

Vector analysis of Morison's equation

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Abstract

For the evaluation of drag force due to wave or wave-current action, the Morison equation is commonly used in conjunction with only the horizontal component of the water particle velocity. However, for a horizontal cylinder in waves, or in waves and current, the vertical component of the water particle velocity is significant. For such a case, Morison's equation should be modified to the vector form. Therefore, the analysis of horizontal cylinder hydrodynamic load due to wave or simultaneous wave-current action, taking into account the vertical component of the water particle velocity as well, has been carried out in the paper. It has been clearly shown that the vertical component of the water particle velocity needs to be considered too, because of the considerable differences in the drag force values in relation to the model that do not take into account the velocity vertical component.

1 Introduction

Wave or wave-current induced forces on submerged cylinders have been investigated mostly for vertical cylinders, for which only the horizontal components of the water particle velocity and acceleration need to be considered. Little research has been done on horizontal cylinders in waves or in waves and current in spite of the common use of horizontal members in many offshore structures and ocean pipelines. A horizontal cylinder normal to the direction of wave or wave-current propagation experiences orbital motion of water particles around the axis of the cylinder, so that the vertical component of the kinematics and dynamics must be considered. For such a case, the Morison equation, which

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is commonly used for the evaluation of hydrodynamic forces, should be modified to the vector form. That makes the problem more complicated.

The purpose of this paper is to investigate and quantify the effects of the vertical component of the water particle kinematics on horizontal cylinder hydrodynamic loading.

2 Assumptions

(a) In the following analysis the linear wave theory is employed to relate surface wave parameters and wave motions.

(b) The current is assumed to be externally generated, in-line with the waves and steady.

(c) It is assumed that the linear superposition principle is valid for estimating the kinematics under waves and current.

(d) It is assumed that on the analyzed section the cylinder is laid horizontally; that it is of a constant outside diameter; rigid; fixed; slender (in the sense that the diameter-wave length ratio is small and diffraction effects are negligible); that it is located in the horizontal co-ordinate $x = 0$; and that the waves and current propagating direction is perpendicular to the cylinder axis.

(e) The drag and inertia coefficients remain constant irrespective whether there is wave-current combination or not, and whether the current is positive or negative.

3 Theory

3.1 Force equation for wave action

The Morison equation¹ is normally used in conjunction with only the horizontal component of the water particle velocity for determining the hydrodynamic forces acting on a submerged cylinder due to wave action. In this equation, the force per unit length, $F(z,t)$, is expressed as the sum of a velocity-dependent drag force, $F_D(z,t)$, and acceleration-dependent inertia force, $F_I(z,t)$.

Horizontal component, $F_x(z,t)$, and vertical component, $F_z(z,t)$, of this force can be expressed, Fig. 1, as:

$$F_x(z,t) = F_{D_x}(z,t) + F_{I_x}(z,t) = u_D v_x(z,t) |v_x(z,t)| + u_I a_x(z,t) \quad (1)$$

$$F_z(z,t) = F_{D_z}(z,t) + F_{I_z}(z,t) = u_D v_z(z,t) |v_z(z,t)| + u_I a_z(z,t) \quad (2)$$

where

$$u_D = 0.5 C_D \rho D \quad (3)$$

$$u_I = 0.25 C_I \rho \pi D^2 \quad (4)$$

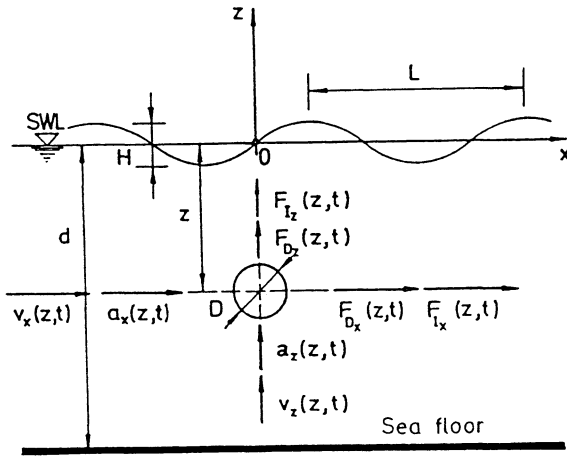


Figure 1: Definition drawing.

In the above equations $F_{D_x}(z,t)$ and $F_{I_x}(z,t)$ = horizontal components of drag and inertia force, respectively; $F_{D_z}(z,t)$ and $F_{I_z}(z,t)$ = vertical components of drag and inertia force, respectively; C_D and C_I = drag and inertia coefficients, respectively; ρ = water density; D = outside cylinder diameter; x and z = horizontal and vertical co-ordinates, i.e. horizontal and vertical axes, respectively; $v_x(z,t)$ and $v_z(z,t)$ = horizontal and vertical components of water particle velocity, respectively; $a_x(z,t)$ and $a_z(z,t)$ = horizontal and vertical components of water particle total acceleration (local acceleration plus convective acceleration), respectively; and t = time.

However, for a horizontal cylinder in waves (or in waves and current), the vertical component of the water particle velocity is significant.^{2,3,4} For such a case, the Morison equation should be modified to the vector form, i.e. the vector sum of the drag force and inertia force, as follows:

$$\vec{F}(z,t) = \vec{F}_D(z,t) + \vec{F}_I(z,t) = u_D \vec{v}(z,t) |\vec{v}(z,t)| + u_I \vec{a}(z,t) \quad (5)$$

The lift force cannot be added to the two terms in Eqn. (5), because the period, the magnitude and the direction of this force are unknown.

Eqn. (5) can be decomposed into the horizontal and vertical directions, respectively:

$$\vec{F}_x(z,t) = [F_{D_x}(z,t) + F_{I_x}(z,t)] \vec{i} \quad (6)$$

$$\vec{F}_z(z,t) = [F_{D_z}(z,t) + F_{I_z}(z,t)] \vec{k} \quad (7)$$

where $\vec{i}$ and $\vec{k}$ are unit vectors in the x -axis and z -axis, respectively, and:

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$$F_{D_x}(z, t) = u_D v_x(z, t) \left| \bar{v}(z, t) \right| = u_D v_x(z, t) \left[v_x^2(z, t) + v_z^2(z, t) \right]^{1/2} \quad (8)$$

$$F_{I_x}(z, t) = u_I a_x(z, t) \quad (9)$$

$$F_{D_z}(z, t) = u_D v_z(z, t) \left| \bar{v}(z, t) \right| = u_D v_z(z, t) \left[v_x^2(z, t) + v_z^2(z, t) \right]^{1/2} \quad (10)$$

$$F_{I_z}(z, t) = u_I a_z(z, t) \quad (11)$$

Eqs. (8) and (10) more correctly describe the components of the drag force acting on a horizontal cylinder.

Thus, using Eqs. (1) and (2), while determining the drag force a certain error, $\vec{E}$, is made, Fig. 2, where the subscript in brackets identifies equation number.

$$\vec{E} = \vec{F}_{D(8),(10)} - \vec{F}_{D(1),(2)} \quad (12)$$

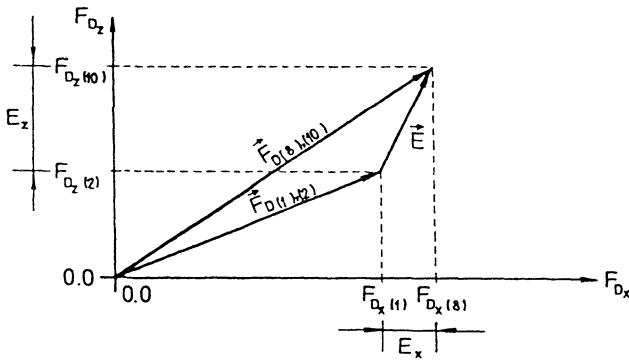


Figure 2: Drag force error vector.

3.2 Force equation for simultaneous wave-current action

For the conditions involving waves as well as a current, the design codes recommend that the drag force term in the Morison equation be based upon the sum of the current velocity and the wave velocity which would be present in the absence of the current.

Accordingly, in the Eqs. (1) and (2) only the term for the horizontal component of the drag force is transformed:

$$F_{D_x}(z, t) = u_D \left[v_x(z, t) + U(z) \right] \left| v_x(z, t) + U(z) \right| \quad (13)$$

where $U(z)$ = current velocity at the depth at which $v_x(z, t)$ is being determined. However, when presenting the Morison equation in the vector form, Eqs. (5) through (11), besides the term for the horizontal component of the drag force, the term for the vertical component of the drag force is also transformed, so that Eqs. (8) and (10) assume the form:

$$F_{D_x}(z, t) = u_D \left[v_x(z, t) + U(z) \right] \left\{ \left[v_x(z, t) + U(z) \right]^2 + v_z^2(z, t) \right\}^{1/2} \quad (14)$$

$$F_{D_z}(z, t) = u_D v_z(z, t) \left\{ \left[v_x(z, t) + U(z) \right]^2 + v_z^2(z, t) \right\}^{1/2} \quad (15)$$

4 Results

For the purpose of quantifying the differences in the calculation of hydrodynamic forces according to the considered models, further analysis will be done on the basis of the calculation of forces for the wave-current parameters according to Table 1, where H = wave height; L = wave length; T = wave period; d = water depth; C_s = shoaling coefficient; and the subscript o identifies deep water wave parameters.

$H_o/L_o/T = 6.0/100/8.0$				
d (m)				
	50	25	15	10
$H = H_o C_s$ (m)	6.0	5.6	5.5	5.6
$L = L_o \tanh 2\pi d/L$ (m)	100	93	82	71
$T = \text{const}$ (s)	8.0			
$U = \text{const}$ (ms^{-1})	-	-	± 1.5	-
C_s (1)	1.0000	0.931	0.913	0.933
d/L (1)	0.50	0.27	0.18	0.14

Table 1: Wave-current parameters.

It is assumed that the waves and current propagating direction is perpendicular to the isobaths (refraction coefficient, $C_r = 1$), so that the wave height, H , is defined as the product of the deep water wave height, H_o , and the shoaling coefficient, C_s . The calculation results are presented in the form of non-dimensional values in Figs. 3 through 6; Figs. 3 through 5 referring to wave action, and Fig. 6 to simultaneous wave-current action.

Relative error, and drag force horizontal component, $F_{D_x(1)}$ and $F_{D_x(8)}$, versus absolute maximum value of drag force horizontal component, $F_{D_x \max}$, for $d/L = 0.50$ and $-z/d = 0.20$ are shown in Fig. 3(a).

Note that the z -axis is positive upward.

Analogous presentation for drag force vertical components is given in Fig. 3(b).

In order to see what the influence of the depth decrease and approaching to the seabed is, the same is done in Fig. 4 for $d/L = 0.18$ and $-z/d = 0.60$.

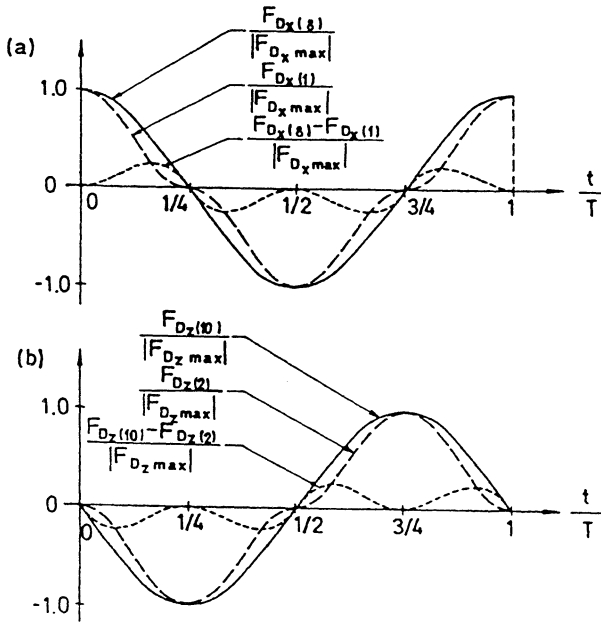


Figure 3: Differences in (a) horizontal and (b) vertical drag force components for the case of wave action; $d/L = 0.50$ and $-z/d = 0.20$.

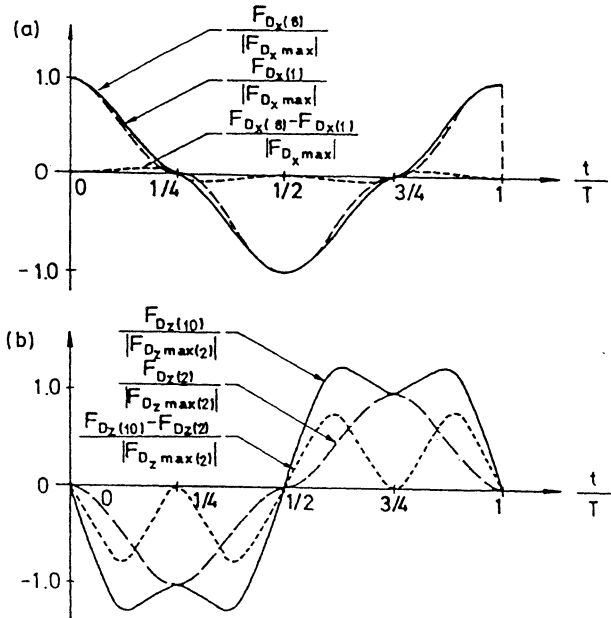


Figure 4: Differences in (a) horizontal and (b) vertical drag force components for the case of wave action; $d/L = 0.18$ and $-z/d = 0.60$.

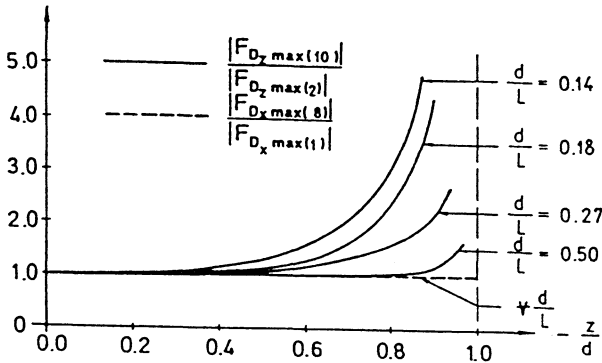


Figure 5: Ratios of horizontal drag force components maxima and ratios of vertical drag force components maxima, for the case of wave action.

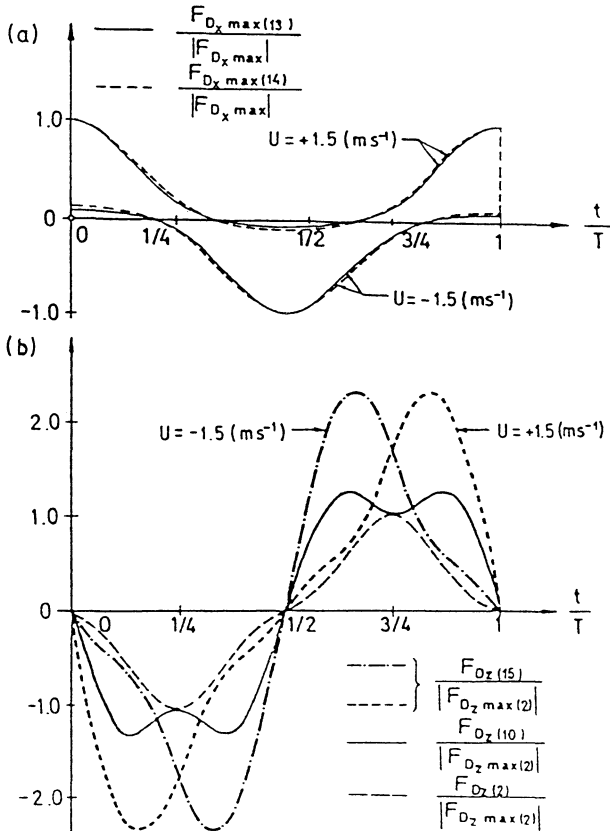


Figure 6: Differences in (a) horizontal and (b) vertical drag force components for the case of wave-current action; $d/L = 0.18$ and $-z/d = 0.60$.

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Ratios of horizontal drag force components maxima and ratios of vertical drag force components maxima, depending on the used model as a function of d/L and $-z/d$ are shown in Fig. 5.

It can be seen from Figs. 3 through 5 that the maximum drag force horizontal components for the case of wave action are the same for both considered models, and that the differences according to Eqs. (1) and (8) are diminishing with the depth decrease and by approaching of the horizontal cylinder to the seabed. However, the situation is reversed when the drag force vertical component is considered. Besides, the frequency of the maximum drag force vertical component changes too, which is very significant fact.

For the case of simultaneous wave-current action, Fig. 6(a), the relative difference in the drag force horizontal components calculated according to Eqs. (13) and (14) is even less compared to the case of wave action (Fig. 4(a)). But, the situation is reversed when the vertical components are considered, Fig. 6(b), because the presence of a current according to Eq. (15) considerably contributes to the increase of the drag force vertical components.

Finally, the relative differences in the drag force components depending on the used model are shown in Fig. 7. There is more pronounced difference in the case of the drag force vertical component than in the case of the horizontal component. Furthermore, when the horizontal component is considered, with the approaching of the horizontal cylinder to the seabed, the mentioned difference disappear, whereas in the case of the vertical component, they increase.

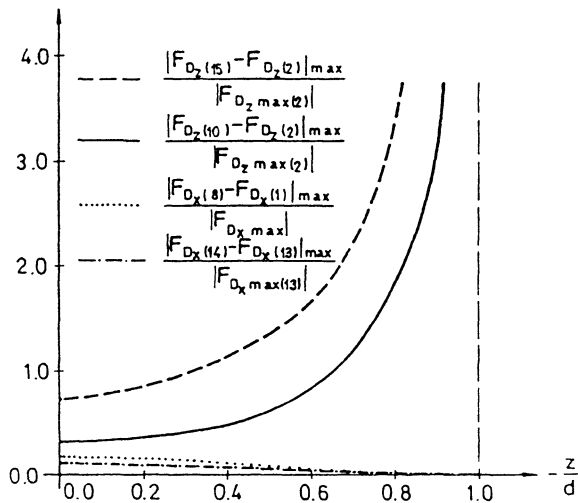


Figure 7: Relative differences in drag force components depending on the used model for the cases of wave action, and simultaneous wave-current action; $d/L = 0.18$.



The presence of a current in the case of the horizontal component contributes to the diminishing of differences, but when the vertical component is considered, it contributes to their increase.

5 Conclusion

In the analysis of wave or wave-current load on a horizontal cylinder located in the drag loading regime, the vertical component of water particle velocity needs to be considered and Morison's equation should be modified to the vector form.

For the case of wave action, with depth decrease and approaching of the horizontal cylinder to the seabed, the difference in the drag force horizontal components according to the considered models diminishes, whereas when the vertical components are considered, it grows. That difference is even more pronounced for the case of simultaneous wave-current action.

References

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