Pricing of options in emerging financial markets using Martingale simulation: an example from Turkey

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Abstract

The objective of this study is to test the applicability of the empirical Martingale simulation approach in pricing of options in the Istanbul Stock Exchange (ISE) market with the aim of providing a workable pricing model that offers realistic solutions for both practitioners and planners in Turkey. Two sets of five different European call options are created by changing the maturity, interest rate, strike price and the volatility. One set of options is assumed to be written on the ISE Composite Index and the other set assumed to be written on the most volatile stock included in the Istanbul Stock Exchange Composite Index, the Yapi Kredi Bank (YKB) stock. The prices for the five different options on the ISE Composite Index and on YKB stock are found by applying the Binomial model, Monte Carlo simulation and Martingale simulation, then they are compared with the results of the Black–Scholes formula. It is found that the Martingale simulation has performed better than the first two models and produced results with smaller or no price differences in most cases. Comparing the coefficient of variation of Monte Carlo simulation versus Martingale simulation revealed that Martingale results were more dependable and the dependability of Martingale results increased for the option contracts that initiated out-of-the-money.
Keywords: empirical martingale simulation, Monte Carlo simulation, equivalent martingale measures.

1 Introduction

The studies in options pricing theory, pioneered by Charles Castelli in 1877, entered a new era with the work of Fisher Black and Myron Scholes called



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"Options Pricing and Corporate Liabilities" published in 1973. The model, known as "the Black- Scholes Options Pricing Formula", for the first time enabled to determine the value of a European-type call option written on a stock with no-dividend payments, using constant volatility and constant interest rate Neftçi [8].

The assumption used in the model for the initial application and for most of the research in options pricing theory thereafter, was that in an efficient market there are no arbitrage opportunities, accordingly probability measures that are free of risk are used.

Until 1990's, the development of options pricing theory has been in the direction of eliminating the unrealistic assumptions of the Black-Scholes formula, which lends itself to analytical solutions. The emergence of exotic options and extension of them, have been the second milestone in the development of options pricing models. Options such as Asian–type options, look-back options, barrier options caused formulas to become more complex and resulted in situations that are impossible to solve Baxter and Rennie [3].

The complexity of analytical solutions to the above model, and the fact that they did not produce any solutions in some situations, have lead practitioners in the market to choose numerical solutions. At this point, a theory that constituted the foundation of the options pricing theory has become important. This is the Martingale Theory that supplies the probability measures free of risk in an arbitrage-free market. This is the interest rate used in options pricing models. Martingale theory, emergence of which is as old as probability theory, provides solutions both for analytical and numerical models and simplifies them.

The appearance of Martingale theory in finance literature is in 1960's with the application of Martingale stochastic processes in a fair game as the standard model. This model explained the stock price movements in an efficient market Bardhan and Chao [2]. In 1979, M. Harrison and D. Kreps formalized the use of Martingale theory in securities and options markets with their work "Martingales and Arbitrage in Multiperiod Securities Markets".

Based on the observation that stock prices usually do not posses Martingale property, Duan et al [5] proved that adjusted probability density function developed using equivalent martingale measures are equivalent to the real probability distribution of the price of the stock. In their study, by imposing Martingale properties to the sample paths of Monte Carlo simulation used in pricing of option contracts, Duan and Simonato have been able successfully remove the disadvantages of the Monte Carlo Simulation approach resulting from the simulation having price predictions that violate the rational option pricing bounds.

The objective of this study is to test the applicability of the Empirical Martingale Simulation model on Istanbul Stock Exchange (ISE) Market in Turkey, with due consideration given to the changes in factors that affect the price of an option. Therefore, our aim is to provide a workable pricing model that provides realistic solutions for both practitioners and planners in Turkey. This is rather vital, especially when the Turkish Options Market is expected to be operational in March 2004.



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To evaluate the accuracy of the results produced by the model, the prices of options on the ISE Composite Index and on YKB stock are found by applying Binomial model, Monte Carlo Simulation and Martingale Simulation, then they are compared with the results of the Black–Scholes formula. It is found that the Martingale simulation has performed better than the first two models and produced results with smaller or no price differences in most cases. Comparing the degree of confidences of Monte Carlo simulation versus Martingale simulation revealed that Martingale results were more dependable and dependability of Martingale results increased for the option agreements that initiated out-of-the-money.

2 Theoretical background

The invention of new options has increased the situations where analytical formulas cannot provide solutions. Especially in path dependent options, most of the time the formulas are not used at all.

In pricing of look-back options, spread options, portfolio options, Asian options etc., two numerical models can be distinguished: Binomial Tree Model, and Monte Carlo Simulation. Of these, Monte Carlo Simulation is criticized of resulting in values that violate the rational option pricing bounds and being expensive due to producing good solutions only for large samples. These disadvantages can be eliminated partially by using variance reduction techniques. Standard Monte Carlo Simulation calculates option value C:

$$C = e^{-rT}\left[E_0 f\left(S_T\right)\right]$$

where $\left[E_0 f\left(S_T\right)\right]$, expected value at initial period of stock price at time *T*. By using the formula:

$$C_{MCS}^{(n)} = e^{-rT}\frac{1}{n}\sum_{i=1}^{n} f\left(S_i, T\right)$$

As the sample size (n) becomes larger and approaches infinity, $C_{MCS}^{(n)}$ value obtained from the simulation approaches the real value C, as a result of law of large numbers.

Empirical Martingale Simulation Model on the other hand, in addition to having the same benefits of the Monte Carlo Simulation used together with the variance reduction techniques, gives better solutions in smaller sample sizes Duan and Simonato [4]. According to this model, the stock price process on which the option is written is not a martingale because of the following property:

$$e^{-rt}\frac{1}{n}\sum_{i=1}^{n} S_{i,t} \neq S_0$$

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The sample paths of the price process $S_{i,t}$ of the security in the standard Monte Carlo simulation is converted into Martingale using the transformation:

$$S^*_{i,t} = \frac{S_0 e^{rt}}{\bar{S}_{n,t}}$$

where $\bar{S}_{n,t} = \frac{1}{n}\sum_{i=1}^{n} S_{i,t}$

The price of the option after transforming is obtained by the formula below:

$$C^{(n)}_{EMS} = e^{-rT}\frac{1}{n}\sum_{i=1}^{n}\left(S^*_{i,T}\right)$$

Duan et [5] proved that the simulation results obtained has an asymptotic distribution of the following form:

$$\sqrt{n}\left(C^{(n)}_{EMS} - C\right) \sim N(0, v)$$

3 The study

The prices of five different options on the ISE Composite Index and on YKB stock are found by applying Binomial model, Monte Carlo Simulation as well as Martingale Simulation, then they are compared with the results of the Black–Scholes formula.

The data relating to the European type call options assumed to be written on the ISE Composite Index and on YKB stock are shown in Table 1 and Table 2.

Table 1: The characteristics of ISE Composite Index Option Contracts.

	Type 1	Type 2	Type 3	Type 4	Type 5
Maturity	30 days	60 days	30 days	30 days	30 days
Interest rate	28.23 %	28.23 %	10 %	28.23 %	28.23 %
Volatility	40.11 %	40.11 %	40.11 %	40.11 %	40.11 %
Stock price: S_0	10.848 TL	10.848 TL	10.848 TL	10.848 TL	10.848 TL
Exercise price	10.848 TL	10.848 TL	10.848 TL	9.763 TL	11.933 TL

TL: Turkish Lira

Table 2: The characteristics of YKB Option Contracts.

	Type 1	Type 2	Type 3	Type 4	Type 5
Maturity	30 days	60 days	30 days	30 days	30 days
Interest rate	28.23 %	28.23 %	10 %	28.23 %	28.23 %
Volatility	67 %	67 %	67 %	67 %	67 %
Stock price: S_0	4.800 TL	4.800 TL	4.800 TL	4.800 TL	4.800 TL
Exercise price	4.800 TL	4.800 TL	4.800 TL	4.320 TL	5.280 TL

TL: Turkish Lira

Data relating to the Type 1 option contract in both Table 1 and Table 2 are taken from the actual market data of the related financial securities. In determining the volatility, historical volatilities of the securities are used.

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Therefore to estimate volatility, the daily closing prices three months prior to the beginning of the maturity period are used.

Risk-free interest rate is determined to be the rate of interest of the term-deposit having the same maturity as the option contracts net of expenses. Hence, risk free rate is calculated as the arithmetic average of the term-deposit interest rates of 48 banks after all the taxes and fixed charges are deducted.

Exercise prices of the option contracts are determined as three different prices: at-the-money, out-of-money, in-the-money; this way, the effect of the initial conditions on the pricing model were tested.

Five option contracts are created using the above data, to reflect the effects of the variables on the option prices obtained. In addition, the fact that the two securities have different volatilities makes the comparisons of the option agreements more meaningful.

4 Results of the analysis

4.1 Type 1 option contract

Using actual data on the market, this option is assumed to have a 30-days maturity and at- the-money at the start of the maturity period. Differences between the prices of the option contract found using Black–Scholes (BS) formula and Binomial model, Monte Carlo Simulation (MCS) and Martingale Approach (MA) are shown in Figure 1.

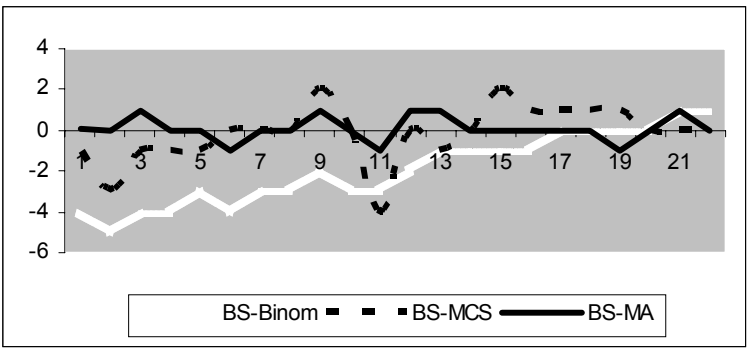

Figure 1: Valuation differences of the ISE Composite Index Type 1 Option Contract.

The results of the YKB Type 1 Option agreement are shown below in Figure 2.

Although MCS produced closer results to the BS analytical solution than the Binomial model, the results of the Martingale simulation for both types of options are very close to the values of the BS solution, having the same values most of the time. Other important difference of MCS and Martingale Simulation

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approach emerges in the levels of variation coefficient of the two methods. The coefficient of variation for MCS is 2.2 times the coefficient of variation of Martingale Simulation approach, indicating that Martingale results are more than twice as reliable.

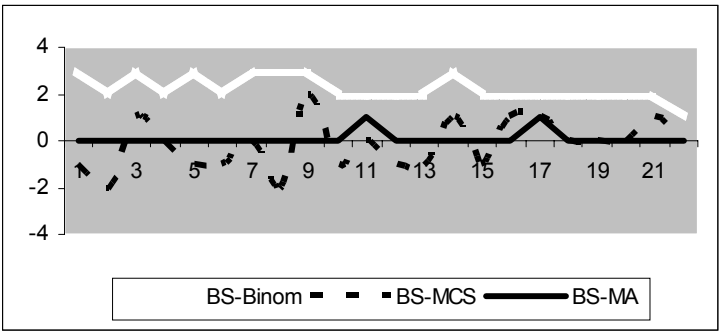

Figure 2: Valuation differences of the YKB Type 1 Option Contract.

4.2 Type 2 option contract

For Type 2 Option Agreement, maturity date has been extended to 60 days. The values of this option written on ISE composite index are presented in Figure 3.

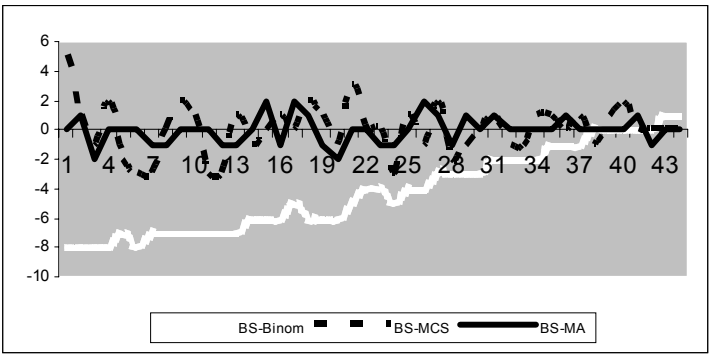

Figure 3: Valuation results of the ISE Composite Index Type 2 Option Contract.

The values of Type 2 option written on YKB are presented in Figure 4.

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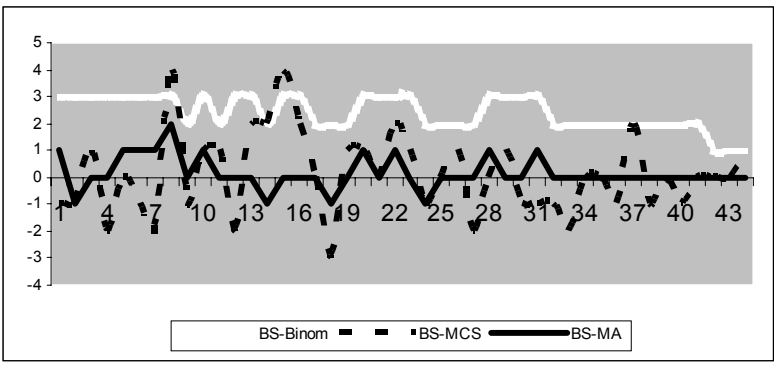

Figure 4: Valuation differences of the YKB Type 2 Option Contract.

For Type 2 options it can be concluded that the lengthening of maturity has not affected the results and once again, MA appeared as the best method in terms of differences in price with respect to analytical model. The coefficient of variation for MCS is 2.5 times the coefficient of variation of Martingale Simulation approach, indicating that Martingale results are again more than twice as reliable.

4.3 Type 3 option contract

Type 3 option contract is formed by only decreasing the 28.23 % interest rate used in Type 1 and 2 options contract to 10%. In this case it can be observed that differences of analytical BS formula and Binomial Model exhibit exactly opposite patterns with respect to Type 1 and 2 contracts.

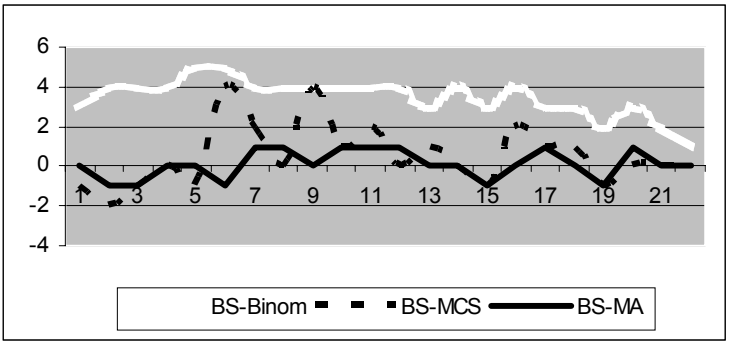

Figure 5: Valuation differences of the ISE Composite Index Type 3 Option Contract.

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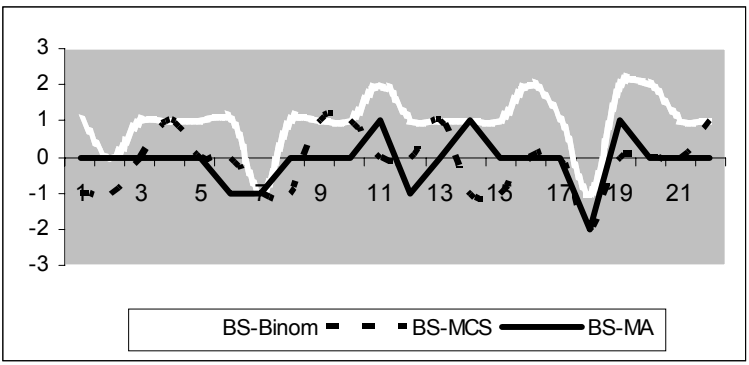

Figure 6: Valuation differences of the YKB Type 3 Option Contract.

An interesting observation in Type 3 Option Contract is that positive differences of analytical BS formula and Binomial Model exhibit exactly opposite patterns with respect to Type 1 and 2 contracts.

Bearing in mind that ISE Composite Index is less volitile than YKB stock, in low volitility situations price differences of Binomial model changed signs with changes in interest rates whereas the differences of MCS and MA did not.

Similar to Type 1 nd 2 Option Contracts, the coefficients of variation of MCS are 2.2 times the coefficient of variation of Martingale Simulation Approach in Type 3 contracts, still making MA twice as dependable.

4.4 Type 4 option contract

The Type 4 option contract is assumed to start out of -money at the beginning of the maturity period with an exercise price 10 % less than the stock price.

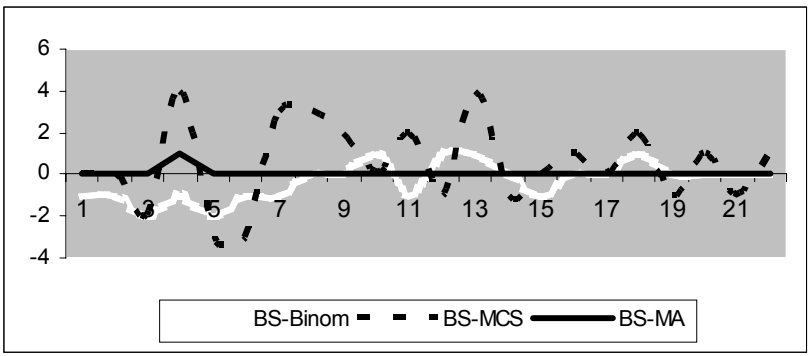

Figure 7: Valuation differences of the ISE Composite Index Type 4 Option contract.

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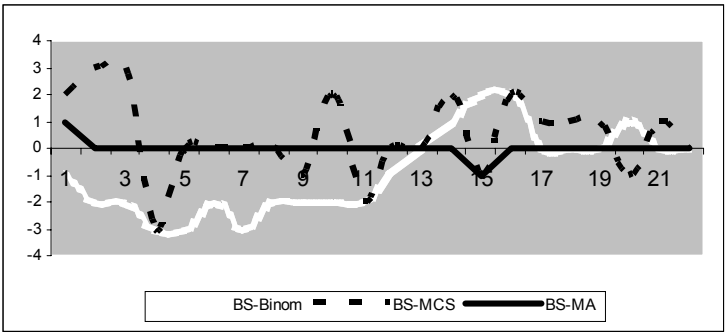

Figure 8: Valuation differences of the YKB Type 4 Option Contract.

The coefficient of variation for MA in ISE Composite Index Type 4 option is quite low, therefore negligible. The coefficient of variation for MCS is 7.4 times larger than the coefficient of variation of ISE Composite Index and 5 times larger than the coefficient of variation of YKB Stock Type 4 option.

4.5 Type 5 option contract

In contrast with the Type 4 option contract, Type 5 Option contract is assumed to start in-the-money at the beginning of the maturity period with an exercise price 10 % more than the stock price.

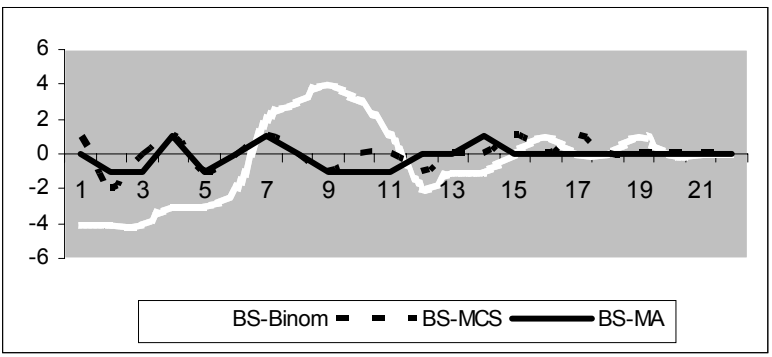

Figure 9: Valuation differences of the ISE Composite Index Type 5 Option Contract.

For both set of Type 5 Option Contracts, the coefficient of variation for MCS is 1.3 times the coefficient of variation of Martingale Simulation Approach. However, unlike other type of option contracts, for ISE Composite Index Type 5 Option Contract, the coefficient of variation is increasing with approaching of

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the maturity date. The fact that this option is in-the-money and has no value at the maturity date would be an explanation for this situation.

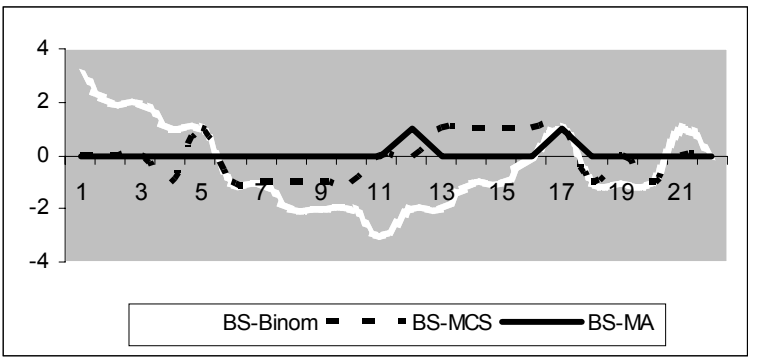

Figure 10: Valuation differences of the YKB Type 5 Option Contract.

Comparing the results of Type 1 and 2 Option Contract written on ISE Composite Index shows that extending the maturity of the option contract does not affect the superiority of the Martingale Approach in terms of price differences. However this superiority is decreasing in case of higher volitility as the comparison of Type 1 and 2 Option Contract written on YKB Stock depicts.

Similar results are obtained for Type 3 option contracts is formed by only decreasing the 28.23 % interest rate used in Type 1 and 2 options contract to 10% and Type 5 Option Contracts with in-the-money exercise price. In terms of price deviations from the analytical BS model, Martingale Simulation Approach performed better than the MCS and Binomial Model in every situation and produced values equal to the analytical model 80% of the time on the average. However, deviations were a little larger in options with greater volatility.

5 Conclusions and recommendations

Martingale theory, by supplying the risk-free probability measures, contributes to the simplification of the probabilistic pricing methods and to the calculation of risk premium in pricing of financial assets. Working with equivalent Martingale measures in the financial asset pricing, appears as the main advantage derived from the theory, allowing the determination of the price of an option without having an average return value (μ) for the option in question Musiela and Rutkowski [7].

In this empirical study, the use of Martingale Simulation Approach in determining the price of options assumed to be written on stocks that are traded in İSE is demonstrated. Two sets of five different European call options are created by changing the maturity, interest rate, strike price and the volatility. One

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set of options is assumed to be written on ISE Composite Index and the other set assumed to be written on the most volatile stock included in the ISE Composite Index, the YKB stock.

In order to evaluate the accuracy of the results produced by the model, the prices for the five different options on the ISE Composite Index and on YKBstock are found by applying Binomial model, Monte Carlo Simulation and Martingale Simulation, then they are compared with the results of the Black–Scholes formula.

According to the results, for both set of options Martingale Simulation Approach exhibits prices closer to the values of the BS solution than MCS and Binomial Method, having the same values with the BS Model most of the time. The results indicate that changing the value of different factors does not affect the superior performance of Martingale Simulation. Only factor that is of importance that causes results to deteriorate a little is the volatility of the stock on which the option is written. But volitility also causes deterioration in the Binomial Model and MCS solutions as well.

Comparisons of the coefficient of variation of MCS and Martingale Approach reveal the superiority of the Martingale results. In first three option contracts for both sets, Martingale Approach has a coefficient of variation two times less than the MCS method. This difference decreases to less than 1.5 times for Type 5 option contracts that are in-the-money and magnifies as much as 7 times for Type 4 Option Contracts that are starting in-the-money. According to these results, it can be concluded that although same number of sample paths are used, Martingale Simulation Approach is clearly superior to Monte Carlo Simulation method and even more so for option contracts that are starting out-of-the-money.

When difficulties in obtaining analytical solutions in pricing Asian option contracts are considered, Martingale Simulation Approach offers an important alternative in determining the prices of such options. Since the price of an Asian option contract is dependent on the path of the price of the stock on which the option is written, use of simulation becomes more meaningful.

In addition to errors that result from the sample size during simulation, the main disadvantage of the Monte Carlo Simulation approach is the presence of sample paths that cause the simulation having price predictions that violate the rational option pricing bounds. Using Martingale Simulation Approach eliminates this disadvantage with more realistic price estimates.

Other advantage of Martingale Simulation Approach is more realistic assumption. The only assumption of the approach is existence of an arbitrage free efficient market. This is a realistic assumption for the study we have conducted. Developments in terms of the efficiency of İstanbul Stock Exchange market in Turkey after Options Market becomes operational in 2004 are expected to support this assumption. In conclusion, validity of the assumption of having an arbitrage free market due to non-existence of an alternative market, and hence, the existence of risk free probability measures make the Martingale Simulation Approach a candidate to successfully provide more realistic prices for option contracts in Turkey.



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