Improved tidal modelling for digital elevation models

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1. Introduction

This paper describes improvements in the elevations provided by a 2D tide and surge model which are employed to height shorelines produced by processing SAR images from the ERS-1 and ERS-2 satellites. The resulting quasi-contours can be used to create a height map, or Digital Elevation Model (DEM) of the inter-tidal zone. Such maps are valuable tools in shoreline management plans \cite{1} for civil engineering and marine conservation. The long term aim of monitoring
sediment movements can be achieved by creating maps both before and after some storm event. The details of the construction of a DEM are described in the paper on the "water-line" method [2].

The accuracy of the DEM depends on the accuracy of the shoreline position and of the model estimates of elevation along it. In regions of gentle beach slopes, model accuracy is particularly important so this paper concentrates on improving the model elevations. A case study has been carried out on The Wash, a large inter-tidal drying region on the East coast of England (see Figure 1). A high resolution model based on a 240m grid has been developed for the Wash (see Figure 2) and has enabled better resolution of the drying process. This, in turn, allows more accurate shoreline heighting since grid boxes close to the actual SAR shoreline can be used. In the Holderness model, defined on a larger, exterior region, with a grid size of 1.2km, errors were introduced by having to use grid cells several km away from the shoreline.

A simple scheme [3] for assimilating tide gauge data at the coast of the Northern open boundary of the Holderness model has been developed. Corrections at the boundary of the Holderness model propagate Southwards and are found to translate into better results from the Wash model.

Data for validation of the model tides is available from a survey carried out by Pugh [4] in the 1970's. Harmonic analysis of this data has been performed and comparisons with corresponding constants from a 6 month run of the model show good agreement. A preliminary DEM has been constructed [5] using 12 SAR images from the period 1992-94. A case study has been carried out on one period spanning the time of overpass of one of the images. Comparisons of the model elevations, including the surge component with observations have been made at West Lighthouse, the only available location with contemporary tide gauge data. Section 2 describes the basis of the model whilst the assimilation scheme for inclusion of tide gauge data at the open boundary is described in section 3. The improvements gained both in the development of the new high resolution model of the Wash and the assimilation scheme are demonstrated in section 4.

Section 5 describes how these model improvements will yield more accurate DEM construction and how future fieldwork and the development of an optimal assimilation scheme should provide still
further benefits.

2. Model Description

The 2D depth averaged momentum and continuity equations describe the model dynamics [6] and are solved using an explicit numerical finite difference scheme on a Arakawa "C-grid". Quadratic laws are assumed for the bottom and surface forcing whilst on open sea boundaries a radiation condition is employed. The inputs for tidal and surge information along the open boundaries of the Wash model are derived from the Holderness model. The tidal input involves 26 constituents, to cater for non-linear effects present in the shallow water regions being investigated.

The model allows for the drying of grid boxes whereby a simple scheme described in Flather and Heaps [7] determines whether the cell is wet or dry. The bathymetry used in the model has been taken from a digitisation of the Kingfisher chart, itself a compilation of Admiralty charts. The surface atmospheric pressure and winds from the Meteorological Office’s LAM model were used to drive the surge component.

3. Description of the Assimilation Scheme

Whilst the best SAR images occur with moderate windspeeds in a range such that surges are not large they are still significant with a magnitude of between 10cm to 40cm at times of satellite overpass. The principal aim of this work is to create a DEM to within 10cm accuracy so clearly the surge is an important component.

Along an open boundary

\[ q_n = q_{ns} + q_{nT} + c(\zeta_s - \zeta_T)/h \]  

where \( q_n \) is the normal flow to the boundary and \( \zeta \) is the elevation at the open boundary and where \( c = \sqrt{gh} \) is the phase speed of a long, progressive wave and \( h \) is the water depth. The subscripts \( T \) and \( S \) denote tidal and surge components. Following Flather and Proctor [8] the surge component of elevation can be corrected using tide gauge data at the coast as follows:
where $\zeta^*(\chi, t)$ is the corrected surge input at longitude $\chi$, where $\zeta(\chi, t)$ is the original surge input at longitude $\chi$ and where $\mu_c(\chi)$ is the normalised distribution of elevation at longitude $\chi$ defined as

$$\mu_c = \frac{\zeta_A(\chi) - \zeta_M(\chi)}{\zeta_A(\chi_0) - \zeta_M(\chi_0)}$$  \hspace{1cm} (3)$$

where $A$ denotes actual value, $M$ the model value and where

$$\zeta_{os} = \zeta_0(\chi_0) - \zeta_s(\chi_0)$$  \hspace{1cm} (4)$$

is the difference between the observation $\zeta_0$ at the coastal station of longitude $\chi_0$ where the model surge is $\zeta_s(\chi_0)$.

The flow is assumed to be that of a Kelvin wave so the elevation decays exponentially out to sea, with the longshore current balancing the elevation geostrophically. So,

$$\mu_c = ae^{-bx}$$  \hspace{1cm} (5)$$

where $a, b$ are constants to be determined.

At the coast, where $\chi = \chi_0$, $\mu_c = 1$, by definition.

Hence

$$a(1-b\chi_0) = 1$$

So,

$$b = (1 - \frac{1}{a})/\chi_0$$  \hspace{1cm} (6)$$

Away from the coast, it has been assumed that in the deep waters, the
model perfectly simulates reality.

\[ \mu_c(\chi_L) = 0 \] (7)

where \( \chi_L \) is the longitude of the end of the open boundary. Thus, substituting for \( b \) from equation 6 gives

\[ a(1 - \chi_L/\chi_0(1 - 1/a)) = 0 \]

which is only valid for

\[ \frac{\chi_L}{\chi_0}(1 - \frac{1}{a}) \leq 1 \] (8)

This yields

\[ a = \frac{\chi_L}{\chi_L - \chi_0} \]

which does not violate equation 8. Consequently,

\[ \mu_c(\chi) = \frac{\chi_L - \chi}{\chi_L - \chi_0} \] (9)

The correction to the normal component of flow is expressed as

\[ q_{ns}'(\chi) = q_{ns}(\chi) + \lambda_c(\chi)\zeta_{os} \]

Geostrophic balance implies that

\[ q_{ns}' = \frac{g}{2\omega \sin \phi \cos \phi} \frac{\partial \zeta_s'}{\partial \chi} \]

Substituting from equation 2 and assuming geostrophy applies before and after the correction yields

\[ q_{ns}'(\chi) = q_{ns}(\chi) + \lambda_c(\chi)\zeta_{os} \]
where

$$\lambda_c(\chi) = \frac{g}{2\omega(\chi_0 - \chi_L)\sin \phi \cos \phi} \frac{1}{(\chi_0 - \chi_L)}$$

Since no suitable data was available at the coast of the Northern boundary to the Wash model, tide gauge data from Whitby on the Northern boundary of the Holderness model was assimilated in the above manner into the Holderness model. The output from the Holderness model is then used to drive the surge input to the Wash model.

A case study was then carried out on the period 4-7 Nov '93 and comparisons between the models and tide gauge data were made in order to assess the improvements gained by the assimilation method.

4. Results

The data which was available to validate the model in the Wash region stems principally from a survey carried out by Pugh in the 1970's. It was harmonically analysed to validate the tidal component of the model with good agreement. The locations for which tidal harmonic constants have been calculated are shown in Figure 2 together with the location, West Lighthouse, where surge information at the time of the SAR image is available.

The development of the high resolution Wash model has resulted in significant improvements in the simulation of the drying regions. The drying regions in the, coarse resolution (1.2km) Holderness model are shown in Figure 3 with the corresponding, much better resolution of the Wash model depicted in Figure 4. The main advantage from the improved resolution in the drying process is that it results in better shoreline heighting using the Wash model. Previously, grid boxes from a suitable wet cell up to 3km away might have had to be used in the Holderness model. This can be seen from Figure 5 where drying occurs in the Holderness model whilst the Wash model follows the tidal curve.

The improvements gained by assimilating the Whitby tide gauge data in the Holderness model are significant and are shown in Figure 6 for the uncorrected and assimilated cases together with the data at
Cromer. An accuracy of 10cm was achieved at Cromer.

Whilst no data exists at the open boundary of the Wash model, runs from the Holderness model with assimilation provide improved surge open boundary inputs to the Wash model. Even though the Whitby gauge is some 160km from the Wash model boundary it has resulted in considerable improvements in the surge simulation, as seen in Figure 7 for West Lighthouse. The remaining discrepancies between the Wash model and the data at West Lighthouse are most likely to be due to the fact that the tide gauge lies 1km along a channel which distorts the harmonic analysis.

5) Future Work

To date, the calculations for the shoreline heighting have been made using tidal information only from the Holderness model in conjunction with tide gauge data in a Kriging method to construct a preliminary DEM of the Wash.

In the near future, 12 runs of the high resolution Wash at the model time of overpass of the SAR images used in the DEM will be performed to generate a new, improved DEM.

A Ministry of Agriculture, Fisheries and Food grant has been obtained which will provide an opportunity to obtain bottom pressure measurements this summer which may allow better calibration of the model drying parameters and facilitate the application of an optimal assimilation scheme to be developed shortly. [9] This has the advantage of being able to assimilate data from more than one location.

6) Conclusions

The results presented show that the high resolution Wash model simulates the drying process well. Moreover, a non-optimal scheme has been developed and significant improvements in model elevations have been achieved which enable accurate shoreline heighting of the DEM.
The fieldwork described above combined with an optimal assimilation scheme should provide greater improvements. The method will be applied in the challenging inter-tidal region of Morecambe Bay.

It is believed that the DEMs produced by this approach will provide useful maps of inter tidal zones in various coastal applications.

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Figure 1 Map to show locations of elevation data in the Holderness region
Figure 2 Map to show locations of elevation data in the Wash region
**Figure 3** Map of drying areas, shaded in light grey, in Holderness model together with flow arrows.

**Figure 4** Map of drying areas, shaded in light grey, in Wash model together with flow arrows.
Figure 5 Comparison of tidal elevations from Holderness and Wash models with harmonically analysed data at Tabs Head.
Figure 6 Comparison of sea elevations from the Holderness model with tide gauge data at Cromer using uncorrected open boundary input and then including assimilation of Whitby data.

Figure 7 Comparison of sea elevations from the Wash model with tide gauge data at West Lighthouse using uncorrected open boundary input and then including assimilation of Whitby data.