Diffraction of wave spectra by two breakwaters at an angle

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Abstract

The present study tackles the problem of random wave diffraction through an opening between two breakwaters forming an arbitrary angle. The numerical scheme used is result-oriented and employs the solution regarding the diffraction coefficient $k$ for the monochromatic case. The study comprises two parts, the first dealing with uni-directional and the second with multi-directional incoming spectra. A simple energy conservation principle applied to a number of components associated with the monochromatic solution of the diffraction problem, produced one-dimensional spectra of the diffracted waves at any point in the lee of the breakwaters. A compound diffraction coefficient $K$ could thus be established in terms of the $k$'s and the characteristics of the incoming spectrum. Graphs of equal $K$-values are presented for the various cases examined. The second part of the study deals with waves presenting spatial distribution of energy. A directional spreading function of the Mitsuyasu-type was assumed. The diffracted spectra yielded a composite diffraction coefficient by integrating through the directions as well as through the frequencies.

1 Introduction

Modelling of wave diffraction is of central importance to coastal and port engineering. Existing models deal with regular or, more rarely, with random uni-directional wavetrains. Analytical solutions have been possible only for monochromatic waves and simple configurations, as e.g. in the case of wave diffraction through a gap in an infinite breakwater. The more practical geometry of two breakwaters at an angle has been treated in a semi-analytical manner by the senior author yielding results
for monochromatic waves. The main assumptions used in that treatment pertain to small-amplitude long-crested surface waves impinging on two semi-infinite reflecting breakwaters.

Diffraction of random sea waves is certainly of greater importance due to their being closer to real life situations. Furthermore random seas with directional energy spread can even more closely represent wind generated surface waves. Thus knowledge of their diffraction characteristics is highly desirable. Goda et al. have produced diffraction diagrams for random directional waves past a semi-infinite breakwater and through breakwater gaps of various widths. The wave approach in all these configurations was assumed normal to the breakwaters. Some of their results are reproduced in the Shore Protection Manual.4

The present study tackles a more general situation of wave diffraction through a gap formed at the intersection of two breakwater legs with axes that are not collinear but intersect at an arbitrary angle (fig. 1). The results presented in the following refer to uni- or multi-directional random waves, and they can be developed for various combinations of wave approach, angle between the breakwaters and wave characteristics. The existing results2 of the associated diffraction coefficient for the monochromatic case have been used in this procedure.

![Diagram](image)

Figure 1: Nomenclature of the problem.

2 Technique applied

In dealing with uni-directional random waves one can establish a diffraction coefficient in the following manner.

$$K = K(\rho, \omega) = \left( \frac{\int S_d(f)df}{\int S_i(f)df} \right)^{1/2}$$

(1)

where, $S_i, S_d$ the energy density of the incident, diffracted one-dimensional spectra respectively
the frequency

\((\rho, \omega)\) the polar coordinates of the point under consideration, non-dimensionalized with respect to wavelength (fig. 1).

Now, a decomposition of the incoming power spectrum into its frequency components can be effected, in such a way that the solution of the monochromatic case can be applied to each individual wavelet of the spectrum.

Expression (1) can be approximated by

\[
K = \left( \frac{\Sigma S_d(f) \Delta f}{\Sigma S_i(f) \Delta f} \right)^{1/2} \tag{2}
\]

Simple energy considerations lead to the following relation holding for any frequency

\[
S_d = S_d(\rho, \omega; f) = k^2(\rho, \omega)S_i(f) \tag{3}
\]

where \(k = k(\rho, \omega)\) the diffraction coefficient for the monochromatic wave.

It is then possible to reconstruct the diffracted spectrum in the harbour area by "adding" appropriately the diffracted individual components. Equation (3) yields

\[
K = \left( \frac{\Sigma k^2S_i(f) \Delta f}{\Sigma S_i(f) \Delta f} \right)^{1/2} \tag{4}
\]

The power spectrum \(S_d\) at any point \((\rho, \omega)\) in the lee of the breakwaters can be constructed through repeated use of eq. (3) for any incoming spectrum \(S_i\), provided the corresponding \(k\)-values are known.

In the case of incident directional spectra an energy spread function \(G(f, \theta)\) was assumed, in the usual way:

\[
S(f, \theta) = G(f, \theta)S(f) \tag{5}
\]

The decomposition of the approaching spectrum was effected for each direction as with the uni-directional case. A composite diffraction coefficient \(K'\) can be defined by extending relation (1) to include the directional spread, following Goda.\(^5\)

\[
K' = K'(\rho, \omega) = \left( \frac{\int \int S_d(f, \theta) d\theta df}{\int \int S_i(f, \theta) d\theta df} \right)^{1/2} \tag{6}
\]

Taking into account eq. (3), the approximate expression of eq. (6) can be written in the following form
Equation (7) gives easily

\[
K' = \left( \frac{\sum k^2 S_i(f, \theta) \Delta f \Delta \theta}{\sum S_i(f, \theta) \Delta f \Delta \theta} \right)^{1/2}
\]  

Relation (5) yields for the diffracted spectrum the following result

\[
S_d(f, \theta) = G(f, \theta) S_d(f) / A^2
\]

where \( A = \int \sqrt{G(f, \theta)} \, df \, d\theta \) is a normalization factor to ensure that

\[
(\int S_i(f, \theta) \, df \, d\theta)^{1/2} = 1, \quad \theta_{\text{max}} \]  

the maximum deviation of the energy spread from the principal wave direction.

In the numerical applications, factor A is approximated by

\[
A = \sum \sum \sqrt{G} \Delta f \Delta \theta
\]

In order to be able to reconstruct the directional spectrum after diffraction, an approximation was made regarding the direction of each wave component in the harbour. To this effect the harbour basin was divided into three distinct zones associated to the direction of the incoming wave component. In these zones the wave propagation is described by a simple geometric law as depicted in fig. 1. The following relations hold for the direction angle \( \alpha \) measured as shown in the same figure.

Zone I  \( \alpha = \varphi - \omega \)

Zone II  \( \alpha = \zeta' - \pi \)

Zone III  \( \alpha = \arcsin \left[ \frac{\delta \sin(\varphi - \omega)}{\rho - \delta \sqrt{1 - \sin^2 \zeta'}} \right] + \varphi - \omega \)

where \( \zeta' = \zeta + \theta \)
It is evident from the above relations that the direction of the diffracted wave on a particular point depends on both the location of the point as well as on the particular wave component of the incoming spectrum.

In both uni- and multi-directional seas the shift of the spectrum peak at each individual point in the harbour basin is presented by the ratio $T_d/T_i$ of the peak periods of the diffracted and incident spectra.

### 3 Results and discussion

The above described procedure was applied to particular cases, of which some representative results will be shown in the following. The Bretschneider spectrum as modified by Mitsuyasu has been assumed for the approaching waves, while a spread function of the Mitsuyasu-type as developed by Goda was taken for the directional seas. It is noted that any other spectral shape can be treated by the same procedure, which in general is insensitive to the type of incoming spectrum. The abovementioned input is expressed as follows

$$ S_i(f) = 0.257 / H_s^2 T_s (T_s f)^{-5} \exp[-1.03 (T_s f)^{-4}] 
$$

(11)

$$ G(f, \theta) = \cos^2 \left( \frac{\theta}{2} \right) / \int \cos^2 \left( \frac{\theta}{2} \right) d\theta
$$

(12)

where,

$$ s = s_{\text{max}} \left( \frac{f}{f_p} \right)^5, \quad f < f_p $$

$$ s = s_{\text{max}} \left( \frac{f}{f_p} \right)^{-2.5}, \quad f \geq f_p $$

$s_{\text{max}}$ number related to the vicinity of the study area to the wave-generating wind field

$f_p$ frequency at peak

$H_s, T_s$ characteristic wave height, period respectively.

In the following applications a value of $s_{\text{max}}=10$ has been assumed related to seas rather than to swell; the decomposition of the directional wave field was effected at $5^\circ$ intervals and the wave energy was assumed to be contained between $-35^\circ$ and $35^\circ$ from the principal direction. Within this angle it was found that 87% of the total energy is contained, which is acceptable for practical applications. The gap width, normalized with respect to the associated wavelength, was kept constant at $\delta=1.38$.

Figure 2 presents diffraction patterns in terms of both $K$ and $T_d/T_i$ values, for the particular case of uni-directional waves and $\zeta = 250^\circ$,
$\phi=120^\circ$, $\delta = 1.38$. In the same figure the corresponding case with $\delta=1.0$, for monochromatic wave has been reproduced for comparison. It can be said that the diffraction coefficient is larger with random rather than with monochromatic waves for this particular case. However, part of the difference is attributed to the larger opening in the configuration with random waves. An interesting variation of the period at peak can also be seen in fig. 2(b). This variation ranges between $\pm 15\%$ of the period at peak of the incoming waves, for the case presented and within a distance of 4L from the opening. It is noted that the periods pattern is common to both uni- and multi-directional incoming waves.

Figure 3 shows results for the case with $\phi = 90^\circ$ and uni-directional ($\zeta=250^\circ$) and multi-directional ($\zeta=235^\circ$) spectra. Comparison of these results with the corresponding diagram for the monochromatic wave shows again some energy spread in the case of random waves. Consequently the wave agitation in the harbour basin can be greater in areas seemingly "protected" when attacked by monochromatic waves. This conclusion is in agreement with the results shown in SPM for the particular case of two breakwater arms along a straight line. The variation of the period at peak is illustrated in fig.3(c) in a manner similar to the previously examined case with $\phi=120^\circ$ (fig. 2).

Finally, figure 4 gives results of the energy non-dimensionalized with respect to $H_s^2T_s$ at particular points in the basin. Figure 4(a) is associated with uni-directional while 4(b) and (4c) with multi-directional incoming waves. The wave direction of the resulting diffracted spectra, denoted by $\alpha$ on the axes of figures 4(b) and 4(c), is measured as shown in fig.1, i.e. between breakwater (B) and wave direction in the harbour. The latter is determined by the vector emanating from A and B for zones I and III respectively, while in zone II the wave propagation is assumed along the main direction of the impinging waves. Comparison of figures 4(a) and 4(b) reveals a similar shape of the 3-D spectrum along the principal direction with the corresponding 2-D one. However, more energy is concentrated in lower frequencies in the three-dimensional case. For all cases the incoming spectrum obeyed eqs (11) and (12), with $H_s=3.3$ m, $T_s=6.5$ sec. The maximum concentration of energy in the diffracted spectrum is observed for $\alpha=41.1^\circ$ (fig.4(b)) and $\alpha=13.5^\circ$ (fig.4(c)) for the two points under consideration.

4 Conclusion

An algorithm has been developed that can predict the diffraction coefficient, the variation of the period at peak and the diffracted spectrum in the cases of random long-or short-crested waves. These results refer to breakwater configurations useful for practical applications. It can be said that random waves spread out more evenly the wave energy within the harbour, thus producing greater agitation in large areas of the basin than those predicted for monochromatic waves. However, more work is needed in order to cover a larger range of cases and to arrive to conclusive results.
Figure 2:(a) $k$-isolines for monochromatic waves; $\delta=1.0$, $\varphi=120^\circ$, $\zeta=250^\circ$
(b) uni-directional spectrum, $\delta=1.38$, $\varphi=120^\circ$, $\zeta=250^\circ$
(b$_1$) $K$-isolines, (b$_2$) $T_d/T_1$-isolines.
Figure 3: Diffraction for $\delta=1.38$, $\varphi=90^\circ$
(a) $K$-isolines, uni-directional spectrum, $\zeta=250^\circ$
(b) $K'$-isolines, multi-directional spectrum, $\zeta=235^\circ$
(c) $T_d/T_i$-isolines, $\zeta=250^\circ$. 
Figure 4: Power spectra for $\delta=1.38$, $\varphi=90^\circ$, $\zeta=235^\circ$, $\rho=3$

(a) one-dimensional spectrum, $\omega=60^\circ$
(b) multi-directional spectrum, $\omega=60^\circ$
(c) multi-directional spectrum, $\omega=80^\circ$. 
Notation

$H_s$  significant wave height  
$L$  wavelength  
$k, K, K'$  diffraction coefficient  
$S_i, S_d$  incident, diffracted spectrum coordinate  
$T_s$  significant wave period  
$\alpha$  direction of the diffracted wave from left breakwater arm  
$\delta$  non-dimensionalized width of the opening  
$\zeta$  direction of incident waves measured from left breakwater arm  
$\zeta'$  $\zeta + \theta$ (positive $\theta$ counterclockwise)  
$\theta$  spreading angle from the principal direction  
$\rho$  non-dimensionalized coordinate of any point in harbour basin  
$\phi$  angle between the two breakwaters  
$\omega$  angle coordinate of any point in the lee of breakwater

References


