Modelling of heat and mass transfer in capillary-porous food products

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Abstract

The present project concerns fundamental analysis of heat and mass transfer (including evaporation and condensation) in animal food products. Mathematical and physical models are formulated and further developed to enable improved understanding of the transport processes and reveal the influence of essential parameters. Prediction and analysis of heat and mass transfer for cooking of meat patty in a double-sided contact fryer are provided.

The governing partial differential equations, which are coupled, are solved by a finite difference technique. In particular, the boundary conditions are treated carefully. The mathematical and physical models, the governing equations and the boundary conditions are presented in detail. The numerical method is briefly described. Results from a few simulations is reported.

Parametric studies were carried out by changing the heat transfer coefficient, effective thermal conductivity for the crust and the core, diffusion coefficient, patty height and temperature of the surrounding. The calculated cooking time was found to be most influenced by the effective thermal conductivity in the core, the height of the meat patty and the heat transfer coefficient, in descending order of importance.

1 Introduction

A substance is said to be capillary-porous and the pores to be capillars if the capillary potential is greater than the gravity potential. The influence of the gravity can then be neglected. Materials or stuff with a moisture content are often
capillary-porous and the binding of the water plays an essential role for their properties. Meat balls and hamburgers are typically capillary-porous substances.

It is of vital interest for the food industry to be able to predict the correct cooking time, i.e., if possible microbials are destroyed and if the quality of the food is sufficient. The objective of the present study is to develop a numerical model of a meat patty that can predict the cooking time adequately.

The calculated results of the model is compared to preliminary experimental measurements on meat patties cooked in a double-sided contact fryer. The experimental data is provided by Kovácsné et al. [1].

2 Numerical model

In order to derive the governing equations for a meat patty cooked in a double-sided contact fryer the following assumptions were made:

1) The energy required for fat melting and denaturation of protein is small compared to the heat required for vaporisation of water.

2) The dimensional change of the patty (shrinking/swelling of diameter and height) is neglected.

3) The thermophysical properties are taken to be constant.

4) The core region is a mixture of water, fat and protein and the crust region consists of water vapour, fat and protein.

5) Due to a large diameter to height ratio the problem is considered to be one-dimensional and symmetry at the central plane of the meat patty is assumed.

The following governing partial differential equations (PDEs) were used to model the heat and mass transfer in the core and crust.

For the heat transfer one has:

$$k_{	ext{eff,co}} \frac{\partial^2 T}{\partial x^2} + J_w c_{p,w} \frac{\partial T}{\partial x} = \left( \epsilon_w \rho_w c_{p,w} + \epsilon_f \rho_f c_{p,f} + \epsilon_p \rho_p c_{p,p} \right) \frac{\partial T}{\partial t}$$  \hspace{1cm} (1)

$$k_{	ext{eff,cr}} \frac{\partial^2 T}{\partial x^2} + J_v c_{p,v} \frac{\partial T}{\partial x} = \left( \epsilon_v \rho_v c_{p,v} + \epsilon_f \rho_f c_{p,f} + \epsilon_p \rho_p c_{p,p} \right) \frac{\partial T}{\partial t}$$  \hspace{1cm} (2)

where the first terms on the left hand side of the equations are the transfer of energy by conduction and the second terms convection due to movement of water. The right hand side specifies the change in internal energy. The boundary conditions are given by:
Temperature symmetry at the centerline

\[ \frac{\partial T}{\partial x} = 0 \quad x = 0, \ t > 0 \]

Water is vaporised at the interface between the core and the crust

\[ T = T_{bp} = 100^\circ C \quad x = I(t), \ t > 0 \]

Convective heat transfer from the surrounding at \( T_\infty = 175^\circ C \)

\[ -k_{\text{eff, cr}} \frac{\partial T}{\partial x} = h(T|_{x=L} - T_\infty) \quad x = L, \ t > 0, \ L = H / 2 \quad (3) \]

An energy balance across the moving interface yields

\[ -k_{\text{eff, cr}} \frac{\partial T}{\partial x} + J_w h_w + k_{\text{eff, co}} \frac{\partial T}{\partial x} - J_v h_v = \]

\[ = \frac{df}{dt} \left[ \varepsilon_s \rho_s (h_{s,cr} - h_{s,co}) + \varepsilon_v \rho_v h_v - \varepsilon_w \rho_w h_w \right] \quad x = I(t), \ t > 0 \quad (4) \]

where the terms with subscript 's' represent the solid phase, i.e., the fat and protein.

For the mass transfer one has:

\[ \frac{\partial c_w}{\partial t} = -\frac{\partial J_w}{\partial x} = D_{ws} \frac{\partial^2 c_w}{\partial x^2} \quad (5) \]

which is the so-called 2nd law of Fick for pure diffusion of liquid water in the core. The mass transfer in the crust is governed by the flux rate of water transported to and evaporated at the interface. Since vapour can only be generated at the interface the mass flux of vapour away from the interface is equal to mass flux of liquid water transported to the interface. Hence the equation for vapour flux in the crust is not needed, Farkas [2].

The boundary conditions are given by:

Absence of diffusion across the centerline

\[ \frac{\partial c_w}{\partial x} = \rho_w \frac{\partial c_w}{\partial x} = 0 \quad x = 0, \ t > 0 \]

No liquid water at the interface

\[ c_w = \varepsilon_w = 0 \quad x = I(t), \ t > 0 \]
Mass balance across the interface

\[ J_v - J_w = \frac{dI}{dt}(\varepsilon_w \rho_w - \varepsilon_v \rho_v) \quad x = I(t), \ t > 0 \] (6)

A combination of eqns (4) and (6) yields a simplified form of eqn (4), which is used to determine the position of the interface:

\[ -k_{eff,cr} \frac{\partial T}{\partial x} + k_{eff,co} \frac{\partial T}{\partial x} - J_w (h_v - h_w) = \]

\[ = \frac{dI}{dt} \left[ \varepsilon_s \rho_s (h_s,cr - h_s,co) + \varepsilon_w \rho_w (h_v - h_w) \right] \] (7)

The one-dimensional grid consists of equally spaced nodes where the distances between the nodes are constant in time. As the interface is progressing continuously during the crust formation it will at every timestep be positioned at a different distance from the nodal points. All the governing heat and mass transfer equations closest to the interface are discretised using a Taylor series expansion with unequally spaced increments.

Interface

\[ \begin{array}{c}
\cdot \cdot \cdot p \Delta x \cdot \cdot \cdot \\\\
\cdot \cdot 1-1 \cdot \cdot \\\\
\cdot i \cdot \\\\
\cdot i+1 \cdot \\\\
\cdot \cdot \cdot \Delta x \cdot \cdot \cdot \\
\end{array} \]

Figure 1: Example of unequally spaced increments.

Figure 1 shows an example of an unequally spaced increment where the interface is behind the node of interest. An example of a Taylor series expansion for the first temperature derivative is given below:

\[ T_{i+1} = T_i + \Delta x \frac{dT}{dx} + \Delta x^2 \frac{d^2T}{dx^2} \frac{2!}{d^2T} \]

\[ T_{i-1} = T_i - p \Delta x \frac{dT}{dx} + \frac{p^2 \Delta x^2}{2!} \frac{d^2T}{dx^2} \]

This equation becomes the well known second order central difference approximation when p = 1. This difference approximation can pose a problem when p is a small value close to zero.

The grid independence was verified by using three different number of nodes and then calculate the time needed to reach a temperature of 72°C in the centre of the meat patty. As can be seen in Table 1 it is sufficient to use 26 nodes.
Table 1. The calculated cooking time for three different set of nodes.

<table>
<thead>
<tr>
<th>Nodes</th>
<th>Time, s</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>119.8</td>
</tr>
<tr>
<td>21</td>
<td>121.0</td>
</tr>
<tr>
<td>26</td>
<td>121.2</td>
</tr>
</tbody>
</table>

The governing partial differential equations, which are coupled, are solved by an implicit finite difference technique. The PDEs are discretised using primarily central difference approximations. It was found that a timestep of $\Delta t = 0.2$ s is sufficient to resolve the dynamic features in the numerical model.

3 Physical properties

Simple formulas for thermophysical properties of liquid water are provided from the work by Popiel & Wojtkowiak [3]. Equations for the thermophysical properties of fat and protein are given by Choi & Okos [4]. The composition of the meat patty is, according to the accredited laboratory at Swedish Meats R&D PI:

- Water 66.1 wt%
- Fat 14.3 wt%
- Protein 19.6 wt%

The meat patty has a diameter $D = 0.10$ m and a height $H = 0.01$ m. With the given dimensions, composition and equations for the thermophysical properties, all data for the food model at the mean temperature of 50°C are given in Table 2.

Table 2. Thermophysical data for the model.

<table>
<thead>
<tr>
<th>Property $\rho$ [kg/m$^3$]</th>
<th>Water</th>
<th>Fat</th>
<th>Protein</th>
<th>Vapour 100°C</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k$ [W/mK]</td>
<td>988.0</td>
<td>904.7</td>
<td>1304.0</td>
<td>0.590</td>
</tr>
<tr>
<td>$c_p$ [J/kgK]</td>
<td>4180.9</td>
<td>2046.9</td>
<td>2065.4</td>
<td>2042.0</td>
</tr>
</tbody>
</table>

Due to the complex mixture of the ingredients only an effective thermal conductivity can be used. In the literature there exists a number of different models describing the effective thermal conductivity for food consisting of two or more different ingredients, see e.g. Becker & Fricke [6]. The simple parallel and perpendicular models have been found to predict the upper and lower limits, respectively, of the effective thermal conductivity of most food items. The weighted geometric mean model gives an effective thermal conductivity that is the mean of the parallel and perpendicular model. This model represents the chaotic nature of randomly distributed media, Urbicain & Lozano [7].
where $\varepsilon$ is the volume fraction ($V_i / V_{tot}$) for the different ingredients. This is the model used in the present investigation. The effective thermal conductivity for the core and crust reads, respectively,

$$k_{eff} = k_w^\varepsilon \cdot k_f^\varepsilon \cdot k_p^\varepsilon$$  \hspace{1cm} (8)

In this investigation a constant heat transfer coefficient is used. The value for the heat transfer coefficient, $h = 260 \text{ W/m}^2\text{K}$ is based on the work by Dagerskog [8], where similar experiments on cooking of a meat patty in a double-sided contact fryer was performed. The diffusion coefficient, $D_{w} = 2.5 \times 10^{-10} \text{ m}^2/\text{s}$, is taken from the work of Motarjemi [9], where mass transport of water in minced meat products at different temperatures was investigated.

### 4 Results

The results of the numerical model is compared with experimental data from measurements on a meat patty in a double-sided contact fryer. The surface temperature was set to 175°C and the initial temperature of the meat patty was 4.7°C. The cooking time is defined as the time needed to reach the temperature 72°C in the centre of the meat patty.

In Figure 2 the temperature history is plotted for the measured and calculated temperature in the centre. The agreement between the measured and predicted centre temperature is reasonable.

![Figure 2: Comparison between measured and calculated centre temperature.](image-url)
Both curves in Figure 2 show similar trends and the difference in cooking time arises during the first 20 seconds. A possible reason for this can be that the assumption of a constant heat transfer coefficient is not valid. It can also be attributed to the fact that when the crust is formed it will form a growing heat insulating barrier due to the lower effective thermal conductivity of the crust. This will lead to a diminishing importance of the convective heat transfer coefficient.

A sensitivity analysis was carried out by changing the heat transfer coefficient, effective thermal conductivity for the crust and the core, diffusion coefficient, patty height and surface temperature of the fryer.

Table 3. Sensitivity analysis.

<table>
<thead>
<tr>
<th>Property</th>
<th>Variation</th>
<th>Value</th>
<th>Time, s</th>
<th>% Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>(h) [W/m^2K]</td>
<td>-</td>
<td>260</td>
<td>121.2</td>
<td>-</td>
</tr>
<tr>
<td>260</td>
<td>-50%</td>
<td>130</td>
<td>143.2</td>
<td>+18.2</td>
</tr>
<tr>
<td></td>
<td>+50%</td>
<td>390</td>
<td>116.4</td>
<td>-4.0</td>
</tr>
<tr>
<td>(k_{\text{eff,cr}}) [W/mK]</td>
<td>-50%</td>
<td>0.024</td>
<td>122.2</td>
<td>+0.8</td>
</tr>
<tr>
<td>0.048</td>
<td>+50%</td>
<td>0.072</td>
<td>120.6</td>
<td>-0.5</td>
</tr>
<tr>
<td>(k_{\text{eff,co}}) [W/mK]</td>
<td>-50%</td>
<td>0.221</td>
<td>224.0</td>
<td>+84.8</td>
</tr>
<tr>
<td>0.441</td>
<td>+50%</td>
<td>0.662</td>
<td>86.8</td>
<td>-28.4</td>
</tr>
<tr>
<td>(D_{ws}) [m^2/s]</td>
<td>-50%</td>
<td>1.25\times10^{-10}</td>
<td>121.0</td>
<td>-0.2</td>
</tr>
<tr>
<td>2.5\times10^{-10}</td>
<td>+50%</td>
<td>3.75\times10^{-10}</td>
<td>121.4</td>
<td>+0.2</td>
</tr>
<tr>
<td>(H) [m]</td>
<td>-0.002 m</td>
<td>0.008</td>
<td>80.2</td>
<td>-33.8</td>
</tr>
<tr>
<td>0.010</td>
<td>+0.002 m</td>
<td>0.012</td>
<td>170.8</td>
<td>+40.9</td>
</tr>
<tr>
<td>(T_\infty) [°C]</td>
<td>-10°C</td>
<td>165</td>
<td>123.4</td>
<td>+1.8</td>
</tr>
<tr>
<td></td>
<td>+10°C</td>
<td>185</td>
<td>119.4</td>
<td>-1.5</td>
</tr>
</tbody>
</table>

The differences in the predicted cooking time for different values of the properties are given in Table 3. A variation by 50% of the effective thermal conductivity in the crust and diffusion coefficient, and also an increase or decrease by 10°C of the surface temperature of the fryer does not affect the cooking time particularly. The heat transfer coefficient has a higher influence on the cooking time if the value is decreased by 50% than when it was increased by the same amount. This suggests some mechanism affecting the transport of energy into the meat patty is present. The cooking time was greatly affected by the height of the meat patty. The largest influence on the cooking time was due to the alteration in the effective thermal conductivity in the core. Hence it is important to determine and accurately model the effective thermal conductivity in the core.
Figure 3 gives an example of how the variation of the effective thermal conductivity affects the calculated cooking time.

The measured and calculated mass losses, approximately 30 wt% and 5 wt%, respectively, and the measured and calculated interface velocity, approximately $1.5 \times 10^{-5}$ m/s and $3 \times 10^{-6}$ m/s, respectively, show discrepancies. This indicates that the present model with purely diffusive mass transport is not sufficient to describe the mechanism of water transport in the meat patty.

Conclusions

The results of the present numerical model show that it can predict the development of the centre temperature reasonably well. The sensitivity analysis of different parameters in the model shows that the largest influence on the cooking time was due to the alteration in the effective thermal conductivity of the core. This implies that a better model can be gained if the thermophysical properties and their mutual influences in the meat patty are better understood. The numerical model can be improved if other mechanisms of water transport is included. This will give better agreement between the measured and calculated mass loss.

References


