A new regression model for wave overtopping data

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Abstract

Random waves with a significant height $H_s$ produce a maximum run-up of $C H_s$ on the face of a coastal structure, where coefficient $C$ is determined by the wave and wall characteristics. Unless this run-up is greater than $R_c$, the freeboard of the structure, then there is no overtopping (apart from wind-blown spray). That is $1 - R_c / C H_s > 0$ for overtopping to occur. The parameter $1 - R_c / C H_s$ is used in an analysis of overtopping data and a new regression model is evaluated which, unlike existing expressions, satisfies the relevant physical boundary conditions. The new model is inherently suitable for representing the small overtopping discharges associated with normal design conditions.

1 Introduction

An important criterion for the design of a seawall is the allowable degree of wave overtopping which depends upon the activities normally performed in the lee of the structure, the need to prevent erosion of the rear face of the seawall, and the economic consequences of flooding. During 1978 and 1979, Owen (at HRS 1, Wallingford) carried out an extensive series of model tests to determine the overtopping discharges for a range of seawall designs subjected to different random wave climates. The modelled seawalls were all of the same general type: a flat-topped embankment fronted in some cases by a flat berm. The tests were aimed at establishing the impact on overtopping discharge of the wave climate, the seawall slope, the crest and berm elevations, and the berm width.

This paper presents a re-analysis of some of Owen’s data: the results for simple seawalls possessing uniform seaward slopes of 1:1, 1:2 and 1:4, subjected to random waves approaching normal to the slope. The purpose
has been to construct a new regression model to represent the data which is more reliable than Owen’s expression. Care has been taken to consider the physical boundary conditions. Regression coefficients determined using the least-absolute-deviations (LAD) method are recommended in preference to those obtained using the least-squares (LS) technique.

2 Dimensional analysis

In general, the mean overtopping discharge per unit length of seawall, $Q$, depends upon the wave motion, the seawall profile, the foreshore characteristics and the water properties. Written in symbolic form:

$$Q = \text{function}(H_s, T_m, \beta, R_c, \alpha, d_s, g, \ldots)$$ (1)

$H_s$ is the significant height of the incident waves; $T_m$ is the mean zero-crossing wave period; $\beta$ is the angle of wave approach measured normal to the seawall; $R_c$ is the seawall’s freeboard (the height of the crest of the structure above the still-water-level); $\alpha$ is the angle of the seawall front slope measured from the horizontal; $d_s$ is the still-water-depth at the toe of the structure; and $g$ is the acceleration due to gravity. Alternatively,

$$\frac{Q}{\sqrt{gH_s^3}} = \text{function}(\frac{R_c}{H_s}, \frac{H_s}{T_m g}, \frac{d_s}{H_s}, \alpha, \beta, \ldots)$$ (2)

$H_s / gT_m^2$ is a measure of the incident wave steepness. Owen combined this dimensionless group both with $Q / \sqrt{gH_s^3}$ and with $R_c / H_s$, to write:

$$\frac{Q}{T_m g H_s} = \text{function}(\frac{R_c}{T_m \sqrt{gH_s}}, \frac{H_s}{gT_m^2}, \frac{d_s}{H_s}, \alpha, \beta, \ldots)$$ (3)

However, other arrangements are possible (Hedges & Reis²), including use of the wave period of peak spectral density, $T_p$, rather than the zero-crossing period, $T_m$. The dimensionless groups are generally related using one of the two following functions:

$$Q_* = A \exp(-BR_*)$$ (4)

$$Q_* = A (R_*)^{-B}$$ (5)

where $Q_*$ is the dimensionless overtopping discharge, $R_*$ is the dimensionless freeboard, and $A$ and $B$ are best-fit coefficients determined from the experimental data.

Dimensional analysis provides no means for determining which sets of dimensionless groups may be especially informative or helpful in dealing with a particular data set. A possible problem in using certain pairings of groups is the potential for spurious correlation. A spurious correlation may
arise when dimensionless groups plotted against one another contain a common variable. Care must be taken in interpreting such plots. Scatter in the data may be suppressed simply by the presence of this variable.

3 Regression analysis

3.1 Introduction

Once experimental data have been collected, they may be used to confirm the validity of some theory or, where no satisfactory theory exists, they may be used to construct regression models. However, it is always useful to have some theoretical basis for choosing amongst the possible models. Furthermore, there are many techniques available for fitting regression models. In describing a regression model, care should be taken to emphasise the range of conditions over which there are data to support its use.

As a start, let us consider the physical boundary conditions to be met in addressing wave overtopping:

i) when the embankment has a large freeboard (i.e. when its crest elevation is well above the level of wave uprush), the predicted overtopping discharge should be zero (assuming that the effects of wind-blown spray are ignored);

ii) when the embankment has zero freeboard (i.e. when still-water-level is at the crest level of the embankment) then the predicted overtopping discharge may be large but should still remain finite.

Equations (4) and (5) represent two of the more common functions used to predict wave overtopping. However, when $R^*$ is large, both expressions suggest that the discharge will be finite rather than zero (though it is small provided that $A$ is not very large and provided also that $B > 1$). When $R^*$ is zero, the first of these expressions gives $Q_* = A$, a finite quantity, whilst the second expression gives $Q_* = \infty$. Thus neither expression satisfies both boundary conditions, with the second of them satisfying neither. Since most seawalls are designed to permit only relatively small overtopping discharges, the first of the two boundary conditions is likely to be the more important to satisfy.

In addition to considering the boundary conditions, we also need to establish the line of “best fit” to the observed data. There are many criteria for defining the best fit. One possibility is to minimise the sum of the squared deviations of the observations from the values predicted from our expression, but real data usually do not completely satisfy the classical assumptions for LS fitting. Reliable inferences may be drawn from regression models fitted by the LS method only if the assumptions are valid. Furthermore, an LS fitting has the disadvantage that the result is not “robust”: it is sensitive to outlying data points. Whilst we could remove “outliers”, such a procedure should only be considered if there is reason to
doubt their validity. Such data must not be removed merely because they do not support our regression model: it may be the model which is wrong.

Minimising the sum of the absolute deviations rather than the sum of the squared deviations does not rely upon the Gaussian error assumption and allows us to retain outliers but prevents these points from exerting a disproportionate influence on the values of the regression coefficients. If the errors are assumed to follow a double exponential distribution, which has thicker tails than the Gaussian distribution, then the parameter values are maximum likelihood estimates. In this paper, we have chosen to fit our regression lines using the LAD method. However, we have also compared these results with an LS fitting.

3.2 A new regression model

3.2.1 A simple overtopping theory for regular waves

Let us step back from the complications of random waves to the simpler problem of regular waves of height \( H \) approaching normal to a seawall. We will assume that the instantaneous discharge of water over unit length of the seawall, \( q \), is given by the weir formula:

\[
q = C_d \frac{2}{3} \sqrt{2g} (\eta - R_c)^{3/2} \quad \text{for } \eta > R_c
\]  

where \( \eta \) is the water surface elevation above still-water-level at the seawall (a periodic function of time); \( C_d \) is a discharge coefficient. Obviously, overtopping occurs only when the water surface is above the structure’s crest. We will also assume that \( \eta = kHF(t) \) in which \( F(t) \) denotes a function of time, \( t \). For simple, sinusoidal, progressive waves, \( k=0.5 \) and \( F(t)=\cos(2\pi t/T) \), where \( T \) is the wave period. However, following Kikkawa et al\(^3\), we will adopt the simpler form for \( F(t) \) shown in Figure 1; \( k \) remains a coefficient determined by the particular wave and wall details. Then, the mean discharge, \( Q \), is determined as follows:

\[
Q = C_d \frac{2}{3} \sqrt{2g} \frac{1}{T} \int_{t_1}^{t_2} \{kHF(t) - R_c\}^{3/2} dt
\]

in which \( t_1 < t < t_2 \) corresponds to the interval during each wave period for which \( kHF(t) > R_c \). Using the form for \( F(t) \) given in Figure 1 then yields:

\[
\frac{Q}{\sqrt{g(kH)^3}} = C_d \frac{2\sqrt{2}}{15} \left\{1 - \frac{R_c}{kH}\right\}^{5/2} \quad \text{for } 0 < R_c < kH
\]

\[
= 0 \quad \text{for } R_c \geq kH
\]

Note that overtopping occurs only when \( R_c < kH \). In other words, \( kH \) represents the run-up on the face of the seawall.
3.2.2 The Hedges & Reis (H&R) overtopping model

The above theory suggests a regression equation for the random overtopping data of the following form:

\[ Q_\ast = A(1 - R_\ast)^B \]

\[ = 0 \quad \text{for} \quad R_\ast < 1 \]

\[ = A \quad \text{for} \quad R_\ast \geq 1 \] (9)

in which

\[ Q_\ast = \frac{Q}{\sqrt{gR_{\max}^3}} = \frac{\varrho}{\sqrt{g(CH_s)^3}} \]

\[ R_\ast = \frac{R_c}{R_{\max}} = \frac{R_c}{CH_s} \] (10)

Coefficient \( k \) in the expression for regular waves has been replaced by \( C \) in this regression model for random waves characterised by \( H_s \). Note that \( CH_s \) represents \( R_{\max} \), the maximum run-up induced by the random waves, not the run-up induced by a wave of height \( H_s \). Consequently, \( C \) will depend upon the duration of the incident wave conditions unless the wave heights in front of the wall are limited by the available water depth. Until the maximum run-up exceeds the freeboard, \( R_c \), there will be no overtopping. It is also clear that coefficient \( B \) is related, in the case of regular waves, to the shape of the function \( F(t) \) which describes the water surface variation on the seaward face of the wall. There will be a similar dependence on the detailed behaviour of the water surface at the face of the wall in the case of random waves. Finally, coefficient \( A \) represents the dimensionless discharge over the seawall when the freeboard is zero. All three coefficients will be influenced by the seaward profile of the structure.

The above model for overtopping has the advantage that \( Q_\ast = 0 \) when \( R_\ast \geq 1 \) and that \( Q_\ast = A \) when \( R_\ast = 0 \), in accordance with our required boundary conditions. Figure 2 shows the influences of coefficients \( A, B \) and \( C \) in the new overtopping model.
Although C (=R_{max}/H_s) is not normally evaluated during tests involving wave overtopping, its value may be estimated from run-up measurements for random waves acting on slopes for which there is no overtopping. We have adopted this option rather than including C alongside A and B as a regression coefficient.

Owen recorded his overtopping discharges during tests involving sets of five different runs, each of 100 waves, characterised by the same value of H_s. Assuming that run-up values may be described by a Rayleigh distribution, then the expected maximum run-up, R_{max}, during each run is related to the significant run-up, R_s, by R_{max} = \sqrt{(\ln 100)/2} \cdot R_s = 1.52R_s. The CIRIA/CUR^4 manual gives two equations for evaluating R_s for smooth slopes without overtopping. Rewritten in our notation and allowing for a printing error, the following expressions for C may be derived:

\begin{align*}
C &= 1.52 \frac{R_s}{H_s} = 1.52(1.35\xi_p) \quad \text{for } \xi_p < 2 \\
C &= 1.52 \frac{R_s}{H_s} = 1.52(3.00 - 0.15\xi_p) \quad \text{for } \xi_p > 2
\end{align*}

Here, \( \xi_p \) is the surf similarity parameter calculated using \( T_p \) (\( \xi_p = \tan \alpha / \sqrt{H_s / L_{op}} \); \( L_{op} = gT_p^2 / 2\pi \)). \( T_p \) was estimated for Owen's data using the relationships between \( H_s, T_m \) and \( T_p \) provided by Isherwood^5.

4 Results of analysis

Figure 3 shows an example of the overtopping data collected by Owen: the results for a simple seawall with a uniform front slope of 1:2. The data are plotted in the formats required for fitting regression equations using both the H&R and the Owen models. Figure 3(a) shows the best-fit lines established using LS and LAD procedures for the H&R model. Comparison of the regression coefficients shows the relatively stronger influence which outlying
data points have on the LS values. For example, the magnitude of B obtained from LAD is only about 92% of the LS result. Similar comments may be made about the regression lines obtained for Owen’s model. Note, that the values of A and B reported in Figure 3(b) for Owen’s model are not those which Owen himself recommended.

Table 1: Regression coefficients obtained for the H&R model and for Owen’s model. Also included are Owen’s recommended values.

<table>
<thead>
<tr>
<th>Slope</th>
<th>H&amp;R MODEL</th>
<th></th>
<th>OWEN’S MODEL</th>
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<tr>
<td></td>
<td>LAD</td>
<td>LS</td>
<td>LAD</td>
</tr>
<tr>
<td>1:1</td>
<td>A</td>
<td>0.00703</td>
<td>0.00581</td>
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<td></td>
<td>B</td>
<td>3.42</td>
<td>3.22</td>
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<tr>
<td>1:2</td>
<td>A</td>
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<td>0.00790</td>
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<td></td>
<td>B</td>
<td>4.17</td>
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</tr>
<tr>
<td>1:4</td>
<td>A</td>
<td>0.0104</td>
<td>0.00792</td>
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<tr>
<td></td>
<td>B</td>
<td>6.27</td>
<td>5.94</td>
</tr>
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</table>
Environmental Problems in Coastal Regions

Table 1 gives the regression coefficients for all three slopes which we have obtained for the H&R model and for Owen’s model, using both LS and LAD fitting. Also included for reference are Owen’s recommended values. Owen restricted his analysis to a particular set of conditions whilst we have included all available data. These data fell within the following ranges:

\[
0 < \frac{Q}{\sqrt{g(CH_s)^3}} < 0.0056 \quad 0 < \frac{Q}{T_m g H_s} < 0.0039 \quad 0.0053 < \frac{H_s^2}{gT_m^2} < 0.0095
\]

\[
0.14 < \frac{R_c}{CH_s} < 0.90 \quad 0.053 < \frac{R_c}{T_m \sqrt{gH_s}} < 0.239 \quad 1.65 < \frac{d_s}{H_s} < 5.20
\]

Earlier, we have mentioned briefly the problem of spurious correlation. Along with most other overtopping models, the H&R model employs a
dimensionless discharge and a dimensionless freeboard which contain a common variable ($R_{max}$ or $CH$). The presence of this common variable may reduce the apparent scatter in the data. Consequently, in Figure 4 we show directly the level of agreement between Owen's measured values of $Q$ (converted by Owen to full-scale discharges for a seawall in 4m water depth) and the predicted values, $Q_{PRED}$. Under random wave conditions, overtopping will be dominated by the few waves with large run-ups: most waves will contribute no overtopping if the seawall has a substantial freeboard. Thus, particularly for short runs of random waves, as in Owen's tests, we can expect some variability in the measured values of $Q$. Indeed, one of the purposes of Owen's tests was to show this inherent variability.

In Figure 4, most data points lie within a range for $Q/Q_{PRED}$ of 3/4 to 4/3, whichever model is adopted. It is not obvious from the figure which model best fits the data, nor is it obvious from the plots for simple seawalls with 1:1 and 1:4 front slopes. However, it should be noted that full-scale discharges greater than about $0.001 \times 10^3 \text{m}^3/\text{s/m}$ will be unsafe for vehicles at high speed. Conditions become dangerous for pedestrians when the discharge exceeds $0.03 \times 10^3 \text{m}^3/\text{s/m}$. Discharges greater than about $2 \times 10^3 \text{m}^3/\text{s/m}$ may damage embankment seawalls (CIRIA/CUR4). Consequently, we have looked in more detail at the data points for discharges in the ranges of practical interest. The H&R model appears generally better than Owen's model for discharges of less than $5 \times 10^3 \text{m}^3/\text{s/m}$, owing to its ability to predict zero overtopping at finite values of freeboard. Furthermore, it tends to give lower required crest levels than Owen's model for small permissible discharges.

5 Concluding remarks

A new regression model has been developed for describing wave overtopping data. The important features of the model are as follows:

i) it satisfies the relevant physical boundary conditions, a feature which is especially important when the model is used near these boundaries;

ii) it explicitly recognises (through its foundations in a simple theoretical model for regular waves) that regression coefficient A depends upon the shape of the structure since the shape, particularly at its crest, affects the discharge coefficient; coefficient A represents the dimensionless discharge when the dimensionless freeboard is zero;

iii) coefficient B depends upon the detailed behaviour of the water surface on the face of the structure; it increases as front slopes become flatter;

iv) coefficient C relates the maximum run-up to the significant height of the incident waves and may be chosen to allow for the influences of the seawall slope, the surface roughness and porosity, and the incident wave steepness; coefficient C can also account for storm duration in influencing $R_{max}$. 
Ideally, any test programme would fix coefficient A by measuring the discharge over a seawall when the freeboard was zero. Likewise, C could be determined from the minimum freeboard giving zero overtopping discharge. In the absence of this information, C has been estimated from available data on the significant wave run-up on smooth slopes.

For the present test results, the H&R model is little different from Owen's model in its ability to represent the data, except for small discharges for which the H&R model is better suited. However, there are a number of ways in which the H&R model could be improved. For example, the period of peak spectral density and the maximum run-up could be measured directly, whereas we have had to estimate these values. We would then expect more reliable estimates for coefficient C and, consequently, a closer agreement between our model and the data.

Finally, one of the purposes of Owen's tests was to show the variability in Q. This property must be considered in design procedures: it is necessary not only to model the expected value of Q, but also the distribution of Q about this value. As a consequence, approaches to coastal engineering design are shifting towards the assessment of the safety of coastal structures using risk analysis rather than using a deterministic procedure. One of the major stages of risk analysis is the formulation of equations to describe the failure mechanisms of a structure. In this connection, the main objective of the present paper has been to improve the mathematical description of wave overtopping of simple seawalls.

Acknowledgements

The authors thank HR Ltd for permission to use Owen's wave overtopping data. Financial sponsorship of Miss Reis's Ph.D. studies by JNICT, Portugal, under the PRAXIS XXI Programme, is also acknowledged.

References