The use of sensitivity analysis for selection of decision variables in machine tool dynamic models identification

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Abstract

A method of creating the dynamic models of machine tool supporting systems and identification of their parameters is presented in this paper. The basic mathematical equations are given. The task of the appropriate selection of the model parameters for the identification is described in detail. This problem is solved with the use of sensitivity analysis of the machine tool model frequency characteristics to changes in the model parameters. This analysis enables the proper selection of the decision variables, i.e. selection of those parameters of the model which can be identified on the basis of information contained in the experimentally determined frequency response characteristics of an object that have been chosen as the bases for the model identification. The results of sensitivity analysis of a real milling machine model are provided and discussed.

1 Introduction

The optimization of the machine tool dynamic properties in order to minimize vibration propagation generated as an effect of the machining process is one of the most challenging problems encountered in designing the new and modernizing existing machines. Effective accomplishment of this task is not possible without appropriate experiments and simulation tests. Both the experimental and simulation approach, however, are complementary with each other. Confining the process to the experimental approach alone is not effective because of a great number of parameters that influence the machine tool behavior under the
dynamic load conditions and their functional dependence both on these loads and the mass - geometrical structure of the machine that changes during the progress of the machining. The simulation tests, in turn, require a physical model of the real object to be available. Such model however, constitutes only a certain approximation of the real object. A measure of this approximation is a degree of compliance of the model with the object which may only be evaluated on the basis of appropriately designed and accomplished experiment.

The dynamic properties of machine tool to a great extent depend on the spring-dissipative properties of slideway joints. Not all slideway joints, however, exert the same influence on the machine tool behavior during the piece machining. Thus the essential problem is a proper, quantitative and univocal, evaluation of the parameters and frequency range in which they influence the cutting process. Such evaluation is possible with the use of sensitivity analysis of the frequency response characteristics of the model to the changes in the model parameters. The results of this analysis are used twice during the machine tool dynamic optimization. At the initial step of the model development, these are used for the selection of such parameters of the model that can be identified on the bases of experimentally determined frequency response characteristics of the tested machine. The subsequent step involves the simulation tests which are carried out using the previously identified model.

2 Physical and mathematical models of machine tool

The physical model of machine tool is constructed in the Rigid Finite Element (RFE) method convention [1] supplemented by the slideway joint modeling option [2]. Its mass - geometrical structure is created with the use of software package DOUNO [3], on the basis of engineering drawings of tested machine. According to the RFE method the machine tool is regarded as a system of spatially arranged rigid finite elements (RFE's) interconnected with the spring-damping elements (SDE's) that have linear characteristics. The division of the machine tool into the rigid bodies is mostly carried out in the natural way, i.e. in the plains of contact of mutually movable sub-assemblies of its supporting systems. In the case of too low rigidity of any, modeled as rigid bodies, parts of machine tool, so called conceived divisions are made. Particular RFE's have six degrees of freedom each. Connecting them SDE's modeling slideway joints and turntables are represented by six weightless springs. These springs model three translational and three rotational stiffnesses and dampings. SDE's modeling lead screws are represented by a single weightless spring situated alongside its axis whilst SDE's modeling vibration isolators are represented by three such springs. The motion equation of the model has the following form:

\[ M \ddot{q} + C \dot{q} + Kq = P(t) \]  

where: \( M \), \( C \) and \( K \) are the inertia, damping and stiffness matrices, respectively; \( P(t) \) is the force vector, and \( q(t) \) is the vector of generalized displacements.
The stiffness and damping matrices are symmetric. They have the form:

\[
K = \begin{bmatrix}
K_{11} & K_{12} & \cdots & K_{1u} \\
K_{21} & K_{22} & \cdots & K_{2u} \\
\vdots & \vdots & \ddots & \vdots \\
K_{u1} & K_{u2} & \cdots & K_{uu}
\end{bmatrix}
\quad \text{and} \quad
C = \begin{bmatrix}
C_{11} & C_{12} & \cdots & C_{1u} \\
C_{21} & C_{22} & \cdots & C_{2u} \\
\vdots & \vdots & \ddots & \vdots \\
C_{u1} & C_{u2} & \cdots & C_{uu}
\end{bmatrix}
\]  

(2)

According to [1] the blocks \(K_{pp}\) of the stiffness matrix \(K\) located on the diagonal and the blocks \(K_{pr}\) located out of this diagonal are determined from the following relationships:

\[
K_{pp} = \sum_{\kappa=1}^{i_p} S_{p\kappa}^T \Theta_{p\kappa}^T K_\kappa \Theta_{p\kappa} S_{p\kappa}
\]

\[
\text{and} \quad K_{pr} = -\sum_{\kappa=1}^{i_p} S_{p\kappa}^T \Theta_{p\kappa}^T K_\kappa \Theta_{r\kappa} S_{r\kappa}
\]  

(3)

where:

- \(i_p\) - the number of SDE’s attached to RFE of number \(p\),
- \(i_{pr}\) - the number of SDE’s that interconnect RFE’s of number \(p\) and \(r\),
- \(S_{p\kappa}\) - the matrix of attaching a SDE of number \(\kappa\) to a RFE of number \(p\),
- \(\Theta_{p\kappa}\) - the matrix of direction cosines of angles between principal axes system of a SDE of number \(\kappa\) and principal axes of inertia of a RFE number \(p\),
- \(K_\kappa\) - stiffness coefficient block of a SDE of number \(\kappa\),

and:

\[
K_\kappa = \text{diag}\{k_{\kappa i}\} \quad i = 1,2,\ldots,6
\]  

(4)

The way of constructing the \(S_{p\kappa}, \Theta_{p\kappa}, S_{r\kappa}\) and \(\Theta_{r\kappa}\) matrices is described by Kruszewski [1]. The structure of damping matrix \(C\) and the procedure of constructing its particular elements are similar to that related to the stiffness matrix.

The elements of matrix \(M\) are assigned on the basis of engineering drawings of the tested machine tool, by the means of proper calculations. So, as a rule, they are not generally burdened by any significant errors. Far more complicated situation is in the case of stiffness and damping matrices \(K\) and \(C\). The creation of any elements of these matrices requires the assumption of specified values of coefficients that characterize particular joints. At the initial step of model development these values are taken from literature data. They can, however, vary in a considerable range. Consequently, the correctness of modeling at this stage of the model development in most cases is not satisfactory.

3 Model and object frequency response characteristics

As outlined above, it is necessary to identify the values of the model parameters on the basis of experimentally determined data. Any element of the stiffness or damping matrices can be thus identified. According to the algorithm developed, the identification is carried out on the basis of experimentally determined amplitude - frequency (A-F) response characteristics of the object. For this reason the motion equation (1) must be transformed into frequency domain. After applying the Fourier transform the following equation is obtained:
where

$$B(j\omega)q(j\omega) = P(j\omega)$$  \hspace{1cm} (5)

$$B(j\omega) = K - \omega^2 M + j\omega C$$  \hspace{1cm} (6)

The searched frequency characteristics \( q(j\omega) \) of the model displacements are obtained by successive solving of equation (5) at fixed excitation amplitude and successively altered frequency \( \omega \).

The dependence of the experimentally determined frequency characteristics \( z \), of the real machine tool, upon the taken for identification parameters \( x \), of the model, is nonlinear. Moreover these characteristics are burdened by systematic and random errors. This relationship, on the assumption, that the systematic errors are removed during the signal processing, may be generally described, as in the case of estimation of state of a time-invariant nonlinear system [4], by the following expression:

$$z = q(x, v)$$  \hspace{1cm} (7)

where:

- \( z \) - vector of experimentally determined characteristics of the object,
- \( x \) - vector of state, that components are to be estimated on the basis of \( z \),
- \( v \) - vector of random errors of observation,
- \( q(x, v) \) - non-linear function of arguments \( x \) and \( v \).

In the case of lack of consistency of the model with the object also their frequency characteristics are not consistent. The above constitutes the main reason for the necessity of identification of the model parameters. The algorithm of the elaborated identification method is described in the work [5].

### 4 Selection of parameters for identification

The effectiveness of the identification calculations is, to a great extent, dependent on the adequate selection of the parameters to be identified. The assumption of all model parameters as the decision variables (i.e. estimated parameters) is theoretically possible but not advisable for practical reasons. This would lead to an excessive bias on some of the resultant estimates, because the identification would have been carried out on the basis of an incomplete information of the object, obtained from the experiment. Each increase in the number of decision variables increases the probability of obtaining ambiguous solution.

For that reason only these parameters should be chosen as the decision variables during the identification process, that in a given frequency band of excitation exert predominant effect on the characteristics taken as the basis for identification. The evaluation of this influence can be made on the basis of the results of sensitivity analysis. The sensitivity of the A-F characteristics to the changes in the model parameters is different for various points of these characteristics. As its measure the value of the mean coefficient, \( M_I \), of the band sensitivity is assumed. This is calculated based on the knowledge of the derivatives of the model characteristics, from the following relationship:
This coefficient evaluates the effect of the parameter \( p_\alpha \) (\( \alpha = 1, \ldots, n \)) on the characteristic \( q_i \) (\( i = 1, \ldots, \beta \)), at the frequency band ranging from \( \omega_1 \) to \( \omega_k \).

As the decision variables should be accepted only those parameters \( p_\alpha \) for which the values of coefficient \( \text{MI} \) are large in comparison with the values of these coefficients for other model parameters.

The derivatives in equation (8) are determined with the use of equation (3). The following dependence is obtained after its differentiation:

\[
\frac{\partial}{\partial p_\alpha} q_i(j\omega) = -[B(j\omega)]^{-1} \frac{\partial B}{\partial p_\alpha} q_i(j\omega)
\]

In this equation, however, do not exist searched derivatives \( \partial q_i(\omega_k) / \partial p_\alpha \) but \( \partial q_i(j\omega_k) / \partial p_\alpha \). The relationship between them can be found as follows:

\[
q_i(j\omega) = \text{Re} q_i + j\text{Im} q_i
\]

after designation \( |q_i(j\omega)| = q_i(\omega) \)

\[
q_i^2(j\omega) = \text{Re}^2 q_i + j\text{Im}^2 q_i
\]

after differentiation of these equations it is obtained:

\[
\frac{\partial q_i(j\omega)}{\partial p_\alpha} = \frac{\partial \text{Re} q_i}{\partial p_\alpha} + j\frac{\partial \text{Im} q_i}{\partial p_\alpha}
\]

and:

\[
q_i(\omega) \frac{\partial q_i(\omega)}{\partial p_\alpha} = \text{Re} q_i \frac{\partial \text{Re} q_i}{\partial p_\alpha} + \text{Im} q_i \frac{\partial \text{Im} q_i}{\partial p_\alpha}
\]

after multiplying equation (a') by the number \( q^*_i \) conjugated with \( q_i \), i.e.:

\[
q_i^* = \text{Re} q_i - j\text{Im} q_i, \quad \text{the following dependence is obtained:}
\]

\[
\frac{\partial q_i(j\omega)}{\partial p_\alpha} q_i^*(j\omega) = \left( \text{Re} q_i \frac{\partial \text{Re} q_i}{\partial p_\alpha} + \text{Im} q_i \frac{\partial \text{Im} q_i}{\partial p_\alpha} \right) + j \left( \text{Re} q_i \frac{\partial \text{Re} q_i}{\partial p_\alpha} - \text{Im} q_i \frac{\partial \text{Im} q_i}{\partial p_\alpha} \right)
\]

Comparing the relationships (b') and (c) it is seen that the right hand side of (b') is equal to the real part of right hand side of (c). From this, the sought relationship between derivatives \( \partial q_i(\omega_k) / \partial p_\alpha \) and \( \partial q_i(j\omega_k) / \partial p_\alpha \) is obtained in the following form:

\[
\frac{\partial q_i(\omega)}{\partial p_\alpha} = \text{Re} \left[ \frac{\partial q_i(j\omega)}{\partial p_\alpha} q_i^*(j\omega) \right] [q_i(\omega)]^{-1}
\]

For the case of calculating the derivatives of \( q_i \) with respect to stiffness coefficients \( k_\alpha \), from equations (6) and (3), the following dependence is found:

\[
\frac{\partial q_i(\omega)}{\partial k_\alpha} = -\left[ K - \omega^2 M + j\omega C \right]^{-1} \frac{\partial K}{\partial k_\alpha} q_i(j\omega)
\]
The block of derivatives \( \frac{\partial \mathbf{K}}{\partial k_a} \) for the SDE \( \alpha \) that interconnects RFE's \( p \) and \( r \) has the following form:

\[
\frac{\partial \mathbf{K}}{\partial k_a} = \begin{bmatrix}
1 & p & r & u \\
0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

\[
= \begin{bmatrix}
p \\
r \\
u \\
\end{bmatrix}
\begin{bmatrix}
0 & \frac{\partial \mathbf{K}_{pp}}{\partial k_a} & \frac{\partial \mathbf{K}_{pr}}{\partial k_a} & 0 \\
0 & \frac{\partial \mathbf{K}_{rp}}{\partial k_a} & \frac{\partial \mathbf{K}_{rr}}{\partial k_a} & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

(12)

In the case when SDE interconnects RFE \( r \) with the base, the following applies:

\[
\frac{\partial \mathbf{K}}{\partial k_a} = \begin{bmatrix}
1 & r & u \\
0 & 0 & 0 \\
\vdots & \vdots & \vdots \\
0 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
r \\
0 & 0 & \frac{\partial \mathbf{K}_{rr}}{\partial k_a} & 0 \\
0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

(13)

From equations (3) the following is obtained:

\[
\frac{\partial \mathbf{K}_{pp}}{\partial k_a} = \sum_{\kappa=1}^{l_p} S_{px}^T \Theta_{px} \frac{\partial \mathbf{K}_{xx}}{\partial k_a} \Theta_{px} S_{px}
\]

(14)

and:

\[
\frac{\partial \mathbf{K}_{pr}}{\partial k_a} = -\sum_{\kappa=1}^{l_r} S_{px}^T \Theta_{px} \frac{\partial \mathbf{K}_{xr}}{\partial k_a} \Theta_{rx} S_{rx}
\]

(15)

For SDE's which model the slideway joints, and under the assumption of the constant dependence \( (k_a/k_t = m) \) between stiffness of contact in normal \( (k_n) \) and tangential \( (k_t) \) directions [6], block \( \mathbf{K}_\alpha \) described by the equation (4) takes the following form:

\[
\mathbf{K}_\alpha = diag\{A_\alpha, mA_\alpha, mA_\alpha, ml_{a1}, I_{a1}, I_{a3}\} k_{na}
\]

(16)

where:

- \( A_\alpha \) - area of contact between the slideway and the guide,
- \( I_{a1}, I_{a2}, I_{a3} \) - principal, central moments of inertia of slideway active area,

thus:

\[
\frac{\partial \mathbf{K}_\alpha}{\partial k_a} = \frac{\partial \mathbf{K}_\alpha}{\partial k_{na}} = diag\{A_\alpha, mA_\alpha, mA_\alpha, ml_{a1}, I_{a2}, I_{a3}\}
\]

(17)

In the same easy way the derivatives for SDE's that model turntables, rolling blocks, lead screws or vibration isolators can be calculated.

5 Numerical example

Among other applications, the method described in this paper has been applied to the identification of the parameters of the milling machine FWD-32J dyna-
mic model. The physical model of this machine is presented in Fig.1. This model comprises seven rigid bodies modeling the main parts of milling machine supporting system. They are interconnected by the spring-damping elements (SDEs) which model particular slideways, lead screws, turntables and vibration isolators. There are 33 SDEs in this model. Not all of these are active. Depending on the selected mode of machining, as well as on the position and weight of workpiece and pretightenings and clearances in particular joints, some slideway joints in the real machine may be temporarily or permanently inoperative. The same is applicable to the corresponding SDEs in the model. In the case analyzed, the identification had to be carried out for the structure which corresponded to a central position of the milling table at the extreme front protrusion of the overarm. The cutting force $P = 4 \text{kN}$. Its direction was defined by the angles $\alpha = 60^\circ$ and $\beta = 81^\circ$ (Fig.1).

Identification should be carried out on the basis of appropriately chosen, experimentally determined A-F characteristics of selected bodies of the tested machine tool. Due to a limited information about tested object and a great number of the model parameters the principal problem before starting identification process was the proper selection of parameters as the decision variables. This was done, using the above described procedure, on the basis of the sensitivity analysis results. The results of this analysis, for translational characteristics of the milling machine table, are given in Figure 2.

![Figure 1: The FWD-32J milling machine model - division into rigid solids and denotation of spring-damping elements modeling tested joints.](image)
Figure 2: Sensitivity of the table A-F characteristics to the changes in stiffness ($k_n$) and damping (psi) of joints.
It is seen from Figure 2 that only some parameters of the model exert essential influence on these characteristics. Particularly significant is the stiffness of slideway joints at the following contact areas: overarm - column, column - knee and knee - lower saddle. A very small influence of the stiffness of the worktable - upper saddle joint is worth noting. It appears to be significant only in the direction of Oz. It is also clearly seen that the influence of damping of particular joints on the translational A-F characteristics of the worktable is very small and for most of joints it is in principle negligible. Only some coefficients of vibration energy dissipation may be identified as so called individual decision variables on the basis of these characteristics. The other parameters should be either properly clustered, and identified as so called grouped variables, or they should be excluded from the identification process.

To increase the number of model parameters that may be acceptable as individual decision variables, and to decrease the probability of obtaining an ambiguous solution, additional information should be provided about the tested machine tool. This can be done in the form of rotational A-F characteristics of the worktable or in the form of A-F characteristics of other bodies, e.g. spindle head or column. The sensitivity analysis of the spindle head A-F characteristics carried out in this work demonstrated, that such operation should indeed bring anticipated results. As an example, the results of sensitivity analysis of A-F characteristic in direction Oz are given in Fig. 3. It is seen from this histogram, that this characteristic provides a new, substantial information about the tested machine, which should enable identification of some additional parameters of the model, e.g. damping coefficients of slideway joints modeled by SDEs No.2 and No.3 (according to designations in Fig.1). Moreover, comparing the results of the sensitivity analysis for the worktable characteristics (Fig.2) and that for

![Figure 3: Sensitivity of the spindle head A-F characteristic in the Oz direction to the changes in stiffness (k_n) and damping (psi) of joints.](image-url)
the spindle head (Fig.3) it is seen that the stiffness coefficients of overarm-column joints should be identified rather on the basis of the spindle head characteristics instead of those related to the worktable.

6 Conclusions

The creation of an adequate model of an object exhibiting multiple degrees of freedom is not, in principle, possible without a proper identification procedure and computations. Not all parameters of the model can be reliably determined on the basis of limited information contained within particular, experimentally determined, characteristics of the object. Consequently, the selection of appropriate characteristics that would constitute the basis for the model identification, together with a proper choice of the model parameters are the key factors for the successful identification.

In the case of dynamic models of machine tool supporting systems, the analysis of sensitivity of characteristics to the changes in the model parameters constitutes the most effective means for facilitating proper selection both of the frequency characteristics and parameters for identification. Information gathered in this manner is essential for beginning the effective computational identification of the model.

References