Non-linear behaviour of thin-walled open section composite beams in lateral stability

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Abstract

An analytical study along with a numerical investigation is conducted to determine the lateral buckling strength of thin-walled open-section I shape composite beams. Based on Vlasov-type linear hypothesis beam stiffness coefficients, which account for a cross section geometry and material anisotropy of the section, are obtained. In this study the axial and bending coupling terms \( (A_{13}, A_{23}, D_{13}, D_{23}) \) and the coupling between bending and twisting terms \( (B_g) \), which refer to anisotropic structural behaviour of the beam are also considered. In order to drive a better estimate of load capacity of composite beams, the transverse shear strain effect on the lateral buckling is included in calculations. To verify the analytical manipulations, the finite element software, ANSYS, is used to present a numerical solution for prediction the buckling load. As a parametric study, different boundary conditions, laminate sequences, fiber orientations are tested in order to find their sensitivity and optimum values to improve the lateral buckling strength of open-section laminated beams. Uniformly distributed, transverse concentrated and end moment loads, being popular loading systems are applied. The results reveal the non-linearity behaviour of general orthotropic beams in flexural torsion instability mode.

1 Introduction

The problem of torsional and torsional-flexural instability of isotropic beams with thin-walled open cross sections has received much attention in the recent literature. In last years, the lateral-torsional instability of isotropic beams has been

2 Analysis

2.1 Kinematic

The basic assumptions regarding the kinematics of thin-walled composite beams are:

A general plate segment of the beam is governed by elastic, classical laminated plate theory. The contour of a cross section does not deform in its plane. Each plate element in a cross section behaves as a thin-walled beam and plane sections originally normal to the beam axis remains plane. The normal stress, $\sigma_z$, in the contour direction $S$ is small compared to the axial stress, $\sigma_x$. From geometric considerations, Fig. 1, these displacements are related as [3]:

$$
\begin{aligned}
    u(z,s) &= U \sin \theta - V \cos \theta - q \phi \\
    v(z,s) &= U \cos \theta + V \sin \theta + r \phi
\end{aligned}
$$

(1)

Figure 1. Kinematics of beam section
The shear strain, $\gamma_{zs}$, of the middle surface is

$$\gamma_{zs} = \varepsilon_{xz} \cos \theta + \varepsilon_{yz} \sin \theta$$  \hspace{1cm} (2)

Thus, (1) and (2) lead to the expression

$$\frac{\partial w}{\partial s} = \varphi_x \cos \theta(s) + \varphi_y \sin \theta(s) - r \phi$$  \hspace{1cm} (3)

After integrating with respect to $s$, longitudinal displacement is obtained as

$$w(z, s) = W + x\varphi_x + y\varphi_y - \omega$$  \hspace{1cm} (4)

where

$$\omega = \int_{s} r \, ds \quad \varphi_x = \varepsilon_{xz} - U \quad \varphi_y = \varepsilon_{yz} - V$$  \hspace{1cm} (5)

The constitutive relations for a general orthotropic laminate are

$$\begin{bmatrix} N_z \\ N_s \\ N_{zs} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_z \\ \varepsilon_s \\ \gamma_{zs} \end{bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} k_z \\ k_s \\ k_{zs} \end{bmatrix}$$  \hspace{1cm} (6a)

$$\begin{bmatrix} M_z \\ M_s \\ M_{zs} \end{bmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{16} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_z \\ \varepsilon_s \\ \gamma_{zs} \end{bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{16} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{bmatrix} k_z \\ k_s \\ k_{zs} \end{bmatrix}$$  \hspace{1cm} (6b)

where

$$A_{i,j} = \sum_{k=1}^{N} \overline{Q}^{k}_{i,j} (h_{k} - h_{k-1}), \quad B_{i,j} = \frac{1}{2} \sum_{k=1}^{N} \overline{Q}^{k}_{i,j} (h_{k}^{2} - h_{k-1}^{2})$$

$$D_{i,j} = \frac{1}{3} \sum_{k=1}^{N} \overline{Q}^{k}_{i,j} (h_{k}^{3} - h_{k-1}^{3})$$  \hspace{1cm} (7)

### 2.2 Constitutive relations

After some mathematical manipulations, the constitutive relations are obtained and to avoid of lengthy paper the reader can refer to references [9]. Equilibrium states are defined by the condition that first variation of the total potential energy $\Pi_T$ is zero. The total potential energy can be represented by the following equation:

$$\delta \Pi_T = \delta \Pi_s - \delta \Pi_w$$  \hspace{1cm} (8)

Using the method of calculus of variation, the equilibrium differential equations at the buckling stage can be obtained as follows.
Computational Methods and Experimental Measures

\[
\begin{align*}
\int (I_{xx} V'' - H_c \phi'' - K_{xx} \varepsilon_{xx}'' + M_w'' = 0 \\
\int (I_{yy} U'' - H_s \phi'' - K_{yy} \varepsilon_{yy}'' + (M_w \phi)' = 0 \\
\int (I_{oo} \phi'' - GJ \phi'' - H_c V'' + H_s U'' + M_w U'' + Q_w \varepsilon_{xx} = 0 \\
\int K_{xx} (V'' - \varepsilon_{xx}'') + K_{yy} (U'' - \varepsilon_{yy}'') + Q_w \phi = 0
\end{align*}
\]

where the definition of coefficients are in appendix.

3 Examples

For simply supported end conditions, the following displacement functions satisfy the forced boundary conditions.

\[
U = \sum_{m=1}^{M} a_m \sin(\frac{m\pi z}{l}), \quad V = \sum_{n=1}^{N} b_n \sin(\frac{n\pi z}{l}), \quad \phi = \sum_{k=1}^{K} c_k \sin(\frac{k\pi z}{l})
\]

in which a, b and c are the unknown coefficients of displacement functions. By differentiating of total energy with respect to these coefficients, the following equilibrium equation are obtained

\[
\begin{bmatrix}
\frac{\pi^3}{2l^3} I_{yy} & 0 & \frac{\pi}{2l} M + \frac{\pi}{l^2} H_c \\
0 & \frac{\pi^3}{2l^3} I_{xx} & -\frac{\pi}{l^2} H_s \\
\frac{\pi}{2l} M + \frac{\pi}{l^2} H_s & -\frac{\pi}{l^2} H_c & \frac{\pi^3}{2l^3} I_{oo} + \frac{\pi}{2l} GJ
\end{bmatrix}
\begin{bmatrix}
a \\
b \\
c
\end{bmatrix} =
\begin{bmatrix}
0 \\
2l/M \\
0
\end{bmatrix}
\]

After some algebraic manipulation, the critical moment can be calculated as:

\[
M_{cr} = \frac{\pi}{l} \sqrt{I_{yy} GJ (1 + \frac{\pi^2}{l^2} \frac{I_{oo} GJ}{GJ} - \frac{4}{\pi^2} \frac{H_c^2}{I_{xx} GJ} - \frac{2}{\pi} \frac{H_s}{\sqrt{I_{xx} GJ}})}
\]

It is observed that both the $H_c$ and $H_s$ reduce the critical load. Equation (12) can be applied to the beams under the central concentrated load and uniform distributed load with different coefficient as indicated by Timoshenko [10]

4 Numerical results

4.1 Material properties

All computations in this study are carried out for glass/polyester materials with the following elastic constants [6]:

Elastic modulus: $E_t=68.9$ GPa, $E_m=3.45$ GPa, Poisson’s ratio: $v_t=0.2$, $v_m=0.3$
For a fiber volume fraction of 60%, the material constants are:
$E_{11}=4.72$ GPa, $E_{22}=8.0$ GPa, $G_{12}=3.1$ GPa, $\nu_{12}=0.24$

Dimensions of this beam is very close to the real beam used by Pandy et al. [6]. The thickness of each layer is 0.05 cm, and since the total thickness of laminates is 1 cm, the number of layers is 20. The length of beam is $l=240$ cm. The warping, torsional and flexural components are defined, [7].

4.2 Deformation coefficients

The coefficients of lateral and vertical deflection and rotation of beam, see Eqns. 10, are plotted versus applied moment in Fig. 2. The angle of fibers is placed at 0° and 45° in web and flanges, respectively. It is observed that the vertical displacement $V$ increases in a linear form, which is reasonable. Fig. 2-b.

4.3 The optimum angle

The critical moment is plotted versus the fiber orientation of web and flanges in Fig. 3. It’s revealed that the optimum angle of fibers in the flange is 0° and in the web is 45°. The results obtained for this special case can be extended for the beams with other boundary conditions and loading systems. The effect of shear strain is tested for a simply supported beam subjected to the gradient moment with previous defined cross section and aspect ration $L/H=3.6$. The critical bending moment versus web fibre angle is shown in Fig. 4. The flange fibre angle is placed at 0°. As it is expected, the shear strain effect makes more flexibility of beam stiffnesses and, therefore, causes a considerable reduction in buckling strength of composite beams.

4.4 Finite element approach

The finite element software, ANSYS 5.4 is used to present a numerical solution for estimating the buckling load. In this direction, the appropriate elements SHELL 91 and SHELL 99 are applicable. The maximum number of layers in SHELL 91 is twelve, and in SHELL 99 is 100. Fig. 5 shows the critical moments versus the angle of fibers in web. It is seen that the differences between analytical and finite element solutions are justifiable. The error percent curve indicates the maximum difference about 12%.

5 Web distortional interactive mode

Attention is focused on examining the optimal fibre orientation in the web for interactive distortional mode, when the beam span becomes shorter. In lateral-torsional buckling it is already known that, due to the rotation of the total section, the web torsional stiffness, $D_{33w}$, has an important effect on the load carrying capacity, hence placing the web fibre angle at $\pm 45^\circ$ leads to considerable
Figure 2. Variation of deformation amplitudes versus end moment.
Figure 3. Variation of flange and web fibre against of critical load

Figure 4. Shear strain effect on critical end moment
Uniform Critical Moment for a 12h Hinged Beam

Figure 5. The critical end moments against web fibre angle

Pcr for a 6hw Hinged Beam

Figure 6. Web fibre orientation versus critical end moment
(Short span, interactive mode occurred)
improvement in the collapse load. In this coupled interactive mode, it is assumed that the web is distorted and the bending stiffnesses, $D_{11^w}$, $D_{22^w}$, $D_{12^w}$ and also the coupling terms $D_{13^w}$ and $D_{23^w}$ becomes important. These terms are sensitive to the fibre orientations. It is, therefore, logical to think of designs that combine the optimum value of several terms with inherent superior performance in the distortional buckling mode for the web. To show this phenomenon numerically, the FEM ANSYS program is used. Applying an initial lateral imperfection on the web, the interactive buckling in lateral mode is observed for short span beams. Fig.6 shows the optimal fibre angle for a beam with 1.2 m span. The results obtained by numerical method verify those presented in the past [1]. It is seen that for short span, the combined action among the bending, torsional and coupling terms, shifts the optimal fibre angle, which is $45^\circ$ in prebuckling, towards $90^\circ$.

6 Conclusion

The study of lateral torsional buckling of general orthotropic composite beams is the main theme of this work. In this direction the mathematical manipulation is derived in detail. The effect of coupling terms, reflecting the anisotropic characteristics of such complicated structural elements, makes difficulties in closed form solution. However, some noteworthy founds are as follows:

The general constitutive force-displacement relations of general composite beams are presented and the differential equations govern to the stability of the problem is obtained. In some special cases, the expression for the buckling loads including coupling terms is derived.

The sensitivity of fibre angle versus buckling load is also tested. As a parametric study, the optimum fibre orientation in web is about $45^\circ$ against the buckling load.

For the distortional buckling mode, it is concluded that the peaks in the distortional failure load have been shifted toward $90^\circ$ and are accompanied by increasing bending stiffness, $D_{22^w}$.

7 Appendix

$$
\begin{align*}
I_{xx} &= \int (A_{11} y^2 + 2B_{11} y \cos \theta + D_{11} \cos^2 \theta) \, ds, \\
I_{yy} &= \int (A_{11} x^2 - 2B_{11} x \sin \theta + D_{11} \sin^2 \theta) \, ds \\
I_{\omega\omega} &= \int (A_{11} \omega^2 + 2B_{11} \omega + D_{11} q^2) \, ds, \\
H_c &= 2 \int (B_{13} y + D_{13} \cos \theta) \, ds, \quad H_s = 2 \int (-B_{13} x + D_{13} \sin \theta) \, ds \\
H_q &= 2 \int (B_{13} \omega + D_{13} q) \, ds, \quad GJ = 4 \int D_{33} \, ds \\
K_{cc} &= \int_s A_{33} \cos^2 \theta \, ds \\
K_{cs} &= \int_s A_{33} \sin \theta \cos \theta \, ds, \\
K_{ss} &= \int s A_{33} \sin^2 \theta \, ds
\end{align*}
$$
8 References