

Numerical simulation of wave overtopping using two dimensional breaking wave model

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Abstract

A two-dimensional breaking wave numerical model capable of simulating regular and irregular wave overtopping over the coastal structures is presented. The model uses the volume of fluid (VOF) algorithm to track the free surface movements. The model is based on Reynolds Averaged Navier-Stokes (RANS) equations for mean flow field and the $(k - \varepsilon)$ equations for turbulent kinetic energy, k , and the turbulence dissipation rate, ε . The results have been compared with other analytical solutions, laboratory data and design empirical formulae for wave overtopping at sloping sea walls. The comparison suggests that the current design formulae for wave overtopping in the breaking zone underestimate the overtopping discharges for the range of cases investigated.

1 Introduction

Breaking waves in surf zone play very important role for early all the coastal processes. The breaking waves generate strong turbulence and are in general accompanied by strong energy dissipation. Breaking waves also modify wave forces on the coastal structures when the wave-structure interaction occurs. Wave overtopping is a complex process to model which involves shoaling, wave reflection, wave breaking and turbulence and in which the random nature of the waves must be taken into account (Ingram et al [1]). The total volume of sea water overtopping in a particular storm is generally well predicted by current methods (Owen [2], Van der Meer & Janssen [3], De Wall et al [4] and Hedges & Reis [5]). However, Goda [6] has showed that current formulae, which do not take full account of the complexity of wave breaking in shallow water, can

significantly underestimate overtopping discharges. Analysis by Besley et al [7] shows that methods that exclude these effects can severely underestimate overtopping under breaking wave conditions, a finding supported by the numerical study of Causon et al [8].

The simulation of breaking wave has been a challenging problem to many coastal researchers due to the complicated flow and turbulence structures. Recently, Lin [9] presented a two-dimensional numerical model which solves the Reynolds Averaged Navier-Stokes (RANS) equations for mean flow field and the $(k - \varepsilon)$ equations for turbulent kinetic energy, k , and the turbulence dissipation rate, ε . In this model the volume of fluid (VOF) algorithm (Hirt and Nichols [10]) method is employed to track the free surface movements. For this study, Lin's model has been extended to describe the corresponding turbulence field by an improved $k - \varepsilon$ model. A nonlinear Reynolds stress model is employed to relate the Reynolds stress and the strain rates of the mean flow. In the following sections the new model, (denoted 2D-BWNM), is described briefly. Results from applying it to regular and irregular wave overtopping over sloped sea wall are presented. The case of regular wave overtopping at sloping seawalls is presented first to demonstrate the validation of the breaking wave model. Numerical solutions have been compared with other analytical and laboratory data and very good agreement has been found. Results for irregular wave overtopping are then presented. The paper concludes with a summary of the main findings.

2 Mathematical formulation of the numerical model

For a turbulent flow, both the velocity field and the pressure field can be split into mean component and turbulent fluctuations as follows:

$$u = \langle u_i \rangle + u'_i \quad (1)$$

$$p = \langle p \rangle + p' \quad (2)$$

The mean flow is governed by RANS equations as follows:

$$\frac{\partial \langle u_i \rangle}{\partial x_i} = 0, \quad (3)$$

$$\frac{\partial \langle u_i \rangle}{\partial t} + \langle u_j \rangle \frac{\partial \langle u_i \rangle}{\partial x_j} = - \frac{1}{\langle \rho \rangle} \frac{\partial \langle p \rangle}{\partial x_i} + g_i + \frac{1}{\langle \rho \rangle} \frac{\partial \langle \tau_{ij} \rangle}{\partial x_j} - \frac{\partial \langle u'_i u'_j \rangle}{\partial x_j} \quad (4)$$

in which $\langle \rangle$ denotes the mean quantities, the prime ' represents the turbulent fluctuations, u_i denotes the i -th component of the velocity vector, p the pressure, ρ the density, g_i the i -th component of the gravitational acceleration, and τ_{ij}^m the molecular viscous stress tensor. The product of the density and the correlation of velocity fluctuation, $\rho \langle u'_i u'_j \rangle$, is also called the Reynolds stress. The correlation had been modelled by a nonlinear eddy viscosity model (modified $k - \varepsilon$ equations) where k is the turbulence kinetic energy, and ε is the

turbulence dissipation rate. More details of the mathematical formulation are given in Lin [9], Lin & Liu [11] and Liu et al [12].

3 Results

To illustrate the validation and performance of the results from 2D-BWNM, two cases are presented here.

3.1 Regular wave overtopping at sloping Seawalls

Extensive small-scale laboratory test data for wave overtopping at sloping sea walls has been collected by Saville [13]. The experiments were based on regular waves overtopping a sloping sea wall with slopes of 1:3. Hu et al [14] summarised this data and used it to test his numerical model (AMAZON) which is based in the non-linear shallow water (NLSW) equations. The profile of tested sea walls are illustrated in Figure 1, where d_b , d_s , and R_c represent water depth below SWL at the seaward boundary, water depth below SWL at the toe of the structure and the crest level of the structure above SWL (freeboard). The configurations and the results for 20 runs are presented in Tables 1 and also illustrated in Figures 2. In this study, the boundary wave conditions were the same as specified by Hu et al [14].

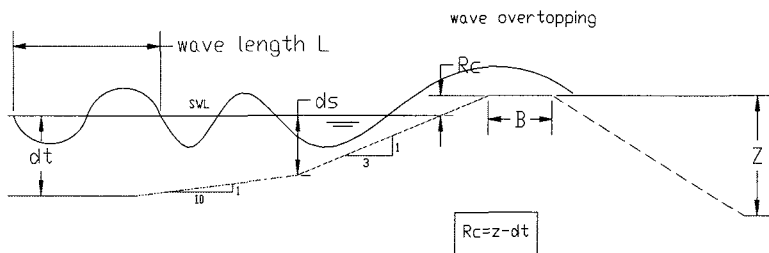


Figure 1: Sketch explains the case study of regular waves overtopping at sloping seawalls.

The dimensionless discharge Q was defined by Hu et al [14] as:

$$Q = \frac{q}{H_o \sqrt{gH_o}} \quad (5)$$

where: q is the dimensional average overtopping discharge, g is the gravitational acceleration and H_o is the “deep-sea” wave height. To be consistent with the results produced by Hu et al [14], an average value of Q was calculated during the fourth and fifth wave period ($4 \leq T \leq 5$). The results produced with 2D-BWNM compared well with the measured data, see Figure 2. Table 1 shows details of each run and the output from our model together with the laboratory experiments and the AMAZON results. As measured by the sum of the modulus of the differences between the laboratory and model results over the 20 cases, 2D-BWNM provides approximately 10% improvement in the performance of AMAZON. On purely theoretical grounds one would expect the Navier-Stokes equations to provide a more robust means for the simulation of wave overtopping than the nonlinear shallow water equations; the latter are derived on the assumption that the vertical velocity is much less than the horizontal velocity, i.e. hydrostatic pressure is assumed. This assumption is not strictly applicable outside of shallow water or in the surf zone.

Run No.	S.W Slope	d_t (m)	D_s (m)	R_c (m)	H_t (m)	T (s)	$Q(10^{-3})$ [-] Amazon	$Q(10^{-3})$ [-] Saville	$Q(10^{-3})$ [-] 2D-BWNM
1	1:3	3.0	0.75	0.50	0.95	4.73	39	66	46
2	1:3	3.0	0.75	1.00	0.95	4.73	15	41	3
3	1:3	3.0	1.50	0.50	0.95	4.73	81	64	72
4	1:3	3.0	1.50	1.00	0.95	4.73	24	36	25
5	1:3	4.0	2.00	0.67	0.99	6.55	86	90	86
6	1:3	4.0	0.75	0.50	1.08	7.98	64	60	66
7	1:3	4.5	0.75	1.00	1.06	7.98	27	17	40
8	1:3	4.0	0.75	1.50	1.08	7.98	11	4	4
9	1:3	4.0	1.50	0.50	1.08	7.98	101	94	103
10	1:3	4.0	1.50	1.00	1.08	7.98	53	40	48
11	1:3	4.0	1.50	1.50	1.08	7.98	24	8	33
12	1:3	6.0	1.00	0.67	1.20	12.8	90	91	115
13	1:3	6.0	2.00	0.67	1.20	12.8	108	130	138
14	1:3	6.0	2.00	1.33	1.20	12.8	41	77	61
15	1:3	6.0	2.00	2.00	1.20	12.8	7.5	25	30
16	1:3	6.0	2.00	2.67	1.20	12.8	0	11	11
17	1:1.5	4.92	0.75	0.50	1.04	7.98	50	49	63
18	1:1.5	4.92	0.75	1.50	1.04	7.98	5.6	13	19
19	1:1.5	4.17	0.0	0.50	1.07	7.98	34	39	39.7
20	1:1.5	4.17	0.0	1.00	1.07	7.98	8.9	20	13

Table 1: Comparison between 2D-BWNM and AMAZON numerical models with the laboratory measured dimensionless overtopping discharges.

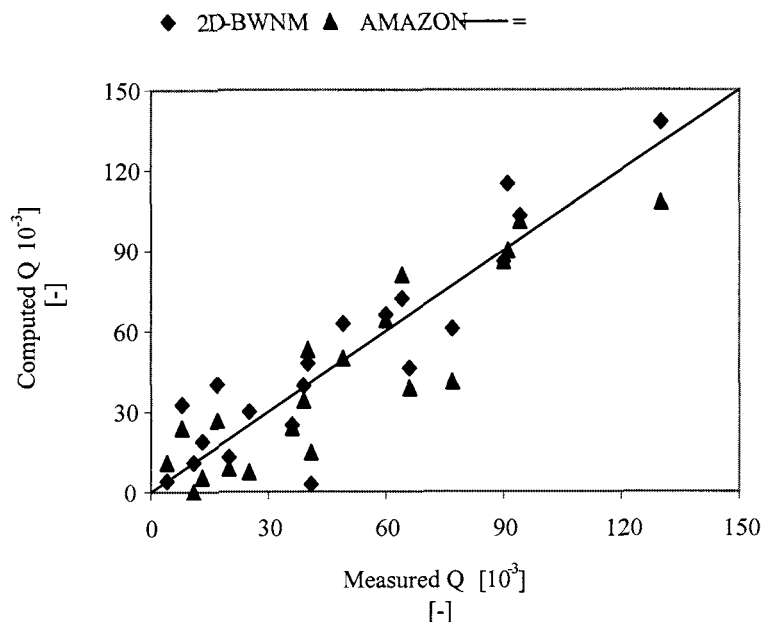


Figure 2: Comparison between 2D-BWNM and AMAZON models with the laboratory measured dimensionless overtopping discharges.

3.2 Irregular wave overtopping at sloping Seawalls

Methods available to predict overtopping rates for irregular waves include numerical modelling, site-specific model testing and empirical formulas. Most numerical models have been validated using small-scale tests with limited structural and incident wave conditions. Site-specific hydraulic model testing is impractical for preliminary design due to the time and expenses involved. As a result, design engineers rely heavily upon empirical overtopping formulae.

Three empirical formulae for wave overtopping of a simple sloped sea wall subjected to random waves approaching normal to the slope are used here to validate the 2D- BWNM: (Owen [2]; Van der Meer & Janssen [3] and Hedges & Reis [5]).

Owen [2] proposed an exponential relationship between dimensionless overtopping discharge and relative crest height. Owen's formula for an impermeable smooth straight with 1:3 slope is:

$$Q = \frac{q}{gH_s T_m} = 0.016 \exp \left(- 32 * \frac{R_c}{T_m \sqrt{gH_s}} \right) \quad (6)$$

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where H_s is significant wave height of incident wave and T_m is the mean wave period.

Van der Meer & Janssen [3] made a distinction between breaking (plunging) and non-breaking (surging) waves on the slope. Their set of formulae is related to breaking waves and are also valid up to a maximum which is in the fact the non-breaking region. Van der Meer's formula for straight impermeable smooth with 1:3 slope is:

For breaking waves

$$Q = \frac{q}{\sqrt{gH_s^3}} \frac{\sqrt{\tan \alpha}}{\zeta_p} = 0.06 \exp(-5.2 * R) \quad \zeta_p < 2 \quad (7)$$

$$R = \frac{R_c}{H_s \zeta_p} \quad \zeta_p < 2 \quad (8)$$

$$\zeta_p = \frac{\tan \alpha}{\sqrt{S_p}} \quad \text{and} \quad S_p = \frac{2\pi H_s}{gT_p^2} \quad (9)$$

For non-breaking waves

$$Q = \frac{q}{\sqrt{gH_s^3}} = 0.2 \exp\left(-2.6 * \frac{R_c}{H_s}\right) \quad \zeta_p > 2 \quad (10)$$

where: $\tan \alpha$ is impermeable seawall slope, ζ_p is the surf-similarity parameter for irregular wave, R is the dimensionless freeboard, S_p is the peak wave steepness and T_p is the wave period corresponding to the peak of the wave spectrum.

Hedges & Reis [5] constructed a model based on a regression against Owen's data subject to the constraint that there is no overtopping if the sea-wall freeboard exceeds the maximum run-up on the face of the sea wall. Hedges & Reis's formula is:

$$Q = \frac{q}{\sqrt{g(CH_s)^3}} = 0.008 \left(1 - \frac{R_c}{CH_s}\right)^{5.22} \quad (11)$$

where: C is the ratio of the maximum run-up(R_{max}) to the significant height of the incident wave [$C = 1.52 (1.35 \zeta_p)$ for $\zeta_p < 2$].

In this case of study, a total of 12 tests were run using 2D-BWNM. Figure 3 gives the cross section of the sea wall with slope 1:3 and shows the breaking wave surface. The water depth is 4.5m, the dimensionless freeboard (R) ranges from 0.29 to 1.37, and the generated random waves accordance with JONSWAP spectrum with significant wave height $H_s=1.22\text{m}$, mean wave period $T_m=3.8\text{s}$, and peak wave period $T_p=5.0\text{s}$. In total 320 cells are used in the x -direction with a cell size of 0.25m. In the y -direction 100 cells are used with a cell size of 0.1m. The basic time step is 0.04s and the simulation time is $t = 60\text{s}$. The value of γ in the JONSWAP spectrum is set to 3.3 and the spectrum is represented by 10 component frequencies between 0.15 and 0.265 Hz.

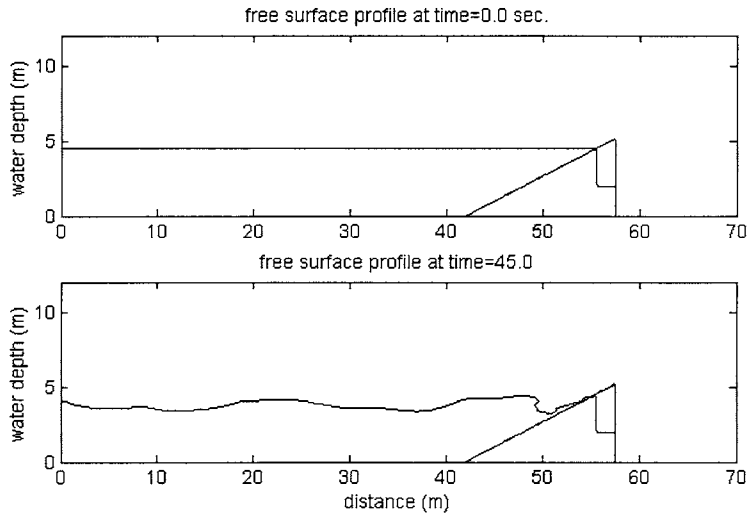


Figure 3: Cross section used for computations with 2D-BWNM Sketch and the breaking wave surface profile after 45 sec.

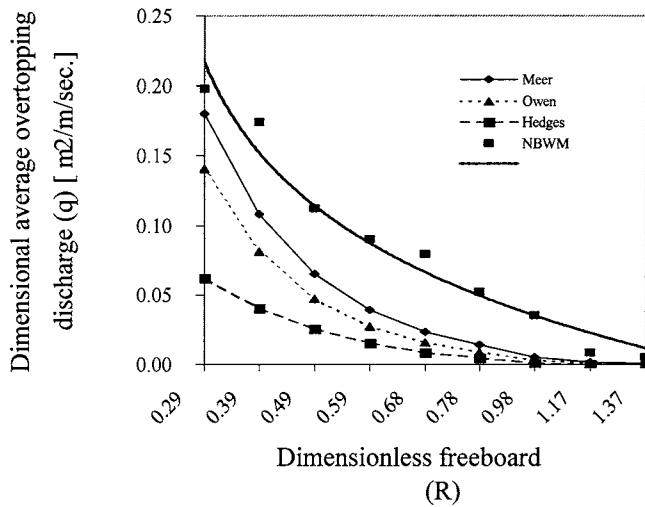


Figure 4: Comparison of 2-D BWNM and the other empirical formulae under JONSWAP Spectrum for 1:3 slope Sea wall.

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Figure 4 shows the comparison between the results produced by 2D-BWNM and the empirical formulae. It can be seen from the figure that the empirical formulae underestimate the amount of wave overtopping under irregular wave attack in comparison with the numerical results. This should perhaps be expected in view of the fact that the range of freeboards considered extends beyond the range investigated in laboratory experiments. The new numerical approach goes some way towards addressing the issues raised by Goda [6], but the model shows some scatter, particularly for lower freeboard.

4 Conclusion

New two-dimensional breaking wave numerical model (2D-BWNM) has been presented and the preliminary results are discussed. The 2D-BWNM is based on Reynolds Averaged Navier-Stokes (RANS) equations for mean flow field and the improved $(k - \varepsilon)$ equations for turbulent kinetic energy, k , and the turbulence dissipation rate, ε . A nonlinear Reynolds stress model is employed to relate the Reynolds stress and the strain rates of the mean flow. The numerical model uses the volume of fluid (VOF) algorithm to track the free surface movements. The case of regular wave overtopping over sloped sea wall has been studied first to validate the model. The output result compares well with the laboratory work and other numerical models. The 2D-BWNM has been used to investigate the case of irregular wave overtopping at sloped structures and the amount of wave overtopping has been calculated and compared with empirical equations used for design purposes (Owen [2], Van der Meer & Janssen [3], and Hedges & Reis [5]). The comparison supports the observation, (see, eg Goda [6]), that the current design formulae underestimate the amount of wave overtopping that arises from irregular breaking waves. Engineers are seeking more accurate means of estimating wave overtopping in these conditions. The preliminary results presented here indicate that the approach used in 2D-BWNM could form the basis of a generic design tool.

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