Using options theory to identify the optimal dispatch strategy for electricity producers in a deregulated environment

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Abstract

The deregulation of the production of electricity has dramatically changed the rules of the electricity market. Before deregulation, to design their optimal dispatch policy, producers had to solve a complicated optimization problem with many operational constraints. Since deregulation their situation is even more complicated. Because they are in a competitive situation, they cannot spread profit and loss over time. Accepting to produce even for a short time at a loss has become a much riskier strategy. One effect of deregulation has been to increase significantly the volatility of the price of electricity. In this research, the decision making of the producers is modeled as a mix of optimization and risk management. To quantify the risks use is made of instruments borrowed from option theory. These tools are put in an optimization framework, which involve the operational and physical constraints involved in producing electricity. Flexibility also comes at a cost. The use of option theory allows a quantification of the premium that the reduction of the risk for the production associated with the failure of plant (reliability) and the ability for the production to adjust instantaneously to sudden changes in the demand (flexibility) are worth.

Keywords: risk management, financial options, deregulation of electricity production, optimization under uncertainty.

1 Introduction

The “unit commitment problem”, i.e. the optimization problem that electricity producers have to solve to optimize their electricity dispatch policy, has been a subject of research for decades [1], i.e. far before electricity production became
Deregulated. Deregulation has made this already difficult problem, even more challenging [2,3,4].

Deregulation means that the price of electricity is determined competitively, in a spot market. The spot price is determined from bids made by producers, in a system of auction. The bids are matched with the demand on an hourly basis. Deregulation has changed the electricity equation significantly. Evidence that risk has become a major preoccupation in the electricity market is the plethora of hedging instruments and bilateral arrangements that has appeared since deregulation. All sorts of exotic options are being now used in the electricity market [3]. Futures, future contacts are heavily discussed in this context too [5].

One reason why the design of an optimal dispatch policy is so challenging is that the spot price $P_{(t,i)}$ ($P_{(t,i)}$ is the spot price for the hourly slice “i”, on day “t”) cannot be described by a simple Geometric Brownian motion. The volatility as well as the average value of the spot price varies during the day. Still it is possible to model the spot price with stochastic processes using some detours. In this work we will use the observation that the spot price can be described by the following system of stochastic differential equations:

$$dP_{(t,i)} = \alpha \left(P_{(t,i)}\right) dt + \sigma_{i} P_{(t,i)} dz$$  \hspace{1cm} (1)

The index “i” refers to the time of the day. The volatility $\sigma_{i}$ is assumed to be a constant (does not depend on “t”), but different for each hourly slice “i”. This description of the price dynamics assumes that day after day, the consumption follows a similar pattern. So the volatility changes during the day, but the volatility at a certain time of the day is the same day after day.

The results do not depend on the specifics of the drift $\alpha \left(P_{(t,i)}\right)$. $\alpha \left(P_{(t,i)}\right)$ can refer to basically any process, including a mean reverting one. It is not limited to a Geometric Brownian motion.

Deregulation has introduced a strong element of risk management in the production of electricity. Electricity producers have to meet a large number of operational and physical constraints, which complicate further their problem. For example, plants cannot change their output instantaneously. There is a “ramp rate” which depends on the plant. Furthermore most plants cannot be completely shut down, or restarted at any time.

Before deregulation producers could more easily plan their production over a long time. In particular they could reasonably confidently produce at a loss during some periods, knowing that their losses would be compensated by benefits at other periods. In the new regime, electricity producers are in competition against each other, temporary losses represent a much higher risk [6].

This paper takes the perspective of the producers and models their dispatch decision as a mix of risk management and optimization. Risk reduction, in our framework is an integral part of the optimization, not a form of sub-optimality. Instead of using a stochastic dynamic programming approach, we borrow from option theory, and base our analysis on a quantity, which measures the value of
the option of producing electricity under “risk neutral” conditions. What is referred to as “risk neutrality” is somewhat different from what is referred to as risk neutrality in finance [7]. Before we precise what we mean by risk neutrality, we provide some background on the conditions in which dispatch decisions are taken.

2 What enters in the dispatch decision of the producers

Producers of electricity have to take decisions involving risks and other forms of uncertainty. They can choose among their plants which ones they want to operate, and to a limited extent choose the power level. But by far the most delicate decision is to start-up and shutdown the plants. This means that if one frames the decision of the producers as an optimization problem, the “control variables” are mostly binary variables: the decisions to start up or shut down the plant. This has implications in the way the problem is solved [8].

2.1 Profit $R(t)$ from selling electricity

In deregulated markets, the production of electricity is allocated by hourly slices. If $P_{(t,i)}$ denotes the spot price for day “t” and hour “i”, the profit $R(t)$ of the producer for day “t” can be expressed as (the notations are borrowed from Eydeland and Wolyniec [4], Appendix B):

$$R(t) = \sum_i q_{(t,i)} (P_{(t,i)} - HR(q_{(t,i)}, \zeta_{(t,i)}) G_t)$$

The summation is over the hours of the day.

$q_{(t,i)}$: is the generation level at day “t” and hourly slice “i”, $HR(q_{(t,i)}, \zeta_{(t,i)}) G_t$ is the marginal cost of producing the electricity with that plant that day at that time. It depends on the price $G_t$ for that day of the fuel used by the plant and on the marginal heat rate $HR(q_{(t,i)}, \zeta_{(t,i)})$. The heat rate is in general a function of production level $q_{(t,i)}$, and of the ambient temperature: $\zeta_{(t,i)}$. In this paper we assume the heat rate constant. We will assume that the marginal cost of producing electricity has the form: $z_{(t)} = HR G_t$, i.e. depends only on $G_t$. Without affecting very strongly the validity of the results, this introduces huge simplifications and clarity.

The choice of expression for the cost of producing electricity in formula (EQ 2) assumes that the plant burns coal or gas. For the other cases, the details are different, but the formula is basically the same.

An important consideration for the bidding strategy of the plant is whether the revenue offsets the costs, i.e. whether $R(t) > 0$, i.e. whether:
There is a very large menu of constraints, which we discuss later in this paper. The operational constraints add substantially to the cost of producing electricity. \( R(t) > 0 \) does not guarantee a profit. A more appropriate requirement is: \( R(t) > \text{operating costs} \).

2.2 Risk neutral value \( \Xi_{(t,i)} \) of producing electricity

In order to quantify the risk associated with committing to produce electricity at a certain price, we introduce a quantity \( \Xi_{(t,i)} \), which measures the “risk neutral” value of the option of producing electricity. More precisely we treat the stochastic variables \( z_{(i)} \) and \( P_{(t,i)} \) as values of two assets, and by definition, \( \Xi_{(t,i)} \) measures the “risk neutral” value of the option of substituting the asset measured by \( P_{(t,i)} \) (producing electricity n day “t” at time “i”), for the asset measures by \( z_{(i)} = HR \ G_t \) (burning fuel) [9].

\( \Xi_{(t,i)} \) is a derivative constructed from the stochastic variables: \( P_{(t,i)} \) (instantaneous revenue from selling one unit of electricity) and \( z_{(i)} = HR \ G_t \) (instantaneous cost of producing one unit of electricity).

Although neither \( P_{(t,i)} \) nor \( z_{(i)} \) are really assets, both are measured in $. \( \Xi_{(t,i)} \) is not a tradable option. It is a decision variable, which measures the “risk neutral” value of the option of buying \(- \frac{\partial\Xi_{(t,i)}}{\partial z_{(i)}} \) of asset \( z_{(i)} \) to be able to sell \( \frac{\partial\Xi_{(t,i)}}{\partial P_{(t,i)}} \) of asset \( P_{(t,i)} \), which here translates into burning some fuel at price \( z_{(i)} \) to produce the amount \( q_{(t,i)} \) of electricity at price \( P_{(t,i)} \), ignoring (for the moment) the operational and physical constraints.

The stochastic evolution of the price of fuel \( z_{(i)} \) can be written:

\[
dz_{(i)} = \gamma(z_{(i)})dt + \eta(z_{(i)})dv
\]  

(3)

The general form of \( \Xi_{(t,i)} \) can be derived using standard techniques [9]:

\[
\Xi_{(t,i)} = q_{(t,i)} \left\{ P_{(t,i)} \Phi \left( d_1 \left( \frac{P_{(t,i)}}{z_{(i)}}, T_i \right) \right) - z_{(i)} \Phi \left( d_2 \left( \frac{P_{(t,i)}}{z_{(i)}}, T_i \right) \right) \right\}
\]  

(4)
In this expression:

$$
\Phi(d) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{d} e^{-z^2} dz
$$

(5a)

$$
d_1(z, T_i) = \frac{1}{\sqrt{2 T_i}} \left[ \log(z) + \frac{T_i}{2} \right]
$$

(5b)

$$
d_2(z, T_i) = \frac{1}{\sqrt{2 T_i}} \left[ \log(z) - \frac{T_i}{2} \right]
$$

(5c)

$T_i = \left( \sigma_i^2 + \chi^2 - 2 \rho \sigma_i \chi \right) \tau$ is the cumulative uncertainty, with time horizon: $\tau$. The time horizon can be whatever is relevant for the producer, i.e. days, weeks, months, years...

$\Xi_{(t,i)}$ depends on the risk, measured here by the cumulative uncertainty $T_i = \left( \sigma_i^2 + \chi^2 - 2 \rho \sigma_i \chi \right) \tau$. Here the time horizon can be taken very short as in the present competitive deregulated environment, utilities cannot anymore afford to lose money for a sizable time with the hope of recouping afterwards.

In the limit where the uncertainty disappeared ($T_i \to 0$), EQ.4 yields:

$$
\Xi_{(t,i)} = q_{(t,i)} \text{Max}\left[ P_{(t,i)} - z_{(t)} , 0 \right].
$$

(6)

2.3 $\Xi_{(t,i)}$ as a decision variable

Producing electricity makes sense if it generates more revenue than the cost of producing it. The cost of fuel is only one cost. There are additional costs involved in operating a plant. The value of the option $\Xi_{(t,i)}$ has to be compared with the additional costs involved in producing $q_{(t,i)}$ amount of electricity, i.e. the hourly aggregated value of the costs $C_{(t,i)}(q_{(t,i)}, P_{(t,i)}, G_{t})$ due to operating and physical constraints. The cumulative value of the option of producing electricity, which is measured by $\sum_{t,i} \Xi_{(t,i)}$, must be larger than the cumulative operating costs over the same period $\sum_{t,i} C_{(t,i)}$, and the difference has to be maximized.

The problem for the decision maker is to maximize:

$$
\sum_{i} \left\{ \Xi_{(t,i)} - C_{(t,i)} \right\}
$$

(7)

The design an optimal dispatch policy is still an optimization problem, but a kind of “risk optimization”. In this optimization problem, the control variables are the
shut down or start up decisions and to a lesser extent the power level $q_{(t,i)}$. The dynamic variables are the electricity price $P_{(t,i)}$ and the fuel cost $G_{t}$. In addition to the cost $C_{(t,i)}$, there are inequality constraints that are introduced later.

### 2.4 Operational costs $C_{(t,i)}(q_{(t,i)}, P_{(t,i)}, G_{t})$

Those refer to the cost of starting up and shutting down plants. Those operational costs can be written as (Eydeland and Wolyniec [4], p.465):

$$C_{(t,i)} = q_{(t,i)} VOM + u_{(t,i)}^{on,off} \left( SC(d_{t,i}^{shutDown}) + FSC(d_{t,i}^{shutDown})*G_{t} \right) + PSC(d_{t,i}^{shutDown})*P_{(t,i)}$$

(8)

In that expression:

- VOM is the variable O&M expenses ($/MWh),
- SC: is the fixed start up cost: it is a function of the time since the last shutdown.
- PSC is the power start up cost: power needed to start the plant.
- FSC: is the fuel start up cost. It corresponds to the amount of fuel necessary to start-up.

- $u_{(t,i)}^{on,off}$ is a binary decision variable, which refers to the start-up (respectively shutdown) decision.
- $d_{t,i}^{shutDown}$ is the time from latest shutdown.

### 2.5 Identifying the optimal dispatch strategy

With our assumptions, the expression to maximize is linear in the yield $q_{(t,i)}$. First, EQ 8 has the general form:

$$C_{(t,i)}(q_{(t,i)}, P_{(t,i)}, G_{t}) = q_{(t,i)} D_{(t,i)}(P_{(t,i)}, G_{t}) + E_{(t,i)}(P_{(t,i)}, G_{t})$$

(9a)

Where:

$$D_{(t,i)}(P_{(t,i)}, G_{t}) = VOM$$

(9b)

And:

$$E_{(t,i)}(P_{(t,i)}, G_{t}) = u_{(t,i)}^{on,off} \left( SC(d_{t,i}^{shutDown}) + FSC(d_{t,i}^{shutDown})*G_{t} \right) + PSC(d_{t,i}^{shutDown})*P_{(t,i)}$$

(9c)

Furthermore EQ 4 has the general form:

$$\Xi_{(t,i)} = q_{(t,i)} \Psi_{(t,i)}(P_{(t,i)}, z_{(t)})$$

(9d)

With:
So the expression to maximize is linear in the “control variable” $q_{(t,i)}$:

$$
\sum_{i} \left[ q_{(t,i)} \left( \Psi_{(t,i)} \left( P_{(t,i)}, z_{(t)} \right) - D_{(t,i)} \left( P_{(t,i)}, G_{t} \right) \right) - E_{(t,i)} \left( P_{(t,i)}, G_{t} \right) \right] \quad (11)
$$

Assuming that the yield has to satisfy: $q_{\text{min}} \leq q_{(t,i)} \leq q_{\text{max}}$, if we neglect the inequality constraints, the optimal dispatch strategy is:

$$
q_{(t,i)} = \begin{cases} 
q_{\text{min}} & \text{when } \Psi_{(t,i)} - D_{(t,i)} \leq 0 \\
q_{\text{max}} & \text{when } \Psi_{(t,i)} - D_{(t,i)} > 0 
\end{cases} \quad (12)
$$

The optimal strategy is to produce as much as possible, when the conditions for profit are met and produce at a loss as little as possible.

### 2.6 Adding the inequality constraints

We have ignored most of the physical constraints. They introduce some additional costs and complicate somewhat the picture:

When a plant has to produce a minimum amount of power even if the price of electricity is lower than the marginal cost of producing it, the revenue of the producer includes a negative term of the form: $q_{\text{min}} \sum_{i} \left( P_{(t,i)} - z_{(t)} \right)$, where the sum is over the number of hours during which this occurs.

Another physical constraint, which adds some cost, is that the yield of a plant cannot jump instantaneously. There is a ramp rate $RR$ (MW /hr), which translates in the inequality constraint:

$$
\left| q_{(t,i)} - q_{(t,i+1)} \right| \leq RR \quad (13)
$$

Another physical constraint is the fact that a certain amount of time $D_{\text{start-up}}$ is required between the actual start-up and the unit is available. There is also a minimum amount of time $D_{\text{shutdown}}$ after a shutdown before the unit can be started again.

### 3 Conclusion: Results of this paper

This paper was about the identification of the optimal dispatch policy by electricity producers operating in the present deregulated environment. The use
of option theory gets us closer to the real nature of the problem in so far as it has a strong element of risk management, furthermore it leads to huge simplifications and ultimately far more transparency in a problem which has its share of complications as it involves some path dependence.

More precisely: this problem is traditionally approached as an optimization under uncertainty, i.e. using some form of stochastic dynamic programming. The fact that the spot price cannot be described by a smooth stochastic process together with the existence of a variety of operating (inequality) constraints, has led to the use of numerical methods using Monte Carlo techniques. Basically the idea is to explore all the space of possibilities to identify the best scenario. Not only is that approach cumbersome, it is also opaque.

By contrast our approach is based on the premise that there is a strong element of risk management in the choice of strategy of the producers. We offer an approach where the producers can estimate the risk neutral value of producing electricity at a certain price. They can compare that value to the aggregated cost of producing electricity. Their optimal policy maximizes this difference.

This approach also makes more obvious an aspect of the problem which is included, but somewhat buried in the stochastic dynamic programming approach: the physical and operational constraints introduce some path dependence in the problem. This means that at some point some numerical computations are required. The advantage of our approach is that they are much simpler and more transparent than a Monte Carlo based algorithm.

References:

