An improved method of approximating frequency characteristics in the problem of modal analysis and its applications

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Abstract

The paper presents an improved method and an algorithm of determining modal parameters of the machine tool model and its application: of continuous description of dynamic characteristics of models of variable structure and variable parameters and of damping identification on the basis of approximated frequency characteristics. The method employs the natural form of the approximating formula for iterative calculations improving initial estimates of modal parameters. The algorithm was prepared as a linear programming problem that is solved in MATLAB system. The effectiveness of the method was evaluated by the comparison of the obtained results with those found in the effect of the application of classical methods, integrated in the MATLAB Optimization Toolbox.

1 Introduction

Complex and diverse problems associated with machine tool dynamics [1, 2] are the subject of multidirectional research works aimed at improvement of their dynamic properties e.g. main drive, feed drive and body system dynamics. These systems, however, are only the components of machine tools, are very complex and their dynamic properties are described by large numerical data. Effective usage of these data and minimization of errors made during data processing lay the directions of the research works tending to the elaboration of the optimal analytic methods.
The elaborated method and algorithm of approximation of the frequency characteristics APRO is applied not only as a tool which enables the construction of modal model of the real object but also reduction of the numerical data for the continuous description of characteristics of machine tool body system within given frequency band and as a function of the parameter which characterize model's non-stationarity as well.

Most of the presented algorithms that construct modal models of real objects on the basis of the experimental characteristics [3, 4, 5, 6] requires initial values of estimates of modal parameters. These values can not be deliberately chosen, because large differences between estimates and actual values of the parameters influence the convergence of the iterative process. Despite of the application of different methods of determination of the initial values of modal parameters estimates [3, 4, 5, 6] resonance frequencies are determined on the basis of searching the set of experimental characteristics and examining if chosen frequency correspond to the maximum in the most of the measured characteristics. Time needed for this task is relatively long in comparison with the time of computations. Therefore an automated approach seems to be very helpful. Module which determines the optimal number of equations that constitute the base for iterative computations of the parameters of approximation equation was elaborated to improve the quality of the obtained results and to shorten time of computations.

2 Method of approximation of modal parameters

Analysis of the equation (1) leads to the conclusion that frequency characteristic of the system of \( n \) degrees of freedom is a superposition of the characteristics of \( l \) degree of freedom. Structure of the approximation equation (1) is basic for the elaboration of the method of frequency characteristics approximation APRO [6] that determines values of modal parameters \( A_{ik} B_{ik} C_{ik} D_{ik} \) in the given frequency band \((\omega_a;\omega_b)\). Equation (1) considers also the shape of the characteristic in the bands \((0;\omega_a)\) and \((\omega_b;\infty)\)

\[
x_i(f\omega) = \frac{1}{m_i\omega^2} + \sum_{k=1}^{n} \frac{A_{ik} + j\omega B_{ik}}{C_k - \omega^2 - j\omega D_k} + x_0
\]

where:
\[
\frac{1}{m_i\omega^2} - \text{inertial element that considers influence of vibration modes from the range } (0, \omega_a)\]
\[
x_0 - \text{element that considers displacements from the range } (\omega_b, \infty)\]

Scheme of the method APRO is shown in Figure 1. Process of approximation of the experimental characteristic using APRO method starts from the searching the set of the characteristics describing the dynamic properties of the analyzed object to indicate its resonance frequencies that are maximum values in the most of the measured characteristics. Then on the basis of the equation (2)

\[
\frac{A_{ik} + j\omega B_{ik}}{C_k - \omega^2 - j\omega D_k} = Rx_{ik} + jIx_{ik}
\]
where:

\[ R_{x_{ik}} = \text{re}(\chi_i(j\omega)) \quad I_{x_{ik}} = \text{im}(\chi_i(j\omega)) \]  

(3)

describes the isolated resonance frequency and then preliminary estimation of the parameters \( A_{ib}, B_{ib}, C_b, D_k \) are determined. Right-hand side of this equation are coordinates of the \( k \)-th point of the approximated experimental characteristics.

![Figure 1: Scheme of APRO algorithm.](image)

It is enough to solve equation (2) to get preliminary estimations of modal
parameters. To obtain good approximation of the modal parameters \( r \) points (5 or 10) on each side of the resonance frequency are used for the computations. Hence, 10 or 20 points in the vicinity of each resonance are considered. Comparing real and imaginary parts of \( 2r \) equations one obtains a system of \( 4r \) equations that can be transformed to the form

\[
\begin{bmatrix}
1 & 0 & -R_{x_{k-r}} & -I_{x_{1}}\omega_{k-r} \\
0 & \omega_{1} & -I_{x_{k-r}} & R_{x_{1}}\omega_{k-r} \\
\vdots & \vdots & \vdots & \vdots \\
1 & 0 & -R_{x_{k+r}} & -I_{x_{k+r}}\omega_{k+r} \\
0 & \omega_{r} & -I_{x_{k+r}} & R_{x_{k+r}}\omega_{k+r}
\end{bmatrix}
\begin{bmatrix}
A_{i_k} \\
B_{i_k} \\
C_{k} \\
D_{k}
\end{bmatrix}
= 
\begin{bmatrix}
-R_{x_{k-r}} - r\omega_{k-r} \\
-I_{x_{k-r}}\omega_{k-r} \\
-R_{x_{k}}\omega_{k} \\
-I_{x_{k}}\omega_{k} \\
-R_{x_{k+r}}\omega_{k+r} \\
-I_{x_{k+r}}\omega_{k+r}
\end{bmatrix}
\] (4)

Solution of the system (4) using least square method leads to the determination of preliminary estimation of the modal parameters \( A_{i_k}, B_{i_k}, C_{i_k}, D_{i_k} \) [6].

Modification of this procedure consists in the sequence of resonance frequencies considered in the computations sorted according to the descending order of the corresponding compliances values \( |x_{i}(j\omega_{k})| \). In each step of the iteration in the examined frequency range \((\omega_{i},\omega_{j})\) frequency \( \omega_{k} \) is searched according to the criterion

\[
|<x_{i}(j\omega_{k})| = \max
\] (5)

In the first step, frequency \( \omega_{1} \) is found in the result of searching the real characteristic. In the next step succeeding values \( \omega_{2} \) to \( \omega_{n} \) are determined in the result of searching the curve which is difference between experimental characteristic and generated components \( x_{i}(j\omega_{A_{ib}} B_{ib} C_{ib} D_{ib}) \). Number of resonance frequencies searched in each experimental characteristic is determined by the user. Figure 2 shows new method of determination of resonance frequencies and the preliminary estimation of modal parameters.

Processed models of complex objects indicated the element of APRO algorithm that should be modified - method of determination of a given number of points in the vicinity of each resonance frequency, to improve its effectiveness [7].

The process of finding accurate values starts after the determination of all preliminary estimations. In each step of the iteration, values of component characteristics are subtracted from coordinates of points belonging to the frequency range \((\omega_{k-r}, \omega_{k+r})\) which surrounds the analyzed resonance frequency \( \omega_{k} \). Determined this way corrected values of points coordinates \( sRx_{ik} \) and \( sIx_{ik} \) are determined by the formulas

\[
sRx_{ik} = Rx_{ik} - \sum_{j=1}^{n-1} Rx_{y_{j-1}} \quad sIx_{ik} = Ix_{ik} - \sum_{j=1}^{n-1} Ix_{y_{j-1}}
\] (6)
Figure 2: Concept of searching maximum values and determination of preliminary estimations of modal parameters

a) experimental characteristic $|x_i(j\omega)|$,

b) generated component characteristic $x_{il}(j\omega, A_{il}, B_{il}, C_l, D_l)$ which describes resonance frequency and satisfies the condition $|x_i(j\omega_k)| = \max$

c) the characteristic $|x_{r1}(j\omega)| = |x_i(j\omega) - x_{il}(j\omega, A_{il}, B_{il}, C_l, D_l)|$ constructed in the result of subtraction of component $|x_i(j\omega_k)|$ and experimental characteristic $|x_i(j\omega)|$.

These values are substituted into the system of equation (7) that assumes form

$$
\begin{bmatrix}
1 & 0 & -sR_{x_{ik-r}} & -sI_{x_{ik-r}}\omega_{k-r} \\
0 & \omega_{k-r} & -sI_{x_{ik-r}} & sR_{x_{ik-r}}\omega_{k-r} \\
\vdots & \vdots & \vdots & \vdots \\
1 & 0 & -sR_{x_{ik}} & -sI_{x_{ik}}\omega_k \\
0 & \omega_k & -sI_{x_{ik}} & sR_{x_{ik}}\omega_k \\
\vdots & \vdots & \vdots & \vdots \\
1 & 0 & -sR_{x_{ik+r}} & -sI_{x_{ik+r}}\omega_{k+r} \\
0 & \omega_{k+r} & -sI_{x_{ik+r}} & sR_{x_{ik+r}}\omega_{k+r} \\
\end{bmatrix}
\begin{bmatrix}
A_{ik} \\
B_{ik} \\
C_k \\
D_k \\
\end{bmatrix}
= 
\begin{bmatrix}
-sR_{x_{ik-r}}\omega_{k-r}^2 \\
-sI_{x_{ik-r}}\omega_{k-r} \\
\vdots \\
-sR_{x_{ik}}\omega_k^2 \\
-sI_{x_{ik}}\omega_k \\
\vdots \\
-sR_{x_{ik+r}}\omega_{k+r}^2 \\
-sI_{x_{ik+r}}\omega_{k+r} \\
\end{bmatrix}
$$

(7)
Solving system of equations (7) using least square method, values of modal parameters $A_{ik}$, $B_{ik}$, $C_i$, $D_k$ are corrected.

Number of iterations depends on the assumed criterion of the process quality assessment. Iterative process ends when the distance between all points from the resonance zones and the experimental characteristic is lower than δ.

$$\left| x_i(j\omega) - \sum_{k=1}^{n} \frac{A_{ik} + j\omega B_{ik}}{C_k - \omega^2 - j\omega D_k} \right| \leq \delta$$

(8)

The effectiveness of the method was evaluated [7] by the comparison of the obtained results with those found in the effect of the application of classical methods, integrated in the MATLAB Optimization Toolbox [8]:

1. Broyden-Fletcher-Goldfarb-Shanno;
2. Gauss-Newton polynomial interpolation line search method.

On that basis, the elaborated method was qualified as comparable with the most effective optimization methods.

3 Application of the approximation method APRO

3.1 Determination of modal local and global parameters

The elaborated algorithm was applied to approximate six frequency characteristics of the head of the FWD-32J milling machine forced by the harmonic excitation. On the basis of the obtained characteristics, modal parameters were determined according to the relationships

$$\omega_k = \sqrt{C_k - \frac{1}{4} D_k^2}$$

$$\vartheta_k = \frac{1}{2} D_k$$

$$V_{ak} = \frac{A_{ak} + B_{ak} \vartheta_k}{2 \omega_k}$$

$$U_{ak} = \frac{1}{2} B_{ak}$$

(9)

(10)

where: $\omega_k$ – natural frequency of the damped system, $\vartheta_k$ – coefficient of modal of the $k$-th mode shape, $U_{ik}$, $V_{ik}$ – modal amplitudes of the $k$-th mode, and global parameters on the basis of the relationships [7]

$$C_{Gk} = \frac{1}{w} \sum_{i=1}^{w} C_{i}^{i}$$

$$D_{Gk} = \frac{1}{w} \sum_{i=1}^{w} D_{i}^{i}$$

(11)

$$\begin{bmatrix}
A_{Gk} \\
B_{Gk}
\end{bmatrix} = \begin{bmatrix}
1 & 0 & \omega_{k-1} & \vdots & \vdots & 1 & 0 & \omega_{k} & \vdots & \vdots & 1 & 0 & \omega_{k+p}
\end{bmatrix} \begin{bmatrix}
Rx_{\omega_{k,l}}(C_{Gk} - \omega_{k,l}^2) - Ix_{\omega_{k,l}} \omega_{k,l} D_{Gk} \\
Ix_{\omega_{k,l}}(C_{Gk} - \omega_{k,l}^2) - Rx_{\omega_{k,l}} \omega_{k,l} D_{Gk} \\
\vdots \\
Rx_{\omega_k}(C_{Gk} - \omega_k^2) - Ix_{\omega_k} \omega_k D_{Gk} \\
Ix_{\omega_k}(C_{Gk} - \omega_k^2) - Rx_{\omega_k} \omega_k D_{Gk} \\
\vdots \\
Rx_{\omega_{k+p}}(C_{Gk} - \omega_{k+p}^2) - Ix_{\omega_{k+p}} \omega_{k+p} D_{Gk} \\
Ix_{\omega_{k+p}}(C_{Gk} - \omega_{k+p}^2) - Rx_{\omega_{k+p}} \omega_{k+p} D_{Gk}
\end{bmatrix}$$

(12)
The obtained results are discussed in [7]. Figure 3 shows the displacements of the FWD-32 milling machine head and their approximation using APRO method.

![Figure 3: Frequency characteristics 4 and 6 and their approximation using APRO method.](image)

3.2 Determination of dynamic properties of pads made of plastic

The elaborated algorithm APRO was also applied to the problems of the identification of dynamic parameters of plastic pads that are used in machine tool foundations [9]. The approximation of the experimental characteristic of relative dynamic compliance of the plastic pad enabled to determine modal damping coefficients $\delta_k$, and the loss coefficients $\eta_k$. Loss coefficient $\eta_1 = 0.0174$. for the resonance frequency $f = 178$ Hz. To verify the correctness of the obtained results, they were compared with the results of identification methods [11] and it turned out that they are comparable, since the obtained value is within the tolerance ($\eta_1 = 0.0228\pm0.0148$)

![Figure 4: Comparison of characteristic of dynamic compliance of plastic foundation pad and its APRO approximation.](image)
3.3 Reduction of data describing characteristics of nonstationary MDS system model

APRO approximation method was also used for the reduction of data that describe dynamic frequency characteristics of mass-damping-spring (MDS) system model of machine tool of varying configuration [10]. The elaborated method of reduction of data that describe characteristics of nonstationary MDS system model consists of three stages:

1. Determination of \( r \) frequency characteristics in a given frequency band for selected configurations of system model as a function of location or cutting force,

2. Approximation using APRO method of determined characteristics aimed at estimation of \( 4 \times n \times r \) modal parameters \( A(p), B(p), C(p), D(p) \), that are functions of parameter \( p \) which corresponds to the successive configurations of machine tool model,

3. Determination of coefficients and number of regression polynomials terms that describe changes of modal parameters \( A(p), B(p), C(p), D(p) \) in a function of parameter \( p \), where

\[
\begin{align*}
A_i(p) &= \sum_{j=1}^{m} a_{i,j} p^{\alpha_{i,j,m}} \\
B_i(p) &= \sum_{j=1}^{m} b_{i,j} p^{\beta_{i,j,m}} \\
C_i(p) &= \sum_{j=1}^{m} c_{i,j} p^{\gamma_{i,j,m}} \\
D_i(p) &= \sum_{j=1}^{m} d_{i,j} p^{\delta_{i,j,m}}
\end{align*}
\]

(13) (14)

The elaborated method of reduction of data that describe characteristics of nonstationary MDS system model is based on APRO approximation method and polynomial regression. It reduces significantly the number of data needed for the description of system characteristics and help to obtain fully continuous description of characteristics within frequency band and for the range of values of the parameter which characterize model nonstationarity.

Figure 5 shows 6 characteristics were recorded for successive locations within the frequency band 1-150 Hz.

The correctness of the method was verified by the computations conducted for the FWD-32J milling machine for the changing location \( \Delta s = 60 \) mm of its table along the distance \( s = 290 \) mm.

As the result of approximation the description of characteristics was reduced from 1800 real numbers to 256, which is 85%. Moreover, the continuous description of the characteristics in the frequency and table location was obtained.
Figure 5: Dynamic characteristics (RFE 1) for successive locations of machine tool table

4 Conclusions

The elaborated approximation method is an effective tool for the deliberately shaped frequency characteristic. The proposed method of determination of global parameters $A_G, B_G, C_G, D_G$ gives good results if all resonance frequencies are clearly shaped.

The method can be applied to the description of the machine tool configuration varying model. A high degree (85%) of data compression was obtained. The method was effectively used for the determination of damping coefficients of vibration isolation pads on the basis of experimental characteristics.

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