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Modelling of wave transformation across the inner surf zone and swash oscillations on beaches

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Abstract

A numerical model based on a high order non-oscillatory MacCormack TVD scheme is presented for the solution of the non-linear Saint Venant equations in surf and swash zones. A comparison with measurements of surface elevations on a beach (Truc Vert Beach, France), shows that the model is capable of predicting accurately the transformation of irregular waves in the surf zone.

1 Introduction

Wave transformation in the surf and swash zones plays a major role in sediment transport and beach evolution. In the inner surf zone spilling breakers have bore-like shape. Lin and Liu [4] have shown, from numerical simulations of Navier Stokes equations, that the pressure distribution under these waves is almost hydrostatic and that the horizontal velocity, excepted in the roller region, is nearly uniform over the depth. Therefore, the non-linear Saint Venant (SV) equations are relevant to describe spilling breaking waves (e.g. Hibbert and Peregrine [2]). Kobayashi, DeSilva and Watson

[3] and Raubenheimer, Guza and Elgar [6] have compared SV numerical simulations with respectively laboratory and field measurements in the inner surf zone and found a very good agreement.

To compute broken wave (bore) propagation it is necessary to use shock-capturing numerical method. Hibberd and Peregrine [2] and Kobayashi et al. [3] have chosen the Lax-Wendroff scheme, which has been successfully applied for solving numerous hyperbolic systems. However, in presence of fronts, the dispersive properties of this scheme introduce spurious numerical oscillations. To reduce these high-frequency oscillations which tend to appear at the rear of the wave front, Hibberd and Peregrine [2] and Kobayashi et al. [3] included an additional dissipative term. An alternative to this approach is to use a TVD (total variation diminishing) scheme, which represents a rational method for the determination of artificial dissipation terms.

In the following section (2), we report the implementation of this method onto the MacCormack time splitting scheme, and the ability of the resulting TVD scheme to simulate broken wave propagation. In section 3, computed solutions are compared with measurements of sea surface elevations on a sandy beach (Truc Vert Beach, France).

2 Model

Our model is based on the nonlinear SV equations with bottom friction. These equations can be expressed in vectorial form as:

$$\frac{\partial \mathbf{q}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x} = \mathbf{S} \tag{1}$$

$$\mathbf{q} = \begin{pmatrix} h \\ hu \end{pmatrix}, \quad \mathbf{F} = \begin{pmatrix} hu \\ hu^2 + \frac{g}{2}h^2 \end{pmatrix}, \quad \mathbf{S} = \begin{pmatrix} 0 \\ -gh\frac{\partial Z_f}{\partial x} - \frac{1}{2}f|u|u \end{pmatrix}$$

where $h(x,t)$ is the total depth of water, $u(x,t)$ is the depth-averaged water velocity, Z_f is the bed elevation, g is the gravitational acceleration and f is a constant friction coefficient. The free surface elevation $\eta(x,t)$ is given by: $\eta(x,t) = h(x,t) + Z_f$.

2.1 Numerical scheme

To solve equation (1), we have implemented the MacCormack scheme which is a classical hyperbolic solving method. The main reason for choosing the predictor-corrector step instead of the one-step Lax-Wendroff formulation is that the former provides a natural way of including the source terms $\mathbf{S}$. Following the work of Yee [8] for the Navier-Stokes compressible equations, we have modified the MacCormack scheme with a TVD flux correction in order to make it non-oscillating in presence of strong gradients.

Let $\mathbf{q}_i^n$ be the numerical solution of equation (1) at $x = i\Delta x$ and $t = n\Delta t$, with Δx the spatial mesh size and Δt the time step; $\lambda = \Delta t/\Delta x$. The TVD MacCormack scheme can be expressed in three steps:

(a) predictor step

$$\mathbf{q}_i^1 = \mathbf{q}_i^n - \lambda(\mathbf{F}_i^n - \mathbf{F}_{i-1}^n) + \Delta t \mathbf{S}_i^n$$

(b) corrector step

$$\mathbf{q}_i^2 = \frac{1}{2}[\mathbf{q}_i^1 + \mathbf{q}_i^n - \lambda(\mathbf{F}_{i+1}^1 - \mathbf{F}_i^1) + \Delta t \mathbf{S}_i^1]$$

(c) TVD step

$$\mathbf{q}_i^{n+1} = \mathbf{q}_i^2 + \frac{\lambda}{2}(R_{i+\frac{1}{2}}^n \phi_{i+\frac{1}{2}}^n - R_{i-\frac{1}{2}}^n \phi_{i-\frac{1}{2}}^n)$$

where R is the right-eigenvector matrix of the flux Jacobian matrix $A = \dfrac{\partial \mathbf{F}}{\partial \mathbf{q}}$. The steps (a) and (b) describe the classical MacCormack scheme whereas the step (c) is a TVD flux correction, which is described in detail in Yee [8] and Vincent et al. [7]. The TVD MacCormack scheme so obtained retains second-order precision in space and time in regular zones and is oscillation-free across wave fronts.

2.2 Boundary conditions

At the seaward boundary we have implemented a method developed by Cox et al. [1], which determines the outgoing Riemann invariant by an implicit scheme. This method allows to specify the measured water depth $h(t)$ directly at the seaward boundary of the domain ($x = 0$).

To manage the swash zone evolution we impose in dry meshes a thin water layer $h_{min} = 10^{-4}m$, with $u = 0$. Thus, SV equations are solved everywhere in the computational domain. However, at the shoreline (wet meshes next to dry meshes) a specific treatment is applied in the momentum equation to the discretization of the horizontal gradient of the surface elevation ($gh\frac{\partial \eta}{\partial x} = gh(\frac{\partial h}{\partial x} + \frac{\partial Zf}{\partial x})$). It consists in omitting the landward spatial differences of this term in the predictor and corrector steps. Vincent et al [7] have shown that this simple numerical treatment gives very accurate results in describing shore-line evolution for a non-breaking wave climbing a beach.

2.3 Energy dissipation

Following the work of Kobayashi and colleagues (Rbreak program), the present numerical model does not include explicitly a physical representation of the energy dissipation due to wave breaking. The modelling of broken wave dissipation is given by the numerical dissipation of our shock-capturing numerical scheme in presence of wave fronts.

Comparisons of measured and computed results (see section 3) confirm previous studies by Cox et al. [1] and Raubenheimer et al. [6], showing that the predicted wave height decay is consistent with field observations in the inner surf zone. Moreover, if Δx is small enough to describe the wave front, the computed dissipation is not sensitive to the spatial resolution.

To illustrate this assertion we have simulated the propagation of a wave packet on a planar bottom in a periodic domain with no friction and a constant water depth h_0. The initial conditions for h and u are given by :

$$h(x) = h_0 + \eta_0 \exp(-\frac{x^2}{2\sigma^2}) \tag{2}$$

$$u(x) = (gh_0)^{1/2}\frac{\eta_0}{h_0} \exp(-\frac{x^2}{2\sigma^2}) \tag{3}$$

where, $\eta_0 = 0.1m$, $h_0 = 1m$ and $\sigma = 5m$. Since SV equations do not contain dispersive terms, unbroken waves are supposed to steepen and form bores. Figure 1 shows the computed solutions obtained for different Δx and a constant CFL ($CFL = (gh_0)^{1/2}\Delta t/\Delta x = 0.31$). Figure 1a shows that the spatial step size affects the steepness of the bore front. Figure 1b shows the time evolution of the total wave energy E ($E = \frac{1}{2}hu^2 + \frac{1}{2}g\eta^2$). At the beginning of the simulation the numerical dissipation of the unbroken wave, as we can expect, is stronger for large Δx. However, when the bore is formed ($t \sim 100$), the energy dissipation becomes independent of the spatial resolution.

3 Comparison of experimental data and numerical results

In order to assess the predictive capability of our model, comparisons have been done with field measurements. The experimental site, the so-called Truc Vert Beach, is a sandy beach NS orientated, located 5 km North of the Arcahon lagoon inlet (SW France), in a mesotidal context. A field study on the morphological evolution of this beach was carried out in May 1998 (Programme National d'Océanographie Côtière). In this paper, we analyse one data run of this fieldwork, corresponding to pressure measurements collected near high tide. Three pressure sensors were deployed on a cross-shore transect, in mean depths between 0.3m and 1.3m (see figure 2). In figure 2, $z = 0$ corresponds to the mean level of the surface elevation at $x = 0$ (P1 location). Data runs were acquired at a 3 Hz sample rate. Water depths were estimated assuming that the measured pressure field is hydrostatic. Offshore waves were irregular and characterized by a broad band spectrum with a spectral peak period of $6s$. Their significant wave height was 0.85m and the wave direction was approximately normal to the

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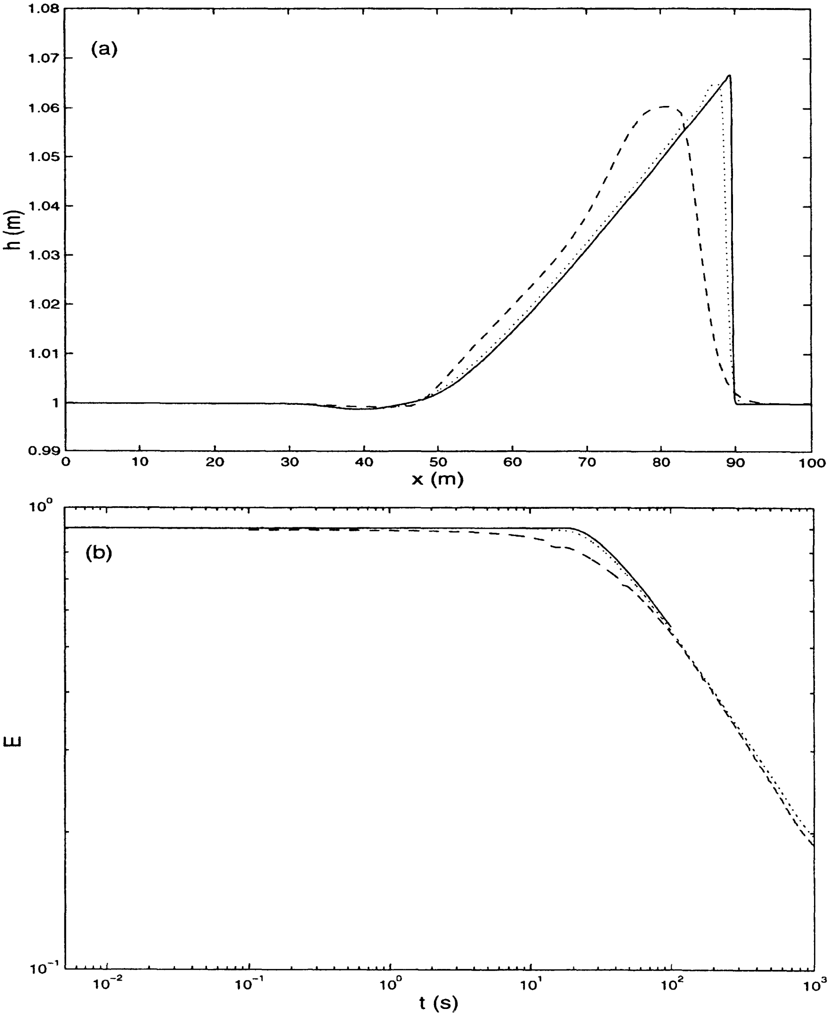

Figure 1: Computed solutions for the propagation of a wave packet (eqns 2 and 3) in a constant water depth. _____, $\Delta x = 0.05$; ...., $\Delta x = 0.25$; $---$, $\Delta x = 1$; (a), Surface elevation at $t = 100s$; (b), Time evolution of the total wave energy E.

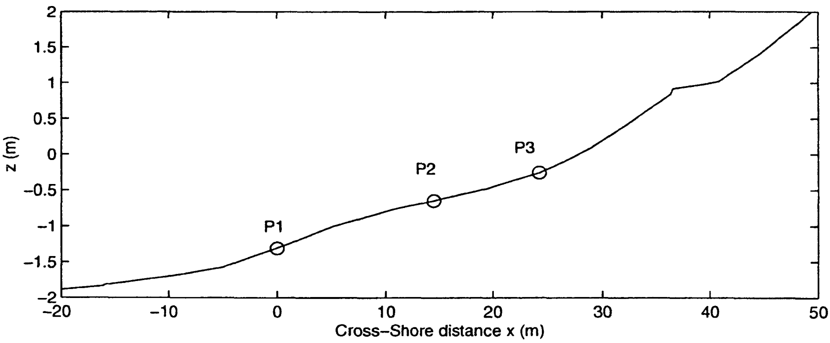

Figure 2: Cross-shore bathymetry for Truc Vert beach and locations of pressure sensors (circles).

beach.

The computation is performed using $\Delta x = 0.024m$ and, following calibrations done by Raubenheimer et al. [5], a friction coefficient $f = 0.015$. The duration of the run was $900s$ $(-50 < t < 850)$, where $t = -50s$ is the start of the computation. The initial condition of no wave motion leads to a transient period $t \in [-50, 0]$, which is eliminated from time series presented hereafter. The seaward boundary condition of the model is given by time series of water depth (P1 sensor), corresponding to the shoreward propagation wave field.

Measured and computed time series of local water depth h at cross-shore locations P1, P2 and P3, are shown in figure 3. The model gives a good prediction of wave transformation in the inner surf zone. For instance, the model predicts the observed decrease of wave height and increase of wave asymmetry as waves propagate shoreward. Discrepancies between minimum values observed with the P3 sensor and computed results (figure 3c) are due to the fact that P3 was located 0.105 m above the bed and thus cannot measure water depths smaller than this threshold. The model can be used to determine time-average quantities, such as the mean water level or the root mean square of the surface elevation (see figure 4), which are useful for sediment transport model based on time-average equations. In figure 4a, we have also reported the minimum and maximum water levels which

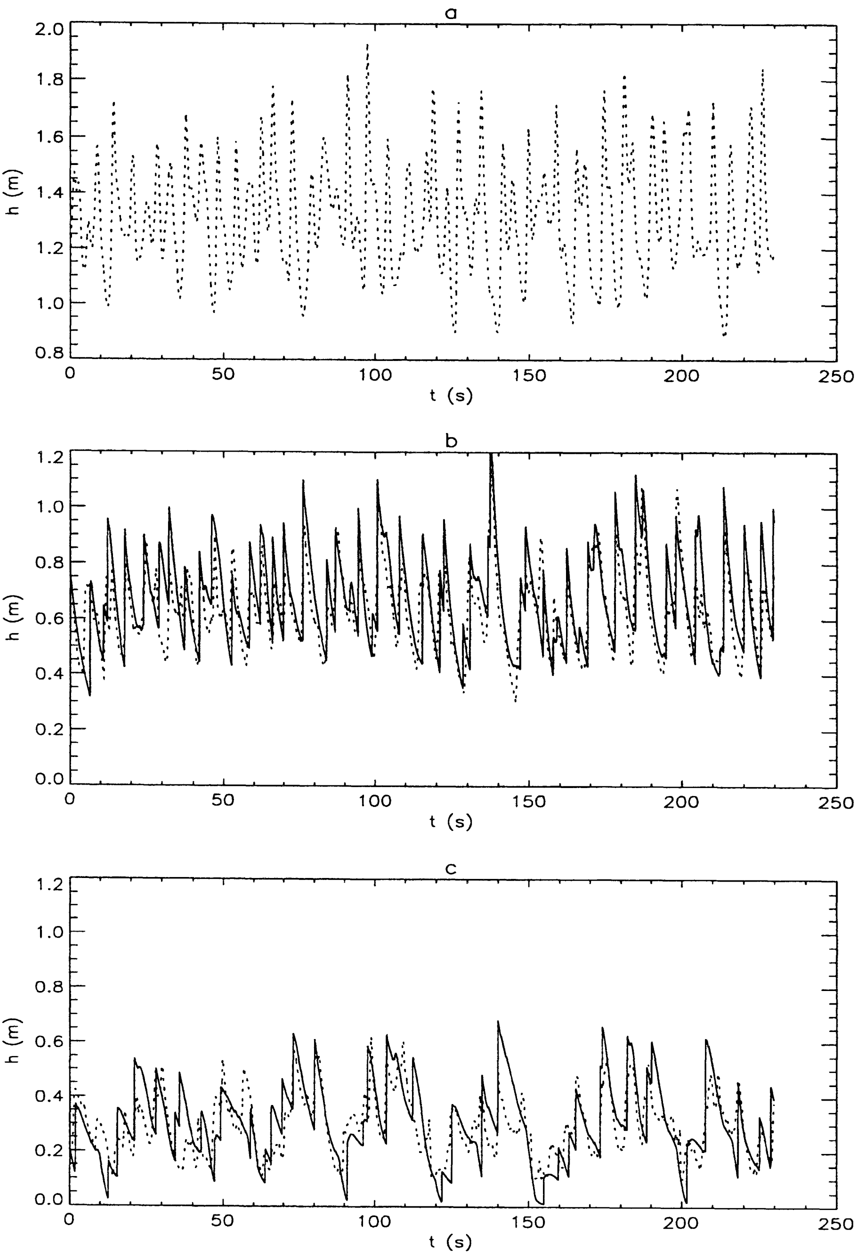

Figure 3: Measured (dashed line) and computed (solid line) time series of water depth h at cross-shore locations: (a), $x = 0$ (P1); (b), $x = 14.5m$ (P2); (c), $x = 24.5m$ (P3).

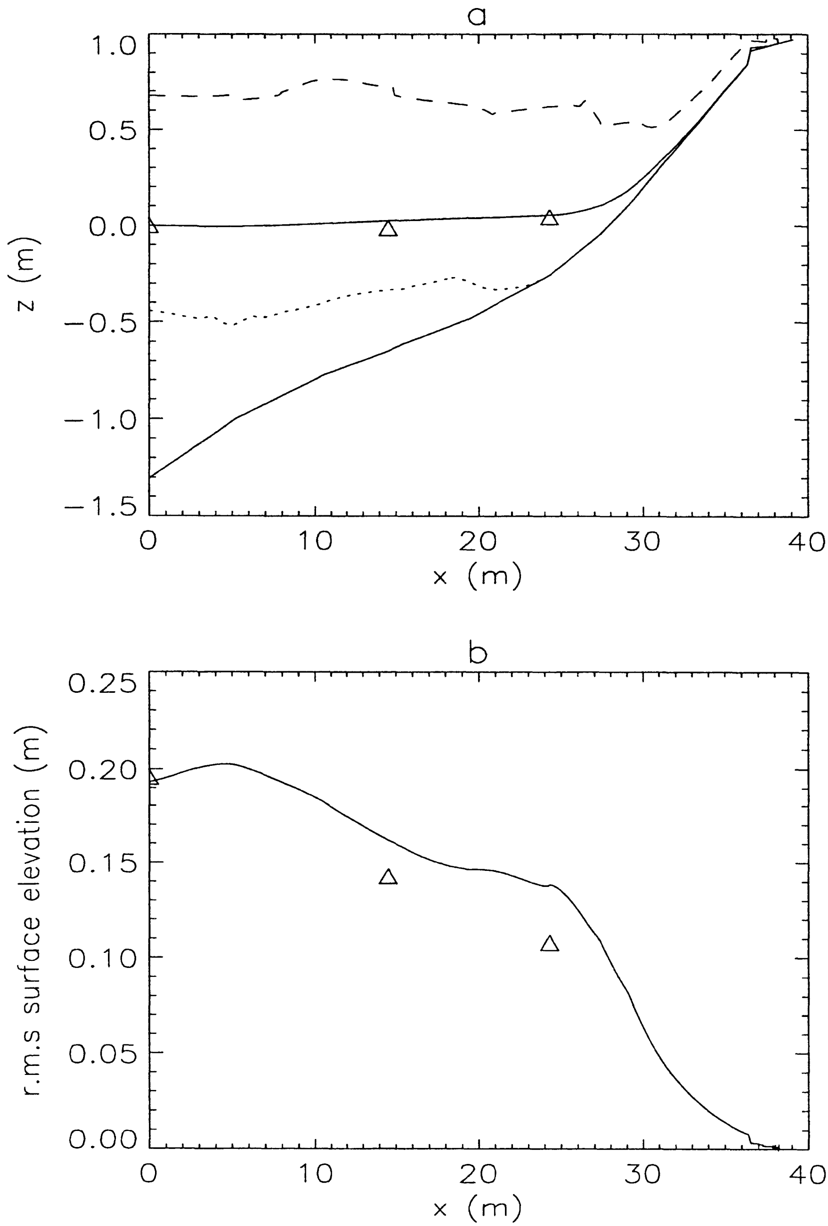

Figure 4: Observed (triangle) and computed (solid line) cross-shore variations of time-average quantities. (a), _____, mean water level; ...., minimum water level; – – –, maximum water level. (b), _____, root mean square of the surface elevation.

show the location of the swash zone. Computed maximum water level is in agreement with video observations, which show that the maximum run-up goes just beyond the berm. We note that the swash zone is characterized by a strong set-up (figure 4a) and a strong decrease of the wave height (figure 4b). This flow regime transition between the inner surf zone and the swash zone is also characterized by the wave skewness S_k. Indeed the observed and predicted skewness is nearly constant ($S_k = 0.4$) in the inner surf zone, but the model shows that it strongly increases in the swash zone.

4 Conclusion

This paper presents the development of a model that solves the non-linear Saint Venant equations, based on a high order MacCormack TVD scheme. This model is able to describe broken wave propagation without introducing numerical oscillations at the rear of the wave front. The numerical model is shown to accurately predict the irregular wave transformation observed along transects spanning the inner surf zone on Truc Vert Beach. At present, we are doing a detail analysis of wave shapes. In particular, we want to characterize the wave asymmetry which plays a important role in sediment transport. A two-dimensional extension of the model is also in progress in order to account for the alongshore variations of incident waves and beach topography.

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