Numerical calculations of ocean surface waves in a coastal zone

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Abstract

Numerical simulation is here used as a tool for investigating sea floor effects on wind generated waves and swells entering a coastal zone. The mathematical model, the Mildslope Equation, is a refraction-defraction equation based on linear nonfrictional theory and assuming a slowly varying sea bed. We present two test cases: A harbour area with breakwaters, and the site of a planned wave power plant. The simulations show promising results. Due to adaptive gridding and incorporation of the effects of an infinite ocean outside the simulation area, the simulation software is able to handle larger areas with a satisfactory resolution than what is commonly feasible. In our conclusion we suggest that tools of this kind be used for investigation of worst case scenarios before and during planning of coastal zone constructions. With some extensions it could also be used for forecasting purposes.

1 Introduction

Numerical simulation is here used as a tool for investigating sea bed effects on wind generated waves and swells in a coastal zone. Swells entering shallow waters will be significantly affected by a varying sea
amplitude. Combined with reflections from shoreline and islets, this means that the state of such long waves near the coastline often differs greatly from what it was in the open ocean areas.

This kind of tool can be of great value in the planning and construction of human installations in the coastal zone, such as marinas, harbours or wave power plants. Information available from the numerical simulator may be local wave height and direction, energy flux, velocity and wave load on the planned constructions at various locations.

The main test case [1] described in the paper is the weather beaten harbour of Hasvik, in Finnmark, Northern Norway. The other case is the site of a projected wave power plant on the Southern coast of Java, Indonesia.

2 Coastal region simulations

A train of wind generated waves and swells can propagate over large ocean areas unaffected by the sea bed. However, when entering the coastal zone or other areas with shallow water, the local wave length will be influenced by the seabed, resulting in a gradual change in wave propagation velocity and direction. In addition, with presence of physical obstacles such as rock awash, headlands, harbour entrances, etc., the resulting diffraction phenomena may cause a major increase in wave amplitude.

2.1 Lenses and Prisms

The focusing effect of banks and underwater slopes is an interesting aspect of coastal zone wave behaviour. These are acting as “lenses and prisms”, and are able to change the direction and amplitude of the waves significantly.

Figure 1 shows how swells with a wave period of 14 seconds are modified coming in over a sea bed varying from 20 to 7 m depth. Part a) shows a sea bed containing two closely located round banks with minimum depth 7 m. Part b) shows the maximum water surface elevation: incoming swells (plane wave from the left) with wavelength 183 m are modified by the bathymetry. The wave of initial amplitude 1 m is focused and defracted by the banks, giving a maximum absolute amplitude of 1.37 m and a minimum absolute amplitude of
0.685 m. Please notice that the plot shows absolute amplitude\textsuperscript{1}, and is not a situational snapshot. In this example, the shortest wavelength occurring (over the banks) is 113 m. This means that a 13 m change of depth corresponds to a 70 m change in wavelength.

In very deep waters, the surface waves propagate independently of water depth. This occurs when the wavelength is very large compared to the water depth. Therefore, in open ocean deep water regions the wave propagation is unaffected of diffraction phenomena due to varying sea bed, and the wave length depends only on the wave period. The typical periods of the waves here studied are from 8 to 25 seconds and they are acting on the air-water interface. These long waves propagate faster and contain more energy than shorter waves. This means that they will reach shore earlier after a storm, and also that they will cause the heaviest loads on constructions along the coast.

\textsuperscript{1}The absolute amplitude is the maximum amplitude in each node, or spatial (xy) point, during a wave period, in other words "as high as it gets" over time.
2.2 Mathematical model

In this choice of description, the surface gravity waves are characterized by the dispersion relation

\[ \omega^2 = \kappa g \tanh \kappa h \]  

which relates the wave period \( T = 2\pi/\omega \) to the local wave length \( \lambda(x, y) = 2\pi/\kappa \). Here \( \omega \) denotes the angular wave frequency, \( \kappa = \kappa(\omega, h(x, y)) \) the wave number and \( g \) the acceleration of gravity. The spatial variation of water depth is denoted \( h = h(x, y) \).

2.2.1 Basic Equations

The equation is a combined refraction-defraction equation, the so-called Mildslope Equation [2], which is derived within nonfriction linear wave theory and with the assumption of a slowly varying seabed. This means that the surface wave amplitude and the horizontal length scale of depth variation \( \nabla h \) are assumed small compared to the wavelength, \( \lambda \). The equation reads

\[ \nabla \cdot (b \nabla \eta) + \omega^2 c \eta = 0 \]  

where \( \eta = \eta_{re}(x, y) + i\eta_{im}(x, y) \) is the complex wave height. The coefficients, \( b(x, y) \) and \( c(x, y) \) depend on the varying depth, \( h(x, y) \).

\[ b(x, y) = c_0 c_g = \frac{gh}{2\kappa h} \left( 1 + \frac{2\kappa h}{\sinh 2\kappa h} \right) \] 

\[ c(x, y) = \frac{c_g}{c_0} = \frac{1}{2} \left( 1 + \frac{2\kappa h}{\sinh 2\kappa h} \right) \]

where \( c_0 = \frac{\omega}{\kappa} \) and \( c_g = \frac{\partial \omega}{\partial \kappa} \) are the local phase and group velocity, respectively.

2.2.2 Boundary Conditions

In the main simulation case, three types of boundary conditions are considered: the open boundary towards an infinite ocean, indicated in Figure 2, the shoreline with varying reflection properties and a total reflecting border towards man made constructions like the moles in Figure 5 or the wave collector in Figure 6. In the case of the wave power plant one more kind of condition is required. At the end of a funnel, water will be running into an elevated reservoir, and thus a radiation condition is used there.
2.3 The Simulation Environment

The numerical method chosen for these simulations is a finite element method (FEM). Due to general complexity of the bathymetry of coastal regions, finite element methods are particularly suitable for this problem, since the elements can be freely varied to discretize the region of irregularity. The software is based on the numerical library Diffpack [3].

In order to handle the phenomena accurately enough to obtain reliable numerical results, the local wave length must be resolved properly. Shallow water regions require finer spatial resolution of the sea bed topography, and hence, denser sampling of the sea bed. Regarding the dispersion relation (1), we can also see that at the same water depth, the local wave length is shorter for small wave periods than for long wave periods. Therefore, when performing numerical calculations of surface waves for various wave periods over the same simulation area, one should discretize the region with respect to the smallest wave period of the calculations.

![Diagram](image)

**Figure 2:** A sketch of a simulation area in a coastal region.

A significant part of the simulation procedure is the construction of the computational grid based on the available geodata. The grid must be dense enough to resolve each present wavelength over the relevant areas, and sparse enough for the numerical computations to be manageable on the full region of interest. In order to achieve...
this, it is necessary to adapt the grid with respect to depth and case relevant wave periods. Sometimes certain sections of the shoreline may require denser sampling to avoid numerical instabilities. In order to obtain a correct picture of bathymetry effects on surface waves it is necessary to have access to reliable geodata, s.a. sea floor topography, shoreline and shoreline characteristics.

An important feature of the simulator is incorporation of an infinite ocean around the actual simulation area. Outside the finite element solution area, a finite difference description is used, and asymptotic theory is applied. The matching between these two solution areas incorporates radiation conditions and other necessary requirements in the infinite exterior, yielding a far more realistic results than a sequential solver\(^2\) would be able to produce.

### 2.4 Results from the Hasvik Simulations

![Figure 3: The bathymetry (water depth in meters)](image)

The Hasvik Harbour is carrying a lot of traffic, and has always been very sensitive to weather conditions. A new breakwater was constructed in 1996, and made the harbour more stable the next year.

\(^{2}\)A sequential solution method means solving for one area first, then using the solution as input when solving for the next.
During a storm on October 31st 1997, the new breakwater was unfortunately destroyed by waves.

In the simulations conducted for the Hasvik case, the main direction of the incoming waves are West North West, while in reaching the harbour area the main direction is changed to South East, directly onto the entrance. The simulated wave height indicates strong reflection from, and thus heavy load on the breakwaters. This is illustrated in Figure 4.

Roughly estimated spectra for incoming waves in the simulations contain three different wave periods, 13, 15 and 17 seconds, and three different directions for each of these, given as 0, 10 and 20 degrees with point of compass as in Figure 4.

These estimated incoming wave spectra are based on wind measurements taken at Torsvåg Lighthouse and Hasvik Airport on the actual day. The nine incident waves have been weighted, with main weight on the 15 second periods and the 10 degrees direction, resulting in a significant wave height of about 10 meters.

Figure 4 demonstrates the focusing effect of the three banks, Silstø, Andottskallen and Knottskallen, combined with the steep slope to the South East. The waves are deflected and trapped to the coastline around Skipperneset. Further, one sees strong reflections from the breakwaters and from coastline areas where steep cliffs go directly into the sea. Inside the harbour area, see Figure 5, the waves seem
to be somewhat more dampened with the third breakwater intact. Note, however, that in these simulations the focus has been on wave propagation towards the breakwaters, and the gridding resolution inside the harbour area is therefore on purpose not dense enough to pick up all effects.

Figure 5: Snapshot of wave height in the coastal region of Hasvik harbour a) with original breakwaters and b) with the additional breakwater constructed in 1996 and destroyed by wind generated waves October 31, 1997.

2.5 Wave Power Plants

A main component in a tapered channel power plant like the ones constructed by the Norwegian company Norwave a.s. [4] is the wave collector, a large funnel which is blasted out into the rock shore. As the waves propagate through this funnel, the amplitude increases as the space between the walls narrows, and at the end the water runs into an elevated reservoir. From this reservoir the water runs out through turbines, creating electrical power. In obtaining optimal design and location for a power plant like this, simulations of surface waves on a realistic sea bed and with modified shoreline including various shapes of the collector are essential. This way one seeks to obtain a design giving high efficiency v.s. low construction costs. These simulations will have to be correlated to wave height measure-
ments made over the year in order to estimate the effect obtainable from the power plant.

Figure 6: Site for construction of wave power plant. The white line indicates the present shoreline, the black a possible geometry for the funnel.

A power plant simulation project is now being carried out at SAM (SINTEF Applied Mathematics) for Norwave a.s, and the first construction site scheduled is close to the village of Baron, located on the southern coast of Java, Indonesia. Figure 6 shows simulations on how the wave amplitude is increased entering the funnel. Incoming wave height is 1 m, and at the end of the funnel, Figure 7, this has been magnified to 3.7 m.

3 Conclusion

In the coastal zone, incoming waves from the outer ocean are often producing heavy loads on constructions and traffic. In foreseeing these, weather data from the open ocean areas are not sufficient. One also needs a description on how the waves will be modified on their way towards the shore.

Currently, the above described simulator produces qualitative results which seem to be quite in accordance with on site observations. Within the assumptions made when choosing numerical and mathematical description (no friction, slowly varying sea floor etc), this
kind of tool can be a cost saving and valuable aid in the planning of constructions along the coast. With some extensions, it can also be used for forecasting purposes.

Adaptive gridding and incorporation of the effects of an infinite ocean outside the outer boundaries makes this simulator able to handle larger areas than what is commonly feasible. This may in many cases turn out to be of great importance, since incoming waves often are modified significantly in amplitude and direction already at some distance from the shore.

References


