Separation of incident and reflected spectra in wave flumes
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Abstract

In wave flumes or basins, when simulating sea waves in physical models with reflective structures, there frequently arrive, on the wave maker paddle, important reflected waves, due to the structure, which affect the incident waves. So that the waves generated by the paddle movement are not perturbed, the paddle movement must be compensated in order that the reflected waves be absorbed and have no effect. For this, the incident and the reflected spectra have to be separated.

There exist several methods for separating incident and reflected spectra both in the time and the frequency domain. The methods considered in this paper belong to the last group. Basically, these methods consist in deducing the incident and reflected wave spectra from the observed wave spectra at two or more points located along a line parallel to the wave propagation direction.

Besides separating the spectra, the model is also able to produce numerically simulated data with given characteristics of wave spectra and physical constraints. It can also be tested by both numerically simulated data and real data recorded by the different wave gages operating in the wave flume.

Comparing the so-called two-gage method and the N-gage method in terms of the ability to reconstruct the incident and reflected wave spectra, and according to the numerical and physical tests already carried out, the N-gage method seems to produce more rigorous results than the two-gage method. For the work presented in this paper these two methods were implemented in computer for a bi-dimensional flume and numerical simulation procedures able to verify the methods were established.

The results of the numerical simulations developed to test these two methods proved both methods to be quite accurate. The two methods were then applied to physical flume waves with satisfactory results. From the work reported in this paper a first version of a software package able to address the problem of unwanted wave reflection in wave flumes was developed.
1 Introduction

The reflection of incident waves produced by physical models is a general problem the engineer must solve when dealing with physical tests in wave tanks or flumes. Actually, reflected waves interact with incident waves, reflecting again on the paddle(s) of the wave generator, in a process that has no similitude to what really happens in Nature, where the energy of those reflected waves is dispersed offshore.

In the case of stability or over-topping physical tests of breakwaters it is thus important to extract the incident waves from those waves recorded in front of the physical model, these being the sum of the incident and reflected waves. This must be done in order to relate the incident waves with the severity of damages in the model.

Knowledge of the characteristics of incident waves is also important in wave agitation studies, for instance for correcting the reflection in both coastal structures or even model’s boundaries.

Incident and reflected wave heights as well as the reflection coefficient can easily be calculated when dealing with non-directional regular (long crested) waves. Also, with bidimensional irregular waves, the incident and reflected wave spectra, measured in different points along the front of the physical model, can be obtained. For that purpose, one uses different methods as the ones described by Thornton & Calhoun, Goda & Suzuki, Morden et al. and Mansard & Funke. Basically, these methods consist in deducting the incident and reflected wave spectra based on the observed wave spectra in several points located along a line parallel to the wave propagation direction. The method used in the work of the first three quoted authors considered two points whereas the last author used three points. An alternate method is to use more than two gages also but with the reflection analysis being performed by averaging the reflection coefficients computed from different pairs of gages with the 2-gage method. Besides these frequency domain methods, there exist an other method able to address the problem of separating the incident waves from the reflected waves by using a time-domain approach as described in Frigaard & Brorsen. This method is, however, not treated in this paper.

The model presented in this paper considers thus the first two methods, with the last model generalised to any number \((N)\) of points. The computer implementation of the model, called REFLEX, has been done in Turbo-Pascal and is able to produce numerically simulated data with given characteristics of wave spectra and physical constraints. Both methods can thus be tested by both numerically simulated data and real data recorded by the different wave gages operating in the wave flume.

Comparing the two methods in terms of the ability to reconstruct the incident and reflected wave spectra, and according to the numerical and physical tests already carried out, the \(N\)-gage method seems to produce more rigorous results than the two-gage method.
2 Theoretical Aspects

In what follows, the main general theoretical aspects concerning the referred two methods are described.

The concept of progressive wave is introduced. A progressive wave is a function $z(t, x)$ two-variable which is sinusoidal in either variable. Thus, the general formula of a progressive wave is given by

$$z(t, x) = a \sin \left( \pm \frac{2\pi t}{T} \pm \frac{2\pi x}{L} + \theta \right)$$

where $t$ is time, in seconds; $x$ is space, in metres; $a$ is the wave amplitude; $T$ is the wave period; $L$ is the wave length corresponding to $T$; $\theta$ is the phase introducing a random location for the $x$ and $t$ references.

For the present problem, the function $z(t, x)$ can be thought of as a sinusoid moving along the $x$ axis. By observing eqn. (1) one can see that the progressive wave moves to the right (positive direction) when the signs in the two terms of the sine argument are different whereas when the signs are equal the wave moves in the opposite (negative) direction.

Consider now in Figure 1 an irregular wave flume where $M (1, 2, ..., M)$ wave gages are located at different distances from one to another. Figure 1 also shows the wave generator paddle and a reflecting object which can be, for instance, a physical model to be tested against wave action. It is assumed also that the observed signal (the time varying wave elevation) is composed of an incident and a reflected signal.

The incident signal is represented by a sum of sinusoids (in the particular case of regular waves, just one) that are functions of $t$ and $x$. This sum represents a progressive wave with a movement that is positive (as in the case of Figure 1) or negative depending on whether the signs in the two terms of the sine argument of eqn. (1) are different or equal.
Hence, a progressive wave moving in the positive direction can be represented as

\[ z_+(t, x) = a \sin \left( \frac{2\pi t}{T} - \frac{2\pi x}{L} + \theta \right) \]  

(2)

where

- \( x \) is the abcissa;
- \( \theta \) is the wave phase,

whereas a progressive wave moving in the negative direction, with the same amplitude, length and period, can be represented as

\[ z_-(t, x) = a \sin \left( \frac{2\pi t}{T} + \frac{2\pi x}{L} + \phi \right) \]  

(3)

where

- \( \phi \) is the phase, different from \( \theta \).

Thus, the observed signal can be represented as follows:

**At gage 1:**

\[
X_i(t) = \sum_{k=1}^{N} I_k \sin \left( \frac{2\pi k t}{T} - \frac{2\pi x_1}{L_k} + \theta_k \right) + \sum_{k=1}^{N} R_k \sin \left( \frac{2\pi k t}{T} + \frac{2\pi x_1}{L_k} + \theta_k + \varphi_{ik} \right)
\]

(4)

**At gage n:**

\[
X_n(t) = \sum_{k=1}^{N} I_k \sin \left( \frac{2\pi k t}{T} - \frac{2\pi (x_1 + x_{in})}{L_k} + \theta_k \right) + \sum_{k=1}^{N} R_k \sin \left( \frac{2\pi k t}{T} + \frac{2\pi (x_1 + x_{in})}{L_k} + \theta_k + \varphi_{nk} \right)
\]

(5)

where

- \( N \) is the total number of harmonics;
- \( k \) indicates the \( k^{th} \) harmonic;
- \( I_k, R_k, L_k \) and \( \theta_k \) are, respectively, the incident amplitude, the reflected amplitude, the wave length and the wave phase of harmonic \( k \), independent of the wave gage;
- \( x_1 \) is the distance from the wave paddle to gage 1;
- \( x_{in} \) is the distance from gage 1 to gage n;
- \( \varphi_{nk} \) is the wave phase shift of harmonic \( k \) due to the wave travelling from gage \( n \) to the obstacle and therefrom to gage \( n \) again.

Thus, in eqns (4) and (5) the first summation represents the incident component of the signal whereas the second summation represents the reflected part.
On the other hand, $X_n(t)$ might be represented by Fourier Analysis as below:

$$X_n(t) = \sum_{k=-N}^{N} A_{nk} e^{\frac{2\pi i k t}{T}}$$

where

- $X_n$ is defined in the interval $(0, T)$;
- $A_{nk}$ are coefficients obtained by Fourier analysis.

Considering now the real and imaginary components of $A^e$, $A^o$, $Z^e$, and $Z^o$, auxiliary complex numbers from which $I_k$ and $R_k$ can be obtained, as

$$A_{nk} = D_{nk} + i E_{nk}$$

$$Z_{jk} = X_{jk} + i Y_{jk}$$

$$Z_{Rk} = X_{Rk} + i Y_{Rk}$$

it can be shown, Capitão & Carvalho,\(^1\) that

$$A_{nk} = Z_{jk} e^{-i\alpha_n} + Z_{Rk} e^{i\alpha_n} \quad \text{for } n = 1, 2, ..., M$$

This is, a system of linear complex equations of size $M$ - the number of gages, where

$$\alpha_n = \frac{2\pi \xi_{1n}}{L_k}$$

$L_k$ is the wave length for each $k$ harmonic, calculated with $T$ substituted by $T/k$;

$D_{nk}$, $E_{nk}$ $(n = 1, 2, ..., M)$ are the parameters found in a harmonic analysis of the signal in gage $n$.

If $M = 2$, that is, if there exist only two wave gages on the flume, one obtains a determined system of equations (actually a system of four equations and four unknowns). It is the so-called 2-gage method.

If, however, $M > 2$, the system is impossible since it is over-dimensioned. An optimal approximate solution can be found if one elects a certain criterion. This criterion can be for instance the least squares method, which is the one used in this work. When $M > 2$ the method is called the $N$-gage method.

Once the system of linear equations is solved, the parameters $X_{jk}$, $Y_{jk}$, $X_{Rk}$, $Y_{Rk}$ can be determined, being these either the exact solution (for $M = 2$) or an approximate solution (for $M > 2$).
With the parameters \( X_{ik}, Y_{ik}, X_{Rk}, Y_{Rk} \), for each \( k \), \( I_k \) e \( R_k \) can be finally found as

\[
I_k = \frac{2 X_{ik}}{\cos W_k} \quad (12)
\]

\[
R_k = \frac{2 X_{Rk}}{\cos Q_{nk}} \quad (13)
\]

where

\[
W_k = \arctan \frac{Y_{ik}}{X_{ik}} \quad (14)
\]

\[
Q_{nk} = \arctan \frac{Y_{Rk}}{X_{Rk}} \quad (15)
\]

Finally, according to Carvalho,\(^2\) since \( I_k \) and \( R_k \) are already known, the incident and reflected spectra are estimated (without smoothing) as below:

\[
p_I \left( \frac{k}{T} \right) = T \frac{I_k^2}{2} \quad (16)
\]

\[
p_R \left( \frac{k}{T} \right) = T \frac{R_k^2}{2} \quad (17)
\]

These spectrum estimates can be smoothed by the customary procedures.

The reflection coefficient is determined as usual. For irregular waves, the reflection coefficient can be computed as:

a) The reflection coefficient for each harmonic

\[
r_k = \frac{R_k}{I_k} \quad (18)
\]

b) The reflection coefficient based on the reflected and incident energy

\[
r_0 = \sqrt{\frac{m_{0R}}{m_{0I}}} \quad (19)
\]

where: \( m_{0R} \) and \( m_{0I} \) are the integrals of the reflected and incident spectra, respectively.

c) The reflection coefficient function obtained by considering reflected and incident spectra with some kind of smoothing.
The computer program allows one also to verify whether or not there is compatibility in proposed distances between wave gages.

3 Verification

For numerically simulated data, the verification of the methods is accomplished by computing:

- the degree of similitude of the computed incident spectrum with the initial incident spectrum
- the comparison of reflection coefficients $r_k$ and $r_0$ with the initially assumed reflection coefficient $Cr$.

For signals from wave gages, the methods are verified by computing:

- the degree of similitude of the computed incident spectrum with the initially wanted wave spectrum. In this case, one must guarantee that the initially wanted wave spectrum will be reproduced in flume adequately as if no reflection would exist. The computed incident wave spectrum is quite irregular, with large fluctuations probably due to non-controlled phenomena, for the case of the 2-gage method. This numerical instability could be smoothed automatically by a routine of moving averages (for example). It has been verified however that each case is a different case. Thus, program REFLEX gives raw results and the smoothing itself will be performed in a later stage.

4 Numerical simulation

In order to check the validity of the procedures a numerical simulation of wave gages signals has been performed. Thus, two (or $N$) sets of wave simulated data were generated, by using formulae (4) and (5), and the 2-gage method (or the $N$-gage method) was applied.

Below an example of a graphical output is presented.

![JONSWAP spectrum](image)

Figure 2: First 60 s of a numerical simulation of 3 gages - $Cr = 40\%$
Thus, in Figure 2, one can see a graphical piece of a numerical simulation of a wave record containing the signals in three different wave gages. For this numerical simulation, a JONSWAP spectrum was used, with parameters: significant wave height \( H_s \) of 0.15 m and peak period \( T_p \) of 1.7 s.

From numerical simulations, one concludes that the computed incident spectrum \( p_{\text{inc}} \) is found to be exactly the same as the initial incident spectrum \( p_{\text{ini}} \). That happens for both methods. However, for very low frequencies, non-realistic reflection coefficients were found. This fact is due to numerical instabilities happening when divisions of very low values occur due to eqns. (12) and (13) but has no effect in the range of frequencies of interest. Therefore, the assumed coefficient of reflection is found to be the same that was assumed for each numerical simulation.

These results prove both methods to be theoretically correct. In practice, however, the methods - mainly the 2-gage method - are not as rigorous as in theory. Several aspects can be pointed out for that loss of exactitude:

- Difficulty in generating the exact computer signal in the flume;
- Measurement errors;
- Transverse water mass oscillation in the flume;
- Re-reflections that can occur either in the direction of wave propagation or any other direction.
- Deviations from linear theory (cross-modal activity presence of locked harmonics, non-linear wave interaction and non-linear propagation, Mansard & Funke). 

5 Physical simulation

Several tests were performed in the flume represented in Figure 1 for different spectra and physical constraints.

![JONSWAP spectrum - Wave gages 1, 2 & 3](image)

Figure 3 - Wanted wave spectrum and computed incident and reflected wave spectra (smoothed) - 3 wave gages with reflecting obstacle
Figure 3 shows an example of a graphical representation of a wanted wave spectrum (JONSWAP with parameters $H_s = 0.15$ and $T_p = 1.7$ s) and the obtained and smoothed incident and reflected wave spectra can be seen. This test considered three wave gages located along a line parallel to the wave propagation in the flume. A reflecting obstacle exists at the end of the flume.

From the physical tests, one can conclude the following:

- The variation of the coefficient of reflection when an obstacle is included in the flume is very small. That means the dissipation beach reflects almost the same as the referred obstacle. The physical conditions of the flume are thus not the best ones mainly if one wants to test dissipation efficacy of the beach. In the tested cases, the high reflection coefficients occurred because the flume water depth was too high. It must be noted, however, that in the case when a model is to be tested, the dissipation efficacy of the beach is not that important since typically one is only interested in what happens in front of the model.

- Out of the range of frequencies of interest, some computed spectrum peaks (from both incident and reflected values) were found. These peaks have no significance since they appear as consequence of divisions of very low values of $X_{ik}$, $X_{rk}$, $Y_{ik}$ and $Y_{rk}$, as defined in eqs. (8) and (9). Thus, when analysing the computed incident and reflected spectra, a good deal of judgement must be exercised in order to consider the right portion of the spectra really important. This is more important when the 2-gage method is used since this method gives much more unstable estimates of spectra. In both methods need, however, a spectrum smoothing is needed.

- The $N$-gage method is found to be more rigorous in what concerns the estimating of the incident and reflected spectra. Actually, that is expected since the more wave information is available when more wave gages are considered.

- For the physical conditions of the flume considered, the reflection is an important phenomenon, mainly for the more energetic frequencies. It must be noted, however, that according to Mansard & Funke the Fourier implementation of reflection analysis that was done generally originates overestimation of the reflection coefficients when noise is present in the data.

6 Conclusions

Numerical simulations performed with REFLEX found both methods to be theoretically correct. The numerical instabilities found in ranges out of the frequency range of interest were found to be unimportant.

Concerning the physical tests, the $N$-gage method was found to produce more rigorous results compared to the 2-gage method. In particular, less numerical instabilities occur and less smoothing is necessary when the $N$-gage method is used. The results get better when more wave gages are used. The limit is however dictated by the physical constraints of the flume.
Reasonable results were obtained in the flume. From those results, one can conclude that, for the physical conditions of the flume considered, the reflection is an important phenomenon, mainly for the more energetic frequencies.

In the future, for the tests without reflection obstacles in the flume, there is a need to perform more flume tests in order to obtain a better energy dissipation at the end of the flume (beach). Conversely, when there are reflecting obstacles in the flume, one needs to compensate the reflected waves on the wave paddle in order to eliminate a re-reflection on those obstacles. This is an important issue of the sea wave simulation that must be analysed.

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Bibliography