Effect of wave breaking on vertical diffusion coefficient

H. Kim, C. Jang

Department of Civil and Environmental Engineering, Kookmin University, Seoul, Korea

Abstract

The effect of wave breaking on vertical diffusion coefficient was described by using a 1DV numerical model with one-equation turbulence closure. The turbulent kinetic energy generated from the wave breaking was supplied at the water surface, and was diffused down into the whole water column. While the breaking-induced turbulence are diffused in the water column, the vertical diffusion coefficient was modified by the additional turbulent kinetic energy. The model was applied to five wave flow cases and three wave and current flow cases to assess the effect of the wave breaking and the current. The eddy viscosity or the vertical diffusion coefficient was obtained from a simple relationship between the eddy viscosity and the mixing length and the turbulent kinetic energy. The model results on the horizontal velocity profile, the turbulent kinetic energy profile, and the diffusion coefficient profile were compared with laboratory measurements, and showed reasonable agreement. The diffusion coefficient was significantly enhanced by about 40000% at the middle of wave boundary layer due to wave-breaking for CASE 1, while it was enhanced by about 200% due to a current for CASE 1-2.

1 Introduction

A wide range of theoretical or numerical models have been proposed to predict detailed flow structure for wave and current flows at coastal zones
by several researchers. These include simple empirical formulas to sophisticated numerical models involving high level turbulence closures. The models can be differently classified by physical processes considered.

O’Connor et al. [1] presented a typical 1DV wave-current boundary layer model for arbitrary intersection angles of waves and current. O’Connor et al.’s model solves two horizontal velocities in a vertical line with a mixing length closure for description of the turbulence.

The effect of wave breaking has been taken into account by a few different ways in existing models (Deigaard et al. [3]; van Rijn [4]).

Deigaard et al. [3] proposed a numerical model to compute detailed flow fields over sloped beaches in the on-offshore direction. However, preliminarily distribution of wave-period-average shear stress due to horizontal gradient of radiation stress was assumed in their model, since the purpose of their model was to describe the undertow in the on-offshore direction which has zero net flow at every section.

Van Rijn [4] proposed another numerical model system to calculate sediment transport rate for non-breaking and breaking waves and currents. However, van Rijn’s model system is in a wave-period-average form, and cannot describe intra wave-period variation of physical properties.

While waves break in shallow areas, a part of the momentum of the water waves is transformed into other forms. Only spilling breakers are dealt with here. A part of the momentum is transformed into the turbulence around the water surface, and the turbulence is diffused down into the whole water column. Consequently the eddy viscosity or diffusion coefficient for other materials like suspended sediment also changes due to the turbulence diffusion.

O’Connor et al.’s [1] intra wave-period 1DV numerical model has been refined for the present work to describe wave-current flow structures for non-breaking and breaking waves and currents introducing a one-equation (turbulent kinetic energy) turbulence closure instead of the previous mixing length hypothesis. The model can handle an arbitrary intersection angle between waves and current, and solves horizontal velocities in the x, y directions, and the turbulent kinetic energy. Some model application cases were chosen for model verification or comparison.

2 Numerical model

The present numerical model is composed of two parts, the main flow module and the turbulence transport module. Both modules are linked, and interact each other at every time step. The flow module solves the two Reynolds momentum equations in the two horizontal directions. Rodi [5]:

\[
\frac{\partial u}{\partial t} + \frac{\partial (u^2)}{\partial x} + \frac{\partial (uv)}{\partial y} + \frac{\partial (uw)}{\partial z} + fu = \]
\[- \frac{1}{\rho_0} \frac{\partial p}{\partial x} + \frac{\partial}{\partial x} \left( A_H \frac{\partial u}{\partial x} \right) + \frac{\partial}{\partial y} \left( A_H \frac{\partial u}{\partial y} \right) + \frac{\partial}{\partial z} \left( A_H \frac{\partial u}{\partial z} \right) \]
\[
\frac{\partial v}{\partial t} + \frac{\partial (uv)}{\partial x} + \frac{\partial (v^2)}{\partial y} + \frac{\partial (vw)}{\partial z} - fu = \]
\[- \frac{1}{\rho_0} \frac{\partial p}{\partial y} + \frac{\partial}{\partial x} \left( A_H \frac{\partial v}{\partial x} \right) + \frac{\partial}{\partial y} \left( A_H \frac{\partial v}{\partial y} \right) + \frac{\partial}{\partial z} \left( A_H \frac{\partial v}{\partial z} \right) \]

where \( u, v, w \) are velocity components in the \( x, y, z \) directions, respectively; \( f \) is the Coriolis parameter, \( p \) is the water pressure, \( A_H \) is the horizontal momentum exchange coefficient, \( A_V \) is the vertical momentum exchange coefficient, \( g \) is the acceleration due to gravity, and \( \rho \) is the fluid density. The model solves two variables from two equations, and need not solve the continuity equation. The water pressure variation in the vertical direction is not of interest in the present work. Adopting some simplification, Equations (1), and (2) are reduced to the following forms (O’Connor et al., 1992):

\[
\frac{\partial u}{\partial t} = - \frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{\partial \tau_{xz}}{\partial x}; \quad \frac{\partial v}{\partial t} = - \frac{1}{\rho} \frac{\partial p}{\partial y} + \frac{\partial \tau_{yz}}{\partial y} \]

\[
\frac{\partial p}{\partial x} = \rho \frac{\partial u_{\infty, x}}{\partial t}; \quad \frac{\partial p}{\partial y} = \rho \frac{\partial u_{\infty, y}}{\partial t} \]

\[
\tau_{xz} = \rho \nu_i \frac{\partial u}{\partial z}; \quad \tau_{yz} = \rho \nu_i \frac{\partial v}{\partial z}; \quad \nu_i = l^2 \frac{\partial V}{\partial z}; \quad V = \sqrt{(u^2 + v^2)} \]

\( u, v \) are the cartesian velocity components in the horizontal \((x)\) and lateral \((y)\) coordinate directions, respectively; \( x, y \) are the horizontal and lateral coordinate respectively; \( z \) is the vertical coordinate; \( g \) is the acceleration due to gravity; \( \partial H/\partial x \) is the mean water surface slope; \( l \) is the mixing length \((=kz)\) and \( u_{\infty, x}, u_{\infty, y} \) are the \( u, v \) components at the top of the wave boundary layer, respectively.

The turbulence is modelled by a one-equation closure. The turbulent kinetic energy \( k \) is convected and diffused by convection-diffusion equation:

\[
\frac{\partial k}{\partial t} = \frac{\partial}{\partial z} \left( \frac{\epsilon}{\sigma_k} \frac{\partial k}{\partial z} \right) + \frac{PROD}{\rho} - C_1 \frac{k^{3/2}}{l} \]

According to Launder and Spalding [6] the Prandtl number \( \sigma_k \) and the dissipation coefficient \( C_1 \) are taken to be \( \sigma_k = 1 \) and \( C_1 = 0.08 \). In a one-
equation turbulence model the length scale of the turbulence is to be prescribed. It is given by:

\[ l = \sqrt[4]{C_\kappa \kappa} \]  

(7)

\( \kappa \) is von Kármán’s constant (\( \kappa = 0.4 \)). Close to the bed, \( l \) has the variation normally used for boundary layer modelling, whereas it attains a constant value away from the bed to describe the conditions of free turbulence. The coefficient of momentum diffusion, the eddy viscosity \( \varepsilon \), is determined by the turbulent kinetic energy and the length scale:

\[ \varepsilon = l \sqrt{k} \]  

(8)

\[ PROD = \tau \frac{\partial V}{\partial z} \]  

(9)

This part of the turbulent production plays the most important role in the wave boundary layer.

Spilling breaker is taken into account through the turbulence module by adding additional turbulent kinetic energy around the water surface, similarly to Deigaard et al. (1991), that is:

\[ k(z_0, t) = \frac{1}{\sqrt{C_1}} \varepsilon \left| \frac{\partial V}{\partial z} \right| \]  

(10)

Existing theories for wave breaking cannot explain the breaking phenomena satisfactorily from the accuracy point of view. Even the satisfactory classification criteria for breaker type has not been obtained yet. The breaking criteria is not just a function of information at a point. In the present paper, only spilling breakers on uniform slopes are dealt with. When waves break in a spilling way, the turbulent kinetic energy is generated from the broken waves, see Figure 1.

Figure 1: Schematic diagramme of wave breaking (\( c = \sqrt{gD} \), \( c \) is the wave celerity, \( g \) is the gravitational acceleration, \( D \) is the mean water depth).
3 Model application results for waves

The model was applied to five typical wave-current flow situations, and CASE 1 has its counterpart of CASE 1-1 for non-breaking condition. The run cases were selected to examine the wave boundary layer flow behaviour, and for comparison with available measurement. The run cases are shown in Table 1.

Table 1: Model run cases.

<table>
<thead>
<tr>
<th></th>
<th>CASE 1</th>
<th>CASE 1-1</th>
<th>CASE 2</th>
<th>CASE 3</th>
<th>CASE 4</th>
<th>CASE 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depth (m)</td>
<td>1.0</td>
<td>1.0</td>
<td>0.0875</td>
<td>0.2125</td>
<td>0.071</td>
<td>0.04</td>
</tr>
<tr>
<td>Wave period (sec)</td>
<td>5.53</td>
<td>5.53</td>
<td>1.79</td>
<td>3</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Wave height (m)</td>
<td>0.5</td>
<td>0.5</td>
<td>0.1312</td>
<td>0.1487</td>
<td>0.05</td>
<td>0.028</td>
</tr>
<tr>
<td>Current speed (m/s)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Wave/current angle</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Bed roughness (m)</td>
<td>0.0009</td>
<td>0.0009</td>
<td>0.0009</td>
<td>0.0009</td>
<td>0.0009</td>
<td>0.0009</td>
</tr>
<tr>
<td>Breaking</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

The computed velocity profile for CASE 1 are shown in Figure 2. The model results show reasonable agreement with Deigaard et al.’s measurements. It should be noted that Deigaard et al’s experiment was on a uniform sloped beach with normal wave direction to the beach. The present model assumes a flat bed (zero slope), so that the model can be used for the description of flows at general coastal places, but it cannot reproduce the effect of the non-uniformity of the flow like the wave-period-average undertow over a sloped beach.

The computed velocity profiles during a wave period for non-breaking condition are almost the same as those for the breaking condition (CASE 1) above the wave boundary layer. However the thickness of the wave boundary layer becomes much larger for the wave breaking case. The overshooting at the top of the wave boundary layer is distinguished for the non-breaking case.
The computed variation of the turbulent kinetic energy profile is shown in Figure 3. Figure 3 explains that the turbulent kinetic energy is supplied at the water surface and is diffused in the water column. The computed turbulent kinetic energy distribution through the water depth is quite larger than measured values by Stive [7], see Figure 4. However, the agreement is still meaningful considering the difficulty in extracting the turbulence from raw measurements.

Figure 3: Distribution of turbulent kinetic energy ($\frac{k}{u^2}$) in time and space (CASE 1: wave length=16.9 m).
The computed distributions of the eddy viscosity were compared with measurements in Figure 5. The computed eddy viscosity agrees reasonably well near the seabed, but overestimates by an order of 2–3 at higher levels.

The model was applied for CASE 2 to see the effect of wave breaking on
the flow and eddy viscosity distribution. The computed distribution of the turbulent kinetic energy for non-breaking wave condition is much smaller than that for breaking wave condition, see Figure 6. The consequent distribution of eddy viscosity for the non-breaking wave condition was compared with that for breaking wave condition in Figure 7, which also shows that the eddy viscosity for breaking wave condition is much larger than that for non-breaking wave condition.

Figure 6: Computed time-averaged turbulent kinetic energy distribution.

Figure 7: Computed time-averaged mean eddy viscosity distribution.
4 Model results for waves with currents

The model was then applied to other wave cases including currents to examine the effect of current speed, see Table 2.

Table 2: Model run cases.

<table>
<thead>
<tr>
<th></th>
<th>CASE1-2</th>
<th>CASE1-3</th>
<th>CASE4-2</th>
<th>CASE5-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depth (m)</td>
<td>1.0</td>
<td>1.0</td>
<td>0.071</td>
<td>0.04</td>
</tr>
<tr>
<td>Wave period (sec)</td>
<td>5.53</td>
<td>5.53</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Wave height (m)</td>
<td>0.5</td>
<td>0.5</td>
<td>0.05</td>
<td>0.028</td>
</tr>
<tr>
<td>Current speed (m/s)</td>
<td>0.336</td>
<td>0.336</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>Wave/current angle</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Bed roughness (m)</td>
<td>0.0009</td>
<td>0.0009</td>
<td>0.0009</td>
<td>0.0009</td>
</tr>
<tr>
<td>Breaking</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

The effect of current on the distribution of eddy viscosity is shown in Figure 8, and Table 3.

The computed horizontal velocity profiles for CASE 1-2 and 1-3 are shown in Figure 8. The effect of wave-breaking for wave and current flows is clearly shown in the figure. The effect of wave-breaking seems similar to that for wave-only cases.
The computed eddy viscosity CASE 1-2 and 1-3 are shown in Figure 9. The wave-period-average eddy viscosities for the wave and current flow are slightly larger than those for wave only flow due to the small current speed.

The computed eddy viscosities are also shown in Table 3. The eddy viscosity is enhanced by about 40000% due to wave-breaking, and further 200% by a current for CASE 1 condition. The enhancement percentage may vary depending on the given conditions. However, these test results on typical laboratory environments demonstrate that the wave-breaking causes significant enhancement of the eddy viscosity.

The effects of strong current on breaking wave are shown in Figure 10. Very strong currents enhance the eddy viscosity by about 250% at mid-depths for CASE 4 and CASE 5 which is relatively small.

Table 3: Computed enhancement eddy viscosity due to current

<table>
<thead>
<tr>
<th></th>
<th>CASE1</th>
<th>CASE1-1</th>
<th>CASE1-2</th>
<th>CASE1-3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eddy viscosity (cm²/s) (height=0.5 m)</td>
<td>0.0475</td>
<td>0.107E-03</td>
<td>0.0898</td>
<td>0.107E-03</td>
</tr>
<tr>
<td>Eddy viscosity (cm²/s) (height=0.07 m)</td>
<td>0.253E-02</td>
<td>0.614E-04</td>
<td>0.493E-02</td>
<td>0.859E-04</td>
</tr>
</tbody>
</table>
Figure 9: The effect of current on the distribution of wave-period-averaged eddy viscosity.

Figure 10: The effect of current on the distribution of wave-period-averaged eddy viscosity.

5 Conclusions

A numerical model was formulated to describe the flow behaviour including the wave boundary layer, the water surface spilling breaker and the depth-mean current. The model reproduced reasonable flow pattern for a typical wave condition. The model was also applied to other flow conditions to examine the effect of wave breaking on the distribution of the
vertical diffusion coefficient. The computed distributions of the turbulent kinetic energy and vertical eddy viscosity reasonably agree with measurements. The model results show that when waves break, the enhanced turbulent kinetic energy significantly increases the vertical diffusion coefficient through the water column. The eddy viscosity is also enhanced by current; but the extent of enhancement is not large for weak current speed.

The model could be used for description of the suspended sediment movement for waves with or without breaking, or currents in the future. The model results need to be parameterised for engineering use, too.

6 Acknowledgements

The present work was financially supported by European Community Directorate under contract number MAS3-CT97-0106, and KOSEF under contract number I-99-005.

References


