Optimal control strategies for portfolios of managed futures

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Abstract

The problem: given only historical trading price-time data for the formally traded financial instrument of interest (portfolio), what trading decisions should be made at the current point in time so that the chosen performance measure of the portfolio is maximized? Advancing computing technology has encouraged development of an extremely large number of computational tools to solve this problem. Most of these tools employ pattern recognition, moving averages, or neural nets. Few, if any, attempts have been made to cast the problem into a single consistent theoretical framework. We use the theory of optimal control of differential equations as the framework and seek an inverse solution for the controller. Given a price-time trajectory, we seek the policies that optimize performance over that trajectory. Assuming that the behavior continues into the future for some time, the optimal control strategies are the result. This paper reports on a program of research that succeeded in that effort.

The essence of a control theoretic approach is to: 1) develop a measure of profitability of the trading portfolio, 2) computationally model the trading process operating on the price-time histories, 3) calculate estimates of the price-time histories using functions with well-known mathematical characteristics, 4) calculate, using the calculated estimates, derived functions of the price-time histories about which control variables are known and about which there is a priori knowledge, 5) simulate, using the derived functions, the trading policies and seek the values of the control variables that maximize the portfolio's trading performance over the life of the trading instruments. A realizable optimal control strategy is a direct extrapolation of the optimal simulation of trading into the future, day by day.

Keywords: managed futures, quantitative analysis, trading systems, trading simulation, dynamic systems, estimation and control.

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1 Introduction

The search for the "Holy Grail" of stock and futures trading entices one to call upon the prayer described by the philosopher F. S. C. Northrup in *The Logic of the Sciences and the Humanities [1]:*

> Were a benign and omnipotent being to appear before the vast body of creative contemporary economists, in religious fever assembled, promising the answer to one request, the prayer offered up would probably be as follows: Most Needed and Welcome being, give us a scientific theory of economic dynamics.

What should the structure of such a theory be? To what extent is market behavior influenced by randomness? Can deterministic components of market behavior be found?

The growth of trading of derivative instruments, formally and informally, has significantly increased in the last quarter of the twentieth century. The complexity and misunderstanding of derivative instruments has also exposed a great deal of risk and loss. As financial markets become more and more complex and participation in market activity becomes broader in the population, the perception of the market must also change from the closed purview of a relatively small group of traders to a truly global market with literally hundreds of millions of participants. As electronic exchanges expand to developing countries around the world, many more markets will become active on a twenty-four hour basis. As more and more people are involved in the markets, the more deterministic the markets will become.

Contemporary investors and traders today have an extremely fast growing digital information infrastructure encompassing almost all of the formal financial markets. Formal markets include open outcry and electronic markets where standardized contracts or instruments are traded and records of each transaction are maintained in an electronic format. These markets have experienced tremendous growth during the last half of the twentieth century that would have been impossible without the development of computing and communications technologies. Everyday there is generated an enormous amount of data on the prices, volumes, and, where applicable, the open interest in actively traded markets. However, the conceptualizations of what to do with all these data are essentially extensions of the work of Charles Dow in the late nineteenth century.

In this paper we describe an approach that is motivated by the ideas of modern systems engineering and the principles of optimal control of processes. This approach is holistic and includes both technical and fundamental analyses, in the sense that both the physical data on the production, supply, storage, transport, and utilization of a commodity as well as the price-time histories of its trading activity.

In this paper we make no assumptions about the form of the mathematical models of the market and let the market data determine the kinds of analytical

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tools that are applicable. The problem considered is limited to trading a portfolio of futures so as to achieve consistent profitability, i.e., the speculator's problem. Market analysis is a process that depends on our conceptualization of what the market is and how it behaves. If we assume that market price movement is random in its behavior, then we seek to apply the tools of probability, statistics, and stochastic processes. If we assume that the market behavior is deterministic, then we seek to use the paradigm of the control of dynamic processes. These several views of the market processes and the search for techniques for the valuation of trading assets have stimulated an endless pursuit of mathematical models of market processes. The efficient market hypothesis and the idea of the random walk described in [2] has been the dominant theoretical construct for almost a half-century. The evidence is not supporting this view as empirical studies have been conducted that cast doubt on the efficient market hypothesis and suggest that a strong causal component exists in market behavior [3]. We take this latter view and believe that the determinism in the markets can be isolated in many individual cases. This result appears to be demonstrated in our research, as each market requires its own unique computational model. There are instances where the controls of the computational model are unique to the particular trading instrument within a given market. Some markets behave better than others and are amenable to our techniques and still others defy us completely.

2 Identification of the problem

Optimal control theory requires that a problem be stated in terms of the fundamental principles of the control paradigm to enable computational modeling and simulation to be effective [4]. It is important to know how and why the computational model works from a scientific or mathematical point of view. Technical analysis tools lack such a mathematical foundation. The methods presented here are based entirely upon fundamental mathematics.

First we develop a measure of performance of the system being modeled and for which an extremum, i.e., a maximum or a minimum, is sought. That measure is, of course, profitability; call it P. Second, express or "model" the profitability, P, in terms of relations, functions, functionals, or processes of the system's observables about which some fundamental knowledge applies. The profitability is the sum of the profit or loss of trades of an instrument over the trading history up to the present; summed for all instruments in the portfolio. Here the term "fundamental knowledge" includes the classical fundamental analysis of the futures industry in addition to the "fundamental first principles" of the applicable mathematics, science, and engineering. The "observables" price time histories of trading activity plus other published data. Third, computational algorithms are applied to calculate filtered estimates of the price time histories and approximate them with functions from a set of known functions with well-known mathematical characteristics. These filtered estimates are used to calculate other characterizing functions of the price time histories about which the behavior of control variables are known. Fourth, using the filtered estimates, values of the control variables are chosen that modify the computational model's performance over the past life of the trading instrument. Finally, those filtered estimates and control values that yield the maximum value for the portfolio up to the current time are the optimal strategies for the portfolio. This extremum is obtained through a sequence of applied simulation runs of trading scenarios applied to the life of the trading instrument.

3 Mathematical considerations

3.1 Mathematical machinery

Most of the mathematical machinery that has been used to investigate issues of trading management is based upon the assumption that the market processes are linear. Here we make no assumption on the form of the dynamic systems. We assume that the market produces continuous time series of prices of the instruments that we wish to trade. This time series is directly observable and we assume the time series $x(t)$ is a continuous function of the variable t on the closed interval $[t_i, t_j]$, for some i, j with $i \neq j$. The interval may extend over several days or months. A portfolio is a set X of futures contracts $x_i(t)$ for $i = 1,...,n$, for some integer n. Each futures contract $x_i(t)$ is traded only for a specific interval of time, say $[t^b, t^e]$, where t^b is the beginning date and t^e is the ending date of trading of a futures contract. Each contract in a futures portfolio may have different beginning and ending dates of trading.

The development of trading for 24 hours a day in many markets makes the assumption of continuous market data a possibility so that these data can be sampled at different times during the trading day with many different sampling schemes. The schemes that we use conform to the standard Dow-Jones format but it is not necessary to do so. For our purposes, we need only to have a good estimate of a mean and median price during the trading day and a price at which execution is possible. In this scheme we take t^o and t^c to be the opening and closing times of trading of futures on day t, so that $x(t^o)$ and $x(t^c)$ are the opening and closing prices of futures contract $x(t)$, respectively. Two other values are sampled, $x^h = \max\{x(\tau)\}$ and $x^l = \min\{x(\tau)\}$ for $\tau \in [t^o, t^c]$.

We define the daily mean price, $\bar{x} = (x(t^o) + x^h + x^l + x(t^c))/4$ and the daily median price, $\tilde{x} = (x^h + x^l)/2$, used to compute characterizing estimates of the daily trading prices that are the basis for the computational model of the decision process. We assume that $x(t)$ has an additive component of random noise and we write $x(t) = y(t) + n(t)$, where $n(t)$ is the noise component that can be removed by linear filters and $y(t)$ is the deterministic component.

The assumption that the market process is deterministic suggests that there exists a vector function $\mathbb{F}(\tau, y, y', y'', u)$ such that the vector differential equation

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$\mathbb{F}(t, y(t), y'(t), y''(t), u(t)) = 0$ has $y(t)$ as a solution with $u(t)$ as a forcing or control function. We define a portfolio $X = \{x_1(t), x_2(t), \cdots, x_n(t)\}$ as a set of n futures contracts with price time histories $x_i(t)$, beginning trading date t_i^b, and ending trading date t_i^e. If $\mathbb{F}(t, y(t), y'(t), y''(t), u(t))$ is a continuous function differentiable in its variables, then we can write the vector differential equation

$$\frac{dy_i(t)}{dt} = f_i(t, y_i(t), u_i(t)) = y_i'(t),$$

where for each portfolio component *i,* the values of the solution of this differential equation is $y_i(t)$. The form of the function $f_i(\tau, y, u)$ is not known nor is it required, since we have the values of $y_i(t)$. We now find polynomials $p_i(t)$ that approximate each $y_i(t)$ as close as we please and then find the derivatives $p'(t)$ and $p''(t)$ that will approximate the derivatives of $y_i(t)$. We then seek the trading policy or set of controls that maximize the profitability of the portfolio, *X*. The profitability $P(X)$ is

$$P(X) = \sum_{i=1}^{n} \sum_{\alpha}^{\beta} \chi(t_\beta^c, t_\alpha^c)(x_i(t_\beta^c) - x_i(t_\alpha^c))\varphi(i)$$

where α is a multi-index of "buy" action times, β is the corresponding multi-index of "sell" action times, $\chi(t_\beta, t_\alpha)$ is a characteristic function, i.e., $\chi(t_\beta, t_\alpha) = 1$ if $t_\beta \geq t_\alpha$ or $\chi(t_\beta, t_\alpha) = -1$ if $t_\beta < t_\alpha$, and $\varphi(i)$ is the contract value per point of the i^{th} portfolio component. Our problem is to maximize $P(X)$.

In this case $y_i(t)$ is not known to be twice differentiable, however, we will assume that it is at least sectionally continuous on a closed time interval. Although we do not know the functional form of the mathematical model for $y_i(t)$, we do know that the polynomial $p_i(t)$ approximating $y_i(t)$ can be found as close as we please. This means that the polynomial $p_i(t)$ will have the same frequency response characteristics as $y_i(t)$ on the interval on which it is defined. As an element in the space of continuous functions $y_i(t)$ can be approximated by polynomials with continuous first and second derivatives defined on the closed time interval $[t_a, t_b]$. For these polynomials, the time rate of change of $y_i(t)$ is approximated by the time rate of change of $p_i(t)$.

In fact, we do not need the actual polynomial coefficients since the formula is not necessary in our computational approach. Figure 1 below shows a graph of the mean of the S&P 500 Index Futures plotted with the mean smoothed with a polynomial filter with end point corrections and the first derivative if the smoothed mean. The end point corrections guarantee that the smoothed function will be in phase with the original data, i.e., the smoothed function does not lag behind the data. The 3-point moving average is an example of a 3-point linear least squares polynomial filter.

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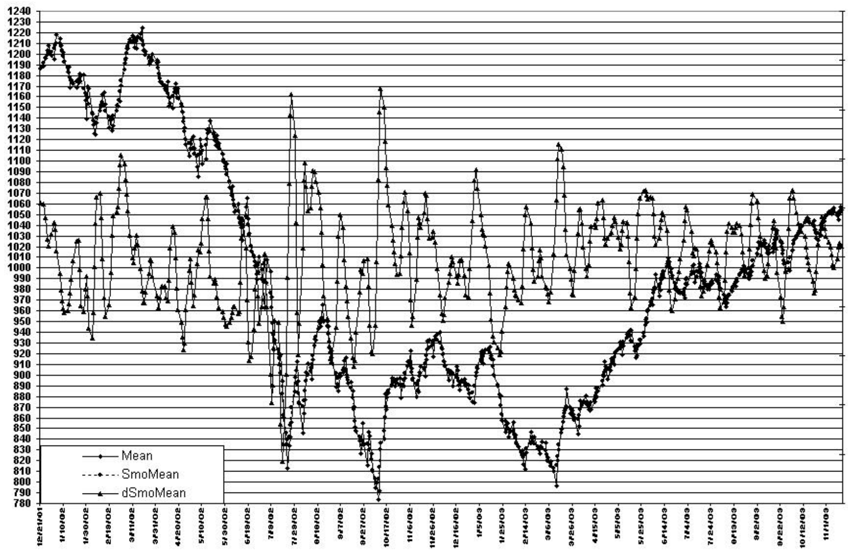

Figure 1: Mean, smoothed mean, and first derivative for S&P 500 Index Futures.

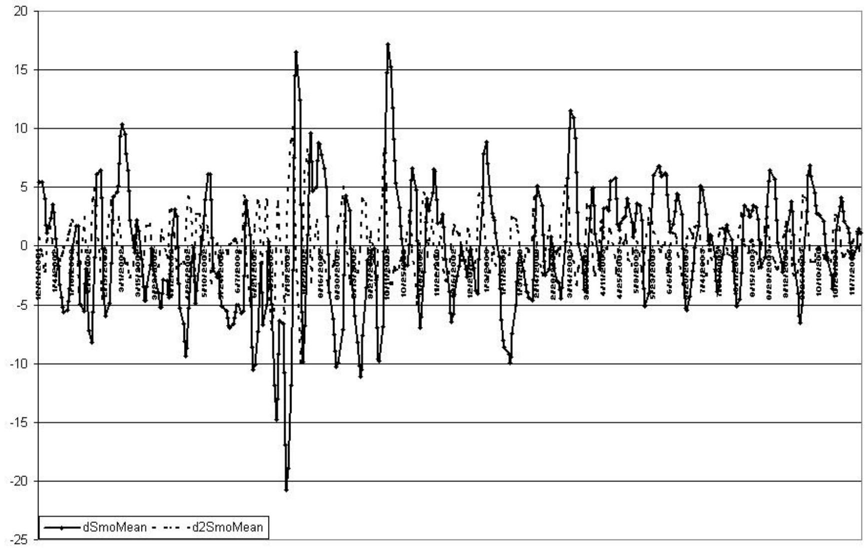

Figure 2: First and second derivatives of the smoothed S&P 500 Index Futures data.

The smoothness of the filter is increased with the length and iteration of the filter and decreased with the degree of the polynomial used to construct the filter. Higher degree polynomial filters follow the variations in the data very closely. A discussion of the theory of least squares polynomial filters is given in [5]. Figure 2 shows the first and second derivatives of the smoothed data.

Further $p'(t)$ and $p''(t)$ approximate $y'(t)$ and $y''(t)$, respectively, in the same manner. We compute these derivatives of *p(t)* with a differentiating filter that is also fitted with the endpoint correction, so that the derivatives are computed up to the boundary of the time interval. The problem can now be formulated as an "inverse" control problem: to find the control functions that maximize the performance of the decisions over the given trajectories.

3.2 Buy low and sell high implemented by an innovative technique

The old traders adage "buy low and sell high" has been a teaser for years because of the difficulty of implementation. Tools and techniques for prediction are improving such that even the weather is predictable. Markets are predictable also and we begin with a first stage analysis required for prediction. First we need to state what we are and are not trying to predict. We are not trying to predict the prices of the time series $x_i(t)$. For the managed futures portfolio, the prediction of the prices is unimportant. We need only estimate the rates of change, $p_i'(t)$ and $p_i''(t)$; and those estimates are needed for each trading day τ in a small left-side neighborhood of the present time, i.e., $\tau \le t$, where $t_i^b \le \tau \le t \le t_i^e$. For each trading day t we create a pushdown table to partition a portion of the interval $[t_b, t]$ in to $\nu + 1$ points $\{t_{-\nu}, t_{-\nu+1}, \ldots, t_{-1}, t_0 = t\}$. This pushdown table is used to compute the values of $p_i(\tau), p_i'(\tau),$ and $p_i''(\tau)$ for $t_{-\nu} \le \tau \le t_0$. Clearly, the values at the near time end point t_0 are the most important. If trading begins at t_0, then for the next trading day the push down table will be $\{t_{-\nu+1}, t_{-\nu+2}, \ldots, t_{-1}, t_0, t_1 = t\}$. Ignoring weekends and holidays, this procedure results in a sequence of functions that are different each day. The function $p_{i,0}(t)$ that approximates $y_i(t)$ on $[t_{-\nu}, t_0]$ is different from the function $p_{i,1}(t)$ that approximates $y_i(t)$ on $[t_{-\nu+1}, t_1]$. In fact, ν is small and needs only to be large enough to permit calculation of $p_i(\tau), p_i'(\tau),$ and $p_i''(\tau)$ at the end point of the interval $\tau = t$. The time series of end-point values generated by this process is substantially different from $y_i(t)$ although it might be difficult to observe. This time series of states of the portfolio is calculated independently for each trading day and is not simply a continuation of an averaging process.

3.3 The control problem analogies

Suppose we make the analogy of the portfolio X with the state S of a space ship that we are required to keep on a trajectory. Then as in the control of space flight, each state contains values of the position, velocity, and acceleration of the system, and the differential equations of motion. We then apply a "bang-bang" control based upon these states to achieve the control objective. Suppose the

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rocket is constrained to a two-dimensional space then we can control pitch by firing two rockets, switching one for up and one for down, hence the term "bang-bang." We will ignore roll and yaw. In this control scenario, the equations of motion and the initial value problems provide the trajectory.

A more complex control problem is the manoeuvrable air-to-air missile-targeting problem. This problem is to follow a target $g(t)$ with a function $r(t)$ as close as possible and to intercept the target, $g(t) = r(t)$, at some time *t*. Clearly, no model is available for the function $g(t)$, it is only known by measurement up to the present. The general class of tracking problems is to follow some desired state *r(t)* throughout the interval $t_0 \le t \le t_1$. Systems with this characteristic are simple servomechanisms and if *r(t)* is a constant it is called a regulator. In this case the trajectory of the target

Now we conceptualize the portfolio trading problem as a one dimensional flight control of a space ship, where we are only concerned with following the trajectory very closely and applying a "bang-bang" switching control, i.e., bang - buy or bang - sell. The simulation process is to optimize the trading performance on every subinterval of length $\nu + 1$ in $[t_b, t]$. Having optimised the performance on subintervals up to the present time t_0, we then allow the simulation to proceed forward in time. The examples included in this paper have been optimised for the year 2002, and allowed to run unaltered for the year 2003.

3.4 Relative maxima and minima

From elementary calculus (see Protter & Morrey [6]) and its theory of maxima and minima we know that: (1) If p is a continuous function defined on a closed interval $[t_a, t_b]$, then there exists at least one point t_{hi} in $[t_a, t_b]$ where p attains its largest value and there is at least one point t_{lo} where p attains its lowest value. (2) A function p is said to be increasing on an interval (t_a, t_b) if $p(t_2) > p(t_1)$ whenever $t_2 > t_1$ and both t_1 and t_2 are in (t_a, t_b). It is decreasing if $p(t_2) < p(t_1)$ whenever $t_2 > t_1$. (3) If $p'(\tau) > 0$ for each τ in an interval (t_a, t_b), then p is increasing on (t_a, t_b). Similarly, if $p'(\tau) < 0$ for each τ in (t_a, t_b), then p is decreasing on (t_a, t_b).

It is further well known that at a maximum or minimum of a function, the first derivative is zero and a point at which the first derivative is zero is called a critical point. The standard tests for determining maxima and minima are as follows:

Test 1(a) If p is increasing $(p' > 0)$ in some interval to the left of t_0 with t_0 as endpoint of this interval, and if p is decreasing $(p' < 0)$ in some interval to the right of t_0 with t_0 as the endpoint of this interval, then p has a (relative) maximum at t_0.

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Test 1(b) If p is decreasing $(p' < 0)$ in some interval to the left of t_0 with t_0 as end point of this interval, and if and if p is increasing $(p' > 0)$ in some interval to the right of t_0 with t_0 as the endpoint of this interval, then p has a (relative) minimum at t_0.

Test 2 If p has two derivatives and p'' is continuous and t_0 is a critical point $(p'(t_0) = 0)$, then (a) if $p''(t_0) > 0, p$ has a relative minimum at t_0, (b) if $p''(t_0) < 0,$ p has a relative maximum at t_0, if $p''(t_0) = 0,$ the test fails.

It is clear that these statements from the calculus assume that we have t_0 as an interior point of some interval. In the application we are considering, t_0 is the right endpoint of the interval so that we have only half of the information in the neighborhood of the boundary. We have only information as we approach the right endpoint from the left side, if time is increasing to the right. But we also have other information, namely that when $p''(t_0) = 0,$ t_0 may be a point of inflection of p. If the graph has a point of inflection at t_0, then the graph is going to be either concave upward on one side of t_0 and concave downward on the other or concave downward on one side of t_0 and concave upward on the other. *In either case, the point of inflection occurs before the maxima or minima are attained*. This means that turning points in the time series can be predicted. Observation of this behavior in the market data confirms that the relative maxima and minima are identified.

We say that p is a monotonic increasing (or strictly increasing) function on an interval when the first derivative is positive $(p' > 0)$ on the interval and p is a monotonic decreasing (or strictly decreasing) function on an interval when the first derivative is negative $(p' < 0)$ on the interval. We say the p is strongly monotonic where p is increasing and both $p' > 0$ and $p'' > 0$. This means that not only is the function increasing strongly, but also the rate of increase is increasing. A similar statement holds for monotonic decreasing functions. The computational model takes advantage of this fact and applying this theory to trading data begins with the identification of strongly monotonic price-time series estimates where $p'(\tau)$ and $p''(\tau)$ in some interval near each trading day t. The pseudocode for this process is what begins the computational trading algorithm.

3.5 Procedure TradeRecommendations

Procedure IdentifyTrend
If $p'(t) > Zero0$ and $p''(t) > Zero4$
Then Call Up_Trend
ElseIf $p'(t) < -Zero0$ and $p''(t) < -Zero4$

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```
                Then Call Down_Trend
        Else
        Call Sideways_Trend
End If
End Procedure
```

Clearly, Zero0 and Zero4 are parameters that control the invocation of the Up_Trend, Down_Trend, or Sideways_Trend procedures. There are several variations of this algorithm as some markets require the strongly monotonic algorithm and in others the monotonic algorithm is effective. Still others require the monotonic algorithm with "bang-bang" acceleration, i.e., they depend only on the sign value (plus or minus) of the acceleration.

Each trend has a separate decision process module in the computational model. Sideways markets are the most complex and have the highest risk because traditional wisdom suggests that positions held through sideways markets almost always loose. The sideways market analysis is more complex than that in either up or down trending markets.

3.6 Up-Trend procedures

The Up-Trend procedure is invoked if the trend is strongly monotonic increasing. If no position is held, then the up-trend process will set the action variable to "buy" to establish a long position and pass that action to the rest of the system. If a short position is held, then the up-trend process will set the action variable to "Buy" to cover the short position. If a long position exists, then Up-Trend evaluates the position to determine if it should be held.

3.7 Down-Trend procedure

The Down-Trend procedure is invoked if the trend is strongly monotonic decreasing. If no position is held, then the Down-Trend process will set the action variable to "Sell" to establish a short position and pass that action to the rest of the system. If a long position is held, then the Down-Trend process will set the action variable to "Sell" to liquidate the long position. If a short position is held, then Down-Trend will evaluate the position to determine if it should be held.

3.8 Sideways-Trend procedure

The sideways trends are to be avoided if at all possible. The decision algorithms invoked in sideways markets depend strongly on combinations of values and inequality constraints on the control.

3.9 Manage & record trade procedure

Each trade has to be managed as to its performance. This procedure invokes stop-loss and take-profit policies that are independent of the exit policies determined from the curvature of the time-history.

4 Simulation results and draw down analyses

The model portfolios described here included ten contracts from each of the currencies, energies, interest rates, grains and oilseeds, stock index futures, meats, metals, and "softs" markets. Included in this paper are results from the stock indices and the currency markets. Complete results from these and other simulations are available on the web site www.ohcriner.com.

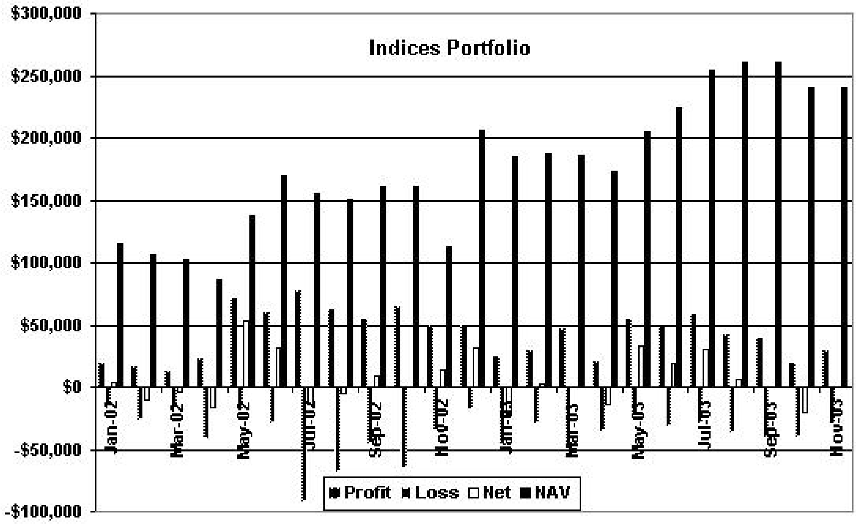

Figure 3: Monthly performance of the stock index portfolio.

Table 1: Stock indices portfolio component summary.

Stock Indices	Beginning Date	Ending Date	Margin	Net P/L	ROR
Dec2003DJ	2/15/2002	12/1/2003	$5,400	$1,645	30%
Mar2004DJ	5/28/2003	12/1/2003	$5,400	$620	11%
Dec2003SP *	1/8/2002	12/1/2003	$17,813	$16,650	93%
Mar2004SP	4/1/2002	12/1/2003	$17823	$32,000	180%
Dec2003ND	4/4/2003	12/1/2003	$11250	$3,975	35%
Mar2004ND *	7/7/2003	12/1/2003	$11250	$9,100	81%
Dec2003RL	1/7/2003	12/1/2003	$15,000	$38,550	257%
Mar2004RL *	4/4/2003	12/1/2003	$15,000	$10,750	72%
Dec2003MD	1/7/2003	12/1/2003	$13250	$12,950	99%
Mar2004MD *	4/4/2003	12/1/2003	$13,250	$4,450	34%

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Figure 3 shows the monthly profit, loss, net profit and net asset value for the portfolio of one contract of ten stock index futures. The initial net asset value is a taken to be the sum of the individual contract margin requirements. Table 1 shows the composition of the stock index portfolio with a percentage gain over the entire period of the simulation.

Figures 4 and Table 2 show the same results for the currency market.

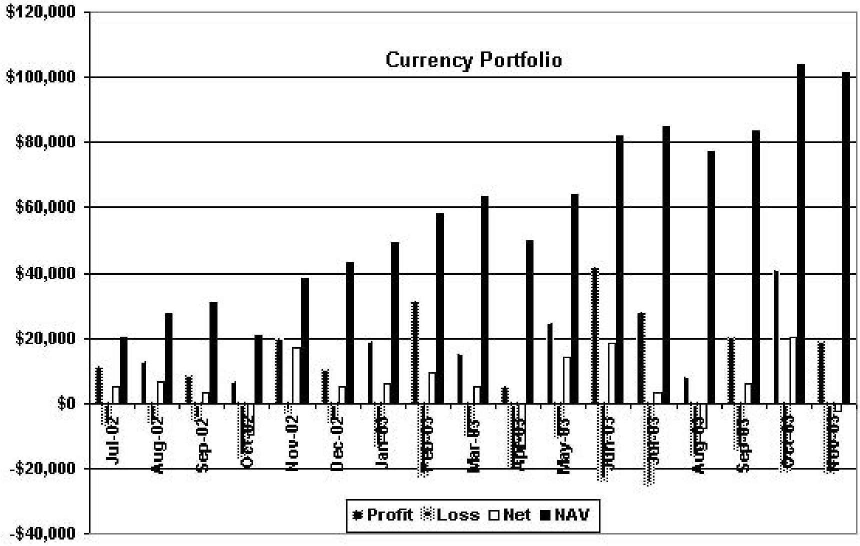

Figure 4: Monthly performance of currency futures portfolio.

Table 2: Currency futures portfolio performance summary.

Currencies	Begin Date	End Date	Margin	ROR
Dec2003BP*	07/01/02	12/01/03	$2,970	-8.42%
Dec2003CD	07/02/02	12/01/03	$1,486	-6.73%
Dec2003EC	07/01/02	12/01/03	$4,320	787.62%
Dec2003JY	07/01/02	12/01/03	$4,590	491.29%
Dec2003AD	07/01/02	12/01/03	$2,160	332.18%
Dec2003SF	07/01/02	12/01/03	$3,510	91.88%
Mar2004BP	09/30/02	12/01/03	$2,970	639.31%
Mar2004EC	09/30/02	12/01/03	$4,320	875.58%
Mar2004JY	09/30/02	12/01/03	$4,590	236.93%
Mar2004SF	09/30/02	12/01/03	$3,510	477.92%

5 Conclusion – approaching determinism

It would indeed be wonderful to have a deterministic theory of dynamic market behaviour that has the same logical basis and validity as, say, physics. Such a

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theory is not likely to be developed in the near future. However, the results presented here make an extremely strong case for determinism in market behavior.

The assumption of deterministic markets has inspired our continuing research activities toward computational "fundamental" analyses modules for classes of formally traded financial instruments. Objectives of this research include:

- Automated optimization of control strategies for managed portfolios utilizing clusters of computers
- Automated stock and option research and analysis systems
- Optimal control modules for spread, arbitrage and hedging strategies for manufacturer's and wholesaler's strategic marketing methodologies
- Commodity risk management and supply chain optimization systems
- Integration of public and proprietary information sources

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