e} \) and condenser \( \dot{M}_{w_c} \), evaporator inlet water temperature \( T_{w_{ei}} \) and condenser inlet water temperature \( T_{w_{ci}} \). The action of a control system is also a boundary condition. It can be cylinder unloading ( \( y \) is the number of unloaded cylinders by holding the suction valve plate in open position) or variable speed control ( \( n \) is the rotation speed). A structure description of components and heat pump, such as constructive data about compressor, evaporator, condenser and TEV are stored in a data file, which can be changed in order to simulate different heat pumps. Data on refrigerant type and charge are also stored in that file. The same data file contains the control of temporal change of boundary conditions as well.
At the calculation start, initial distribution of refrigerant in the entire heat pump is established. The dynamic calculation follows. The compressor time step is variable, depending on valve dynamics, while the time step for condenser, evaporator and TEV is one degree of the compressor crankshaft angle. Compressor subroutine gives suction and discharge pressures $p_{Reo}$ and $p_{Re}$ as well as compressor inlet and outlet mass flows of refrigerant $M_{Reo}$ and $M_{Rel}$, which are boundary conditions for evaporator and condenser. Evaporator and TEV subroutines have the same boundary conditions for $p_{Re}$ and $p_{Reo}$. For the refrigerant leaving the TEV and entering the evaporator, pressure and mass flow are determined, taking into account the pressure drop. This means that the evaporator and TEV are solved simultaneously, and iterations proceed until a total pressure drop across TEV and evaporator becomes equal to the pressure difference between $p_{Re}$ and $p_{Reo}$. In that case, the conservation equations are satisfied for the entire heat pump.

The results of the calculation are dynamic changes of pressures, temperatures and densities in the evaporator, compressor cylinders and condenser. Distribution of all properties is known for the entire evaporator [10]. Refrigerant distribution throughout the entire heat pump is calculated as well, which facilitates transient analysis of refrigerant loss. Compressor motor consumed power $\dot{P}_c$ is calculated, while condenser and evaporator heat rates are evaluated with known water mass flows $M_{wc}$ and $M_{we}$, water inlet boundary temperatures $T_{wei}$ and $T_{we}$, and calculated values for outlet water temperatures $T_{wco}$ and $T_{weo}$.

![Figure 7: Information flow for a heat pump model](image-url)
3 Experimental setup

An experimental test rig has been built in order to validate the computer model. A schematic layout of the experimental setup with locations of measurement points is shown on figure 8.

Figure 8: Schematic layout of the experimental setup

All data were acquired using the Hewlet Packard HP 3852S data acquisition system. Interconnection between the PC and the data acquisition system was established. General Purpose Interface Bus protocol was used for communication.

J type thermocouples were used for temperature measurements. A short time constant was the reason for the choice of thermocouples. Refrigerant pressures were measured with mechanical–inductive pressure transmitters for ranges 0-25 bar and 0-10 bar. Output signal was 0-10 V. Water flow was measured with ultrasonic flow meter, with 4-20 mA output signal. Electric motor power rate was measured using the transmitter for electric current range 75/5A and voltage 380 V, with output signal 0 - 3 V.

Software package LabVIEW® was used to control data acquisition. That software enables a simple graphic programming and thus exempts the user from rigid syntax and complexity of the data acquisition system. The data were transferred to the external memory of the personal computer, to a file convenient for post processing. Data necessary to control the heat pump operation were converted to physical units and displayed on the computer monitor. Those were refrigerant pressures, refrigerant temperatures, water temperatures, heating rate,
cooling rate, and compressor electric power. Refrigerant superheating at evaporator outlet, as well as refrigerant subcooling at condenser were calculated and monitored during the data acquisition.

The benefit of the presented control through the front panel was obvious during the measurement. Heat pump working conditions at a moment when refrigerant pressure approached the setting point of the security equipment (either low or high-pressure) could be controlled. The same control possibility was established for low water temperature.

4 Model verification and results

Measurements were performed on a 140 kW water-to-water heat pump working with R22. The operation of the same heat pump was simulated using the heat pump model adjusted for the above-mentioned machine and boundary conditions achieved from measurements. An example of measurement results is shown on figures 9, 10 and 11, for a single transient run with 100% compressor capacity during the 2061 s. Initial water inlet temperatures in the condenser and evaporator were $T_{wci} = T_{wei} = 298$ K At $t = 64$ s the compressor was started with 100% capacity. Temperatures in hot and cold storage tank changed until $t = 2125$ s, when the compressor was switched off. At the end of the compressor run, water inlet temperatures were $T_{wci} = 321.55$ K and $T_{wei} = 281.26$ K.

Comparison of measured and calculated values for evaporator and condenser energy rates, as well as for the compressor electric power is shown in figure 9. Refrigerant pressures are given in figure 10, while refrigerant temperatures are given in figure 11.

Figure 9: Measured (ms) and calculated (sim) values for heating and cooling rate and compressor power consumption
Figure 10: Measured (ms) and calculated (sim) values for refrigerant pressures

Figure 11: Measured (ms) and calculated (sim) values for refrigerant temperatures

Satisfactory compatibility between measured values and simulation results is obvious from figures 9, 10 and 11. A difference between simulation results and
measured data for compressor outlet pressure and temperature is considered to be a result of neglected heat accumulation in the compressor casing. An extensive set of simulations and measurements was performed for a partial load dynamic operation of a heat pump, and compatibility between measured and simulated values for considered properties was found [6].

5 Conclusion

The presented dynamic model is suitable for analyses of a heat pump response to a control system action, but it can be used as a design model as well. Results of simulations for dynamic heat pump operation in both full and partial load are compatible with measurements. The model is modular and flexible and it can be changed in order to simulate different types of heat pumps or refrigeration machines. With certain modifications, it can be used for numerical constructive optimization of heat pumps and refrigeration machines. Mass inventory facilitates analyses of dynamic behavior under refrigerant leakage conditions. The future work on model development, which can bring radical changes in the model design, is related to use of refrigerant mixtures as a working fluid, analysis of oil migration under dynamic operation conditions and development of models for different types of heat pump components.

List of symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$A$</td>
<td>area, m$^2$</td>
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<tr>
<td>$c$</td>
<td>specific thermal capacity, Jkg$^{-1}$K$^{-1}$</td>
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<td>$F$</td>
<td>force, N</td>
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<td>$V$</td>
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<tr>
<td>$x$</td>
<td>coordinate</td>
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<tr>
<td>$\dot{x}$</td>
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<td>$\alpha$</td>
<td>heat transfer coefficient, Wm$^{-2}$K$^{-1}$</td>
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<tr>
<td>$\varphi$</td>
<td>crankshaft angle</td>
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<tr>
<td>$\epsilon$</td>
<td>void fraction</td>
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<tr>
<td>$\rho$</td>
<td>density, kgm$^{-3}$</td>
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Subscripts

\begin{align*}
&b \quad \text{border} & & m_x \quad \text{maximum} \\
&c \quad \text{condenser} & & o \quad \text{outlet} \\
&d_a \quad \text{damping} & & p \quad \text{due to pressure} \\
&e \quad \text{evaporator} & & R \quad \text{refrigerant} \\
&e_l \quad \text{electric} & & s_{im} \quad \text{simulation} \\
&i \quad \text{inlet} & & S \quad \text{pipe wall} \\
&i_n \quad \text{inertial} & & s_{p} \quad \text{spring} \\
&l_k \quad \text{leakage} & & v \quad \text{valve, vapor} \\
&m_s \quad \text{measured} & & W \quad \text{water}
\end{align*}

References


